Notices from the ISMS

May 2007

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SCIENTIAE MATHEMATICAE JAPONICAE
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INTERNATIONAL SOCIETY FOR MATHEMATICAL SCIENCES
Scientiae Mathematicae Japonicae, Notices from the ISMS

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What is General Algebra?

KLAUS DENECKE

February 27, 2007

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1 Historical Remarks The word "algebra" was mentioned first in the title of the book "Hirab aljabr w'almuqabalah" of the Arabic mathematician Al-Hwarazmi who lived and worked around 800 in Bagdad. During this time, especially during the regency of the caliph Harun ar Raschid, Bagdad was a well-known center of science and art. Mathematics and natural sciences were considered as important parts of human culture. The Arabic mathematicians could follow the traces left from the Greek and Indian mathematicians. From the Indians they took over the representation of integers by ciphers and the decimal system. The decimal system was described in Al-Hwarazmi's book. For this reason even today we speak on arabic ciphers. Probably the word "algorithm" appeared also in this book. The Arabic mathematicians could solve quadratic equations and asked for the solutions of algebraic equations of higher degree. But only after the translation of the Arabic mathematical works in European languages (Hermann v. Kärnten, 1143) and only after it became clear that the economical development needed new methods for calculations, there was more progress in solving algebraic equations of higher degrees.

In the sixteenth century Italian mathematicians solved algebraic equations of third and fourth degree by radicals (Tartaglia, Cardano) and discovered the complex numbers.

A milestone in the applications of algebraic methods in other fields of mathematics was the algebraization of geometry contained in R. Descarte's work "Discours de la méthode".

On the nineteenth century the problem of solving algebraic equations of degree greater than four was attacked from new. Based on former results of Lagrange, Ruffing and Gauss in 1824 N. H. Abel proved the impossibility to solve the general algebraic equation of degree five by radicals and only a short time later E. Galois was able to give necessary and sufficient conditions to solve the general algebraic equation by radicals. This was at the same time the birth of group theory. Algebra changed from being a theory of solutions of algebraic equations to the theory of algebraic structures. Galois' paper "Memoirs sur les conditions de résolubilités des équations par radicaux" is perhaps the most famous and most controversial paper in the history of algebra. Almost everybody dealing with the concept of a Galois connection cites the fundamental theorem of Galois theory as the classical prototype of such connections. However, that theorem, stated in its modern form as a dual isomorphism between the subfield lattice of a Galois extension and the subgroup lattice of the corresponding Galois group was explicitly formulated quite some time after Galois.

New classes of algebraic structures were considered by British algebraists in the nineteenth century. The discovery of the quaternions by Hamilton, the calculation with matrices by Cayley and the definition of associative algebras by Peirce were important steps to general algebra. G. Boole's books "The mathematical analysis of logic" and "An investigation of the laws of thought" linked algebra with propositional logic. Boolean algebras and the algebra of all Boolean functions are fundamentals of computer science.

In the first half of the last century important classes of algebraic structures as groups, fields, rings, lattices and semigroups were developed. In the twentieth and thirtieth of the last century many mathematicians became aware of a radical reorganization of algebra. This reorganization, specifically, the transformation of algebra into a set-theoretical axiomatic science having as its primary object the study of algebraic operations performed on elements of an arbitrary nature- had, of course, been prepared by the whole preceding development of algebra. It began at the end of the nineteenth century and continued with increasing momentum during the first decades of the twentieth. The publication of the two volumes of van der Waerden's "Modern Algebra" ([80]) made the ideas, results, and methods of this new algebra accessible to all mathematicians, including non-algebraists. It is common knowledge how decisive the impact of this modern algebra was on the development of other domains of mathematics. We mention here only topology and functional analysis. Van der Waerden himself decided, following a suggestion of H. Brandt, from the fourth edition on to change the title of his book into "Algebra". In his review on the book (Annual report of DMV 55) H. Brandt wrote: *"Was den Titel anbetrifft, so würde ich es begrüßen, wenn in der vierten Auflage der schlichtere, aber kräftigere Titel "Algebra" gewählt würde. Ein Buch, das so viel an bester Mathematik bietet, wie sie war, ist und sein wird, sollte nicht durch den Titel den Verdacht erwecken, als ob es nur einer Modeströmung folgte, die gestern noch unbekannt war und vielleicht morgen vergessen sein wird"*.

During the following decades the stormy development of algebra itself as well as the discovery of several links with disciplines in the neighborhood created generalizations of groups and rings and other "classical" algebraic structures. "Modern algebra" changed to "general algebra". During these decades, the older branches of general algebra - the theory of fields and of associative and associative-commutative rings - to which van der Waerden's book was mainly devoted - have undergone far-reaching changes. Even more crucial was the reorientation in the theory of groups, the oldest of all branches of general algebra. The theory of rings became more a theory of non-associative rings, incorporating as a constituent part the theory of Lie rings and Lie algebras. Topological algebra sprang up and soon occupied a very prominent position and a parallel development happened in the theory of ordered algebraic structures. Lattice theory appeared and developed rapidly. Within the framework of the classical parts of general algebra, independent trends arose: homological algebra leading to numerous results in topology and algebraic geometry, projective algebra, including the elements of projective geometry, and differential algebra. The theory of semigroups, that of quasigroups and of n-groups ceased to be simply theories of "generalized groups" and found their own paths of development and their own areas of application. Eventually, the general theory of universal algebras came into being, as well as model theory, which is interwoven with mathematical logic. Category theory allows a wide overview on algebraic thinking.

Unfortunately, the fundamental ideas and the most important results of present-day general algebra are not a part of the scientific equipment of every well-educated mathematician. A wide circle of mathematicians today has an acquaintance with the achievements of general algebra that remains rather on the level of the early thirties of the last century. This conflicts with the need of algebraic-structural thinking in applied areas, especially in computer science.

In the following sections we describe some trends in the development of modern general algebra. After a while the reader will agree that completeness is impossible since the topic is mushrooming into many fields. Sometimes it is not clear what general algebra is and what the applications are.

2 Semigroups, Semirings, Nearrings Semigroups are algebras having one binary associative fundamental operation. Semigroups generalize groups. The binary operation of a semigroup has not to be invertible.

There are several points of common interest in group theory and in semigroup theory ([56]). Cayley's theorem enables us to view groups as groups of permutations of some set. An analogous result in semigroup theory represents semigroups as semigroups of functions from a set to itself. Groups act on a set or a space as permutations, semigroup actions are by functions. Finite dimensional linear representations of groups are representations by invertible matrices, while finite dimensional linear representations of semigroups are representations by arbitrary, not necessarily invertible matrices.

Nevertheless, the structure theories for groups and semigroups are different. Congruences of semigroups are not determined by one congruence class. This makes it difficult to study semigroup homomorphisms. Properties of a semigroup can be studied by the ideal structure of a semigroup. During the past decades group theory and semigroup theory have developed in different directions. However there are several directions of nowadays semigroup theory which are related to group theory. Important problems in finite semigroup theory which is closely connected to automata theory and to the theory of formal languages are related to problems about profinite groups.

Semigroups with an identity (neutral) element are called monoids. Without loss of generality one can always assume that the semigroup under consideration has an identity element since one can always adjoin an identity element from outside. By definition, regular monoids are very close to groups. A monoid M is called regular if for each $a \in M$ there exists some $b \in M$ such that $a = aba$ and $b = bab$. The element $b \in M$ is called an inverse of $a \in M$. If b is an inverse of a, then ab and ba are both idempotents of M. A monoid M is called an inverse monoid if for each $a \in M$ there exists a unique inverse a^{-1} in M such that $a = aa^{-1}a$ and $a^{-1} = a^{-1}aa^{-1}$. M is inverse if and only if it is regular and the idempotents commute. Thus the idempotents $E(M)$ of M form a submonoid of M which is a semilattice (a commutative idempotent semigroup). Every inverse monoid can be equipped with a partial order defined by $a \leq b$ if and only if $a = eb$ for some idempotent e of M. If e is an idempotent of a monoid M, then the set

$$G_e := \{a \in M \mid ea = ae = e \text{ and there is an element } b \in M$$
$$\text{such that } ab = ba = e\}$$

is a subgroup of M with identity e. The subgroups G_e are referred to as the maximal subgroups of M. The semilattice of idempotents and the maximal subgroups G_e give us a lot of information about M but do not completely determine the structure of M.

The full transformation monoid on a set X, which consists of all functions from X to itself with respect to composition of functions is an important example of a regular monoid. Idempotents in this monoid consist of functions that are identity maps on their ranges, and the maximal subgroup corresponding to such an idempotent is isomorphic to the symmetric group on the range of the map.

Every group is an inverse monoid. The set of all partial mappings of a set X forms together with the composition defined by $(f \circ g)(a) = f(g(a))$ whenever a belongs to the domain of g and $g(a)$ belongs to the domain of f. The idempotents of this inverse monoid are the identity maps on subsets of X, so the semilattice of idempotents is isomorphic to the lattice of subsets of X. The maximal subgroup corresponding to the identity map on the subset Y of X is the symmetric group on Y. The partial order $\leq$ on this monoid is given by $f \leq g$ if and only if $domf \subseteq domg$ and $f = g|domf$.

There are five equivalence relations, known as Green's relations $\mathcal{R}, \mathcal{L}, \mathcal{J}, \mathcal{H}$ and $\mathcal{D}$ that play a prominent role in semigroup theory. For a monoid M we define

$$\begin{aligned}
\mathcal{R} &= \{(a,b) \mid \exists\, c,d \in M\ (ac = b \wedge bd = a)\},\\
\mathcal{L} &= \{(a,b) \mid \exists\, h,f \in M\ (ha = b \wedge fb = a)\},\\
\mathcal{J} &= \{(a,b) \mid \exists\, l,m,n,o \in M\ (lam = nbo)\},\\
\mathcal{H} &= \mathcal{R} \cap \mathcal{L},\\
\mathcal{D} &= \mathcal{R} \vee \mathcal{L} \ (\vee \text{ denotes the join in the lattice of equivalence relations} \\
&\quad \text{on } M).
\end{aligned}$$

As an example for the importance of Green's relations we give the following result: Let M be a monoid. Then the following statements are equivalent:

(i) M is an inverse monoid;

(ii) every $\mathcal{L}$ - class and every $\mathcal{R}$ - class contains exactly one idempotent;

(iii) every element of M has a unique inverse.

The class of all inverse monoids forms a variety in the sense of universal algebra, i.e. the model class of a set of equations, in this case associativity and the identities $a = aa^{-1}a, (a^{-1})^{-1} = a, (ab)^{-1} = b^{-1}a^{-1}, aa^{-1}bb^{-1} = bb^{-1}aa^{-1}$ are satisfied. As a consequence, free inverse monoids exist.

A semigroup (or a monoid) M is called completely regular if $(a^{-1})^{-1} = a, a = aa^{-1}a$ and $aa^{-1} = a^{-1}a$. Therefore they form also a variety. The subvariety lattice of this variety is well-studied (see [65], [66], [67]).

The investigation of varieties of semigroups is a problem which is related more to universal algebra than to group theory. There are an uncountably infinite number of subvarieties of the variety of all semigroups, forming a lattice under inclusion, with the variety of all semigroups as largest element and the trivial variety consisting of all one-element semigroups as the smallest. This lattice is complete, this means that it is a partially ordered set in which suprema and infima not only for two-element subsets but for arbitrary subsets exist. The infimum (meet) of any collection of semigroup varieties is simply their intersection and their supremum (join) is the smallest variety which includes all of them. The structure of this lattice $\mathcal{L}^{sg}$ is complicated and widely unknown. A first description was given by T. Evans ([26]). The lattice $\mathcal{L}^{sg}$ is the disjoint union of two large parts: the ideal Per of all periodic varieties (that is, varieties of semigroups where all elements have finite order) and the filter OC of all overcommutative varieties (that is, varieties containing the variety of all commutative semigroups). For more information see [76], [77].

In [46] a new method is presented to study complete sublattices of this lattice (the lattices of M-solid varieties).

In the fiftieth of last century, the development of automata theory brought new motivations for the study of finite semigroups. That motivation started to give remarkable fruits when it became clear that combinatorial properties of rational languages can be tested by checking algebraic properties of the respective syntactic semigroups. It turned out that there is a correspondence between certain families of languages and certain classes of finite semigroups. The precise formulation of this correspondence is due to S. Eilenberg ([22], [23]) and needs the concept of a pseudovariety. Finite parts of varieties are pseudovarieties. The converse is not true. The model theory of pseudovarieties can be developed by using filters of equations, or alternatively, by using pseudoidentities. The definition of pseudoidentities needs implicit operations and their topological algebra. The collection of all pseudovarieties of semigroups is a complete lattice too. A new method to determine complete sublattices of this lattice was described in [62] or in [20].

Semirings are algebras $(S;+,\cdot)$ with two binary associative operations $+$ and $\cdot$ such that the semigroups $(S;+)$ and $(S;\cdot)$ are connected by four distributive laws. Usually the concept is used in the narrower sense that the addition is commutative (see [34]). The notion of a semiring appeared first in [75]. Implicitly semirings had appeared earlier in studies on the theory of ideals of rings (Dedekind, Macaulay, Krull, Noether, Lorenzen) and in studies on the axiomatization of the natural numbers and the non-negative rational numbers (Hilbert, Huntington). N. H. Abel considered in 1826 some special kind of a positive semifield of real numbers. For more bibliographical sources see [32].

In our opinion, especially high school teachers of mathematics should be informed on semirings and their extension to semifields.

Intensive investigations of the algebraic theory of semirings began in the 1950s by A. Almeida-Costa, S. Bourne, M. Henriksen, H. Zassenhaus, L. Rédei, O. Steinfeld, K. Iseki ([41]), K. Iizuka, H. J. Weinert, H. Lugowski, W. Słowikowski, W. Zawadawski and their collaborators (see[32]). The following topics were considered: homomorphisms, congruences, ideal- and radical theory, regularity with respect to both operations, inverse semirings, extensions of semirings, semirings of differences and quotients, semifields.

If A^* is the free monoid over the set A and if S is a semiring, then the semiring of formal power series is defined as the set S^{A^*} where addition is defined componentwise and multiplication is the so-called Cauchy product. Formal power series over various semirings are very important tools in the theory of formal languages and useful in several other areas of applied mathematics and computer science.

The class of all semirings forms a variety. Parts of the subvariety lattice of this variety were studied by several authors (see e.g [59], [60]). But also pseudovarieties, quasivarieties and M-solid varieties of semirings became interesting (see e.g. [39], [17]).

A nearring is an algebra $(F; +, \circ)$ of type $(2, 2)$ if $(F; +)$ is a group, $(F; \circ)$ is a semigroup and for all $a, b, c \in F$ the distributive law $(a + b) \circ c = a \circ c + b \circ c$ is satisfied. The concept of a nearring generalizes that of a ring (commutativity of the addition and the second distributive law are missing). Let $(G; +)$ be a group with neutral element 0. Let G^G be the set of all mappings $f : G \to G$, let $\circ$ be the composition of mappings and let $+$ be its pointwise addition. Then $F(G) = (G^G; +, \circ)$ is a nearring. A nearring is said to be a nearfield if $(F \setminus \{0\}; \circ)$ is a group. Every nearring can be embedded in a suitable $F(G)$ whereas every ring is a ring of endomorphisms (linear maps). Starting from works of Dickson, Veblen, Wedderburn, Zassenhaus, Wieland, Betsch, Blacket, and others, a sophisticated theory of nearrings was developed, and many connections to areas like geometry, group theory, design theory, ring theory, universal algebra, finite automata and coding theory were established. The following areas are most actively studied:

- special classes of mappings on groups,
- connections to geometry,
- matrix nearrings,
- near fields,
- codes and designs from nearrings.

For more information see [63].

3 Quasigroups A quasigroup is an algebra $(Q; \cdot)$ of type (2) where the binary operation satisfies the rules of left and right cancellation: $xy = xz \Rightarrow y = z$ and $yx = zx \Rightarrow y = z$. These conditions ensure that the maps $L(q)$ and $R(q)$ defined for a given $q \in Q$ by $L(q)(x) = qx$ and $R(q)(x) = xq$ are permutations on Q. The permutation group generated by the set $\{L(q), R(q) \mid q \in Q\}$ is the mapping group of Q. Much of the significance of quasigroups derives from the way in which they cover a far broader range of phenomena than groups, while still retaining many of the familiar algebraic properties of groups, albeit in revised form. The multiplication table of a finite non-empty quasigroup is a latin square, conversely any latin square is the multiplication table of a finite quasigroup, which need not to be associative in general. T. Evans ([25]) showed that they can be defined in universal-algebraic fashion using three binary operations and four identities. Therefore quasigroups form a variety. Quasigroups form a Mal'cev variety, i.e. any two congruences θ_1, θ_2 in a quasigroup satisfy $\theta_1 \circ \theta_2 = \theta_2 \circ \theta_1$. This is equivalent to the existence of a Mal'cev term t satisfying the identities $t(x,x,y) = t(y,x,x) = y$. Therefore a rich algebraic theory including centrality and cohomology is immediately applicable to quasigroups. Recently, a new definition was presented, using heterogeneous (multi-based) algebras, known as hyperquasigroups. Then only two identities are needed. The representation theory of groups can be extended to quasigroups (see [44]). For more information on quasigroups see [61].

4 Algebraic Logic and Algebras in Logic The study of relationships between algebra and logic goes back to the investigations of G. Boole and his followers. The result was a theory which we now call Boolean algebra. Links between classical logic and the theory of Boolean algebras have been known for a long time. Lindenbaum and Tarski considered propositional formulas or equivalence classes of propositional formulas as elements of an abstract algebra and logical connectives as operations on sets of logical formulas. Another milestone was the treatment of formulas as algebraic functions in certain algebras. This is a generalization of truth-tables in classical logic and can be found in the formulation of many-valued logics by Łukasiewicz and Post. Stone and Tarski established the connections between the intuitionistic propositional calculus and the algebra of open (closed) subsets of topological spaces. McKinsey and Tarski presented connections between the modal propositional calculus and the algebra of subsets of topological spaces. Mostowski extended the algebraic methods of Tarski and McKinsey to cover the intuitionistic predicate calculus. Rasiowa and Sikorski presented a first proof of Gödel's completeness theorem using algebra and topology. In [71] and in [70] Rasiowa and Sikorski presented in a systematic and uniform manner an algebraic approach to classical, intuitionistic, modal, and positive logic. The most fundamental notion of algebraic logic is that of a class of algebras algebraizing a logic ([7]). The basic idea of a class of algebras algebraizing a logic involves uniform tansformations from formulas to equations and from equations to formulas, that are inverse to each other up to equivalence. More generally, the translations may be from formulas to sets of equations and from equations to sets of formulas. The natural habitat of the concept is that of consequence relations rather than of logics. The requirement that the translations are each other's inverse, is expressed on the logical side by means of the consequence relation, and can equivalently be described on the algebraic side using quasiequations.

Classes of algebras and of algebras of relations arising in this way are intensively studied. Relation algebras have a long history going back to de Morgan, Peirce, Schröder, Tarski, Lyndon, Monk, Andréka, Nemeti, Maddux, Hirsch and Hodkinson. Relation algebras have practical applications in computing, e.g. as databases, in artificial planning and in specification theory. The standard reference on relation algebras is [35]. Certain modal logics have algebraic counterparts in Boolean algebras with operators (see e.g. [29]). Boolean algebras with operators or more generally bounded distributive lattices with operators are an intensively studied field of algebraic logic.

An important class of algebras in logic is the class of BCK-algebras. BCK-algebras are algebras $(X; *, 0)$ of type $(2,0)$ satisfying the following axioms:

(i) $x * y) * (x * z) \leq z * y$,

(ii) $x * (x * y) \leq y$,

(iii) $x * x = 0$,

(iv) $0 \leq x$,

(v) $(x \leq y \wedge y \leq x) \Rightarrow x = y$,

where $x \leq y$ is defined by $x * y = 0$. They were introduced by Imai and Iseki ([40], [42]) and are intensively studied by many authors. For a comprehensive source about BCK-algebras we recommend the monograph [57]. There are two important classes of BCK-algebras, commutative BCK-algebras and bounded commutative BCK-algebras. The latter class is categorically equivalent to the class of MV-algebras introduced by Chang [13], to Wajsberg algebras and to bricks ([9]).

5 Universal Algebra Universal algebra can be regarded as the study of finitary operations on a set. The main object are algebras $\mathcal{A} = (A; (f_i^A)_{i \in I})$ where $f_i^A : A^{n_i} \to A$ is n_i-ary and $\tau = (n_i)_{i \in I}$ is the type of $\mathcal{A}$. The first purpose of research was to find and develop the properties in the way what such diverse algebras as rings, fields, semigroups, semirings, nearrings, quasigroups, groups, lattices, BCK-algebras, etc. have in common. The first papers along this line were written by G. Birkhoff in the thirtieth of the last century ([4], [5]). During the period 1935 - 1950 the concepts of a free algebra, the homomorphism theorem, isomorphism theorems, congruence lattices and subalgebra lattices were considered. The logical and model-theoretic aspect of universal algebra was mainly developed by A. Tarski's school (C. C. Chang, L. A. Henkin, B. Jónsson, H. J. Keisler, R. C. Lyndon, R. L. Vaugh), by A. I. Mal'cev and others.

Any algebra has associated with it the lattice $Con(\mathcal{A})$ of all congruence relations on $\mathcal{A}$. Properties of the algebra $\mathcal{A}$ itself can be used to deduce properties of the associated congruence lattice, and sometimes properties of $Con(\mathcal{A})$ can be used to find structural properties of $\mathcal{A}$. Thus one wants to relate properties of lattices, such as permutability of the elements, distributivity, modularity, and so on, to properties of algebras and varieties, The first result in this direction was given by A. I. Mal'cev in 1945 ([50], [51]): he showed that any two congruence relations of any algebra in a variety are permutable with respect to the relational product if and only if the variety satisfies certain identities. The special term used in these identities is called a Mal'cev term (see section 3), and theorems like this one which relate properties of the congruence lattices of all algebras in a variety to the identities of the variety are usually called Mal'cev type conditions.

As it happens quite often in science, having a big universe like $Alg(\tau)$, the class of all algebras of type τ, one tries to collect the objects of the universe into the classes of some equivalence relation so as to reduce their number and then finding invariants - simpler mathematical structures or properties - which identify the nature of the original objects up to equivalence. In universal algebra one uses identities to classify algebras into collections called varieties. The collection of all varieties of algebras of type τ forms a complete lattice and an important area of activity has been to try to describe this lattice. Even for simple types as for type (2) such project is almost hopelessly complicated.

Just as identities classify algebras into varieties, one can use hyperidentities to classify varieties into collections called hypervarieties. Hyperidentities are formulas of second order logic containing operational variables but no relational variables, and a generalization of them called M-hyperidentities, lead us to solid and M-solid varieties. All M-solid varieties of a given type form a complete sublattice of the lattice of all varieties of type τ (see [46], [21]).

Using quasiidentities instead of identities, as model classes one obtains quasivarieties. All quasivarieties of algebras of type τ form also a complete lattice.

An old question of universal algebra is either or not the identities of a finite algebra can be derived (using the five algebraic rules of consequences) from a finite subset of identities of the algebra. R. C. Lyndon proved in [48] that every two-element algebra is finitely axiomatizable, but also in 1954 ([49]) he constructed a seven-element algebra with one binary and one nullary operation whose identities are not finitely based. The smallest such example is a non-finitely axiomatizable three-element algebra of type (2) found by V. L. Murskij in [58].

Let V be a variety. Let $\kappa(V)$ be the least cardinal number λ such that every subdirectly irreducible algebra in V has cardinality less than λ. If there is such a cardinal number, we say that V is residually small. If no such cardinal exists, we let $\kappa(V) = \infty$, and V is said to be residually large. The cardinal number $\kappa(V)$ is called the residual bound of V. A variety V is called residually finite if all its subdirectly irreducible algebras are finite. An important result in this area is K. Baker's theorem ([2]) saying that any congruence distributive variety of finite type which is residually finite, is finitely based. R. McKenzie proved in [52] that a locally finite variety V having only finitely many subdirectly irreducible elements and having an additional property called definable principal congruences, is finitely axiomatizable. McKenzie also proved in [53] that there are only countably many values possible for the residual bound of a finite algebra. The residual bound must be either ∞ or one of the following cardinals: $0, 3, 4, \ldots, \omega, \omega_1, (2^\omega)^+$ where ω is the cardinal number of the set of natural numbers, $\omega_1 = \omega^+$ is the next largest cardinal number after ω and $(2^\omega)^+$ is the successor cardinal of the cardinal of the continuum. R. Willard ([79]) proved the following generalization of Baker's theorem: If a variety is both, congruence meet-semidistributive and locally finite, then it is finitely axiomatizable. In 1993 R. McKenzie resolved several longstanding and challenging problems concerning varieties generated by finite algebras. One of these was the problem known as Tarski's finite basis problem, which asked whether it is algorithmically decidable whether any finite algebra is finitely axiomatizable. The results are the following ones: there is no algorithm to decide whether any given finite algebra is finitely axiomatizable. There is no algorithm to decide whether any given finite algebra generates a residually finite variety ([53], [54], [55]).

Any algebra $\mathcal{A}$ is related to two algebras of operations defined on A, the clone of all term operations and the clone of all polynomial operations. Clones are sets of operations defined on a set A, closed under composition and containing all projection operations. The clone of all term operations of the algebra $\mathcal{A}$ is the clone generated by all fundamental operations $\{f_i^A \mid i \in I\}$ of $\mathcal{A}$ and the clone of all polynomial operations of $\mathcal{A}$ is the clone generated by $\{f_i^A \mid i \in I\} \cup \{c_a \mid a \in A\}$ where c_a is the unary constant operation with value a. Clones can be regarded as algebras of type $(2,1,1,1,0)$ (Mal'cev clones) or as multi-based algebras. The operations of those clones describe the superpositions of operations on A. Let O_A^n be the set of all n-ary operations defined on A and let $O_A = \bigcup_{n\geq 1} O_A^n$. Clearly, O_A is the largest clone of operations defined on A and J_A, the set of all projections is the least one. All subclones of O_A form a complete lattice. For $|A| = 2$ this lattice $\mathcal{L}_A$ was completely described by E. L. Post ([68], [69]). It is countably infinite and each clone is finitely generated. For $|A| > 2$ this lattice is uncountably infinite. For finite sets A the lattice $\mathcal{L}_A$ is complete, algebraic, atomic and dually atomic. It has finitely many dual atoms which were completely determined by I. G. Rosenberg ([73]). The determination of all atoms is for $|A| > 5$ a longstanding open problem.

A finite algebra is called primal if its clone of term operations is equal to O_A and preprimal if this clone is a dual atom in $\mathcal{L}_A$. Varieties generated by primal algebras were considered in [27] and varieties generated by preprimal algebras were studied in [45] and in [16].

Let $f \in O_A^n$ be an n-ary operation defined on the set A and let $\varrho \subseteq A^h$ be an h-ary relation defined on A. Then f preserves ϱ if from $(a_{11}, \dots, a_{1h}) \in \varrho, \dots, (a_{n1}, \dots, a_{nh}) \in \varrho$ there follows $(f(a_{11}, \dots, a_{n1}), \dots, f(a_{1h}, \dots, a_{nh})) \in \varrho$. Let R_A be the set of all relations defined on A. Then the relation defined by "f preserves ϱ" defines a Galois connection (Pol, Inv) between operations and relations. The Galois closed sets on one side are clones of operations and the Galois closed sets on the other side are clones of relations.

Clones regarded as multi-based algebras belong to a variety of multi-based algebras which is defined by three identities. The most important of them is the so-called superassociative law. The algebras of this variety are also called abstract clones and clones of operations are called concrete. Every concrete clone is an abstract one and every abstract clone is isomorphic to a concrete one. This generalizes Cayley's theorem for monoids.

Clones play an important role in multiple-valued logic and in automata theory (switching circuits). Clones can also be applied to the "constraint satisfaction problem". Any $\Gamma \subseteq R_A$ is called a constraint language over A. The constraint satisfaction problem over Γ is defined to be the decision problem with instance (V, A, C), where V is a finite set of variables, A is a set of values, and C is a set of constraints$\{c_1, \dots, c_q\}$ in which each constraint is a pair (s_i, ϱ_i) with s_i a list of variables of length m_i, called the constraint scope and ϱ_i an m_i-ary relation over the set A, belonging to Γ, called the constraint relation. The question is whether there exists a solution to (V, A, C), that is, a function from V to A such that, for each constraint in C, the image of the constraint scope is a member of the constraint relation. There is a correspondence between the expressive power of a constraint language and the algebraic notion of a clone of relations (see [10]).
For more information on clones see [64] and [47].

Finite algebras are important in many areas where finiteness plays a crucial role, for instance in computer science. A major area of research activity has been to try to classify all finite algebras of a given type. For instance, the classification of all finite simple groups ([30]) has been a longstanding mathematical problem. In the early 1980's, R. McKenzie and D. Hobby developed a new theory called "Tame Congruence Theory," which offers a structure theory for finite algebras ([36]).

Multi-based algebras have not only one universe, but a sequence of universes and fundamental operations which operate between different sets of the sequence of universes. Examples of multi-based algebras are vector spaces over the field K or finite deterministic automata. A finite deterministic automaton is a quintuple $(I, S, O, \delta, \gamma)$ where I is the finite set of inputs, S is the finite set of states and O is the finite set of outputs. The multi-based operations $\delta : I \times S \to S$ and $\gamma : I \times S \to O$ are called state transition and output function, respectively. The theory of multi-based algebras can be developed in full analogy to one-based algebras.

A partial algebra of type τ is a set A together with a sequence $(f_i^A)_{i \in I}$ of partial (not everywhere defined) operations $f_i^A : A^{n_i} \multimap A$. The domain of $f_i^A, dom f_i^A$, is a subset of A^{n_i}. Therefore every algebra is a partial algebra. For partial algebras there are several ways to define homomorphisms and subalgebras. The model theory of partial algebras is much more complicated as that of total algebras (see [11]). Hyperidentities for partial algebras were considered in [78] and [12]. Clones of partial operations belong also to a variety of multi-based algebras. At the second conference for young algebraists in Potsdam 1987 H. J. Hoehnke asked the question for giving a system of defining identities of this variety. This problem was solved by F. Börner ([8]) for the one-based case. In [37] H. J. Hoehnke solves the problem for the multi-based case. Hoehnke's book gives a unified approach to total, partial and multi-based algebras, their identities, hyperidentities, theories and clones and covers a good deal of general algebra on a challenging high level based on *dht*-symmetric categories.

6 Category Theory A category **C** consists of a class of objects and a class of morphisms between them. Each morphism has exactly one object as its source and one object as its target. The objects of a category form a class which is not necessarily a set. A category in which the objects are sets with additional structure (such as total operations, partial operations or relations) is called a concrete category. The following properties have to be satisfied by the objects and morphisms of such a category:

(i) If A is the base set of an object, then $id_A : A \to A$ defined by $id_a(x) = x$ for all $x \in A$ is a morphism.

(ii) The class of morphisms is closed under composition: if A, B and C are objects and $f : A \to B$ and $g : B \to C$ are morphisms, then $g \circ f : A \to C$ is a morphism.

An important example is the category **S***et* where the objects are sets and the morphisms are mappings between them. The objects of the category **P***ar* are sets and the morphisms are the partial mappings between sets. The objects of the category $Alg(\tau)$ are algebras of type τ and the morphisms are homomorphisms between them. Every variety is a category with the algebras of the variety as objects and with homomorphisms between them as morphisms. One of the most important concepts of category theory is that of a functor. A functor from a category **C** to a category **D** maps the objects of **C** to the objects of **D** and the morphisms of **C** to the morphisms of **D**, in a way that is compatible with the composition of morphisms and preserves the identity morphism.

The notions of category theory are applied everywhere in general algebra. As an example we consider categorical equivalences between varieties. Two varieties **V** and **W** are categorically equivalent if there exists a functor from **V** to **W** such that for all algebras $\mathcal{A}, \mathcal{B}$ in **V** the functor F defines a bijection between the set of all morphisms from $\mathcal{A}$ to $\mathcal{B}$ and the set of all morphisms from $F(\mathcal{A})$ to $F(\mathcal{B})$ and such that for all $\mathcal{C}$ in **W** there is an algebra $\mathcal{A}$ in **V** such that $F(\mathcal{A})$ is isomorphic to $\mathcal{C}$. There are several properties of algebras and varieties which invariant under categorical equivalence. There is a close connection between the categorical equivalence of varieties and the clones of all term operations of some algebras. Let $\mathcal{A}, \mathcal{B}$ be two finite algebras and let $T(\mathcal{A}), T(\mathcal{B})$ be their clones of term operations, i.e. the clones generated by their fundamental operations. Then the varieties $V(\mathcal{A}), V(\mathcal{B})$ generated by $\mathcal{A}$ and by $\mathcal{B}$, respectively are categorically equivalent if and only if the relation algebras $Inf(T(\mathcal{A}))$ and $Inv(T(\mathcal{B}))$ are isomorphic ([18]). Term equivalence of two varieties, that is, the operations of each variety are expressible as terms of the other, is a special case of categorical equivalence.

Marshall Stone in 1937 created from each Boolean algebra a topological space, now called Stone or Boolean space. More precisely, the category of Boolean algebras is dually equivalent to the category of Boolean spaces whose morphisms are the continuous maps between spaces. Let $V(\mathcal{P})$ be the variety generated by a primal algebra (primal variety).

T. K. Hu extended Stone's dual equivalence to varieties of the form $V(\mathcal{P})$. Varieties of this form are also dually equivalent to the category of Stone spaces. An easy consequence of the main result of [18] is that any variety which is generated by a single finite algebra is categorically equivalent to a primal variety if and only if $\mathcal{A}$ is primal. Stone's and Hu's results were the starting points of duality theory which belongs certainly to the main trends in general algebra. A standard reference is [14].

7 Lattices and Ordered Algebraic Structures There are two lattices belonging to each algebra $\mathcal{A}$, the congruence lattice $Con(\mathcal{A})$ and the subalgebra lattice $Sub(\mathcal{A})$. The concept of a lattice is due to E. Schröder (1870). In 1877 R. Dedekind, motivated by group- and ideal theoretic investigations, defined those lattices which are today called modular and distributive lattices. The full development of lattice theory started in the thirtieth of the last century with the work of G. Birkhoff. During this time and after this M. H. Stone, O. Ore, O. Frink, S. Mac Lane, R. P. Dilworth and J. v. Neumann contributed a lot to lattice theory.

It is well-known that lattices can be regarded as algebras $(L; \wedge, \vee)$ of type $(2,2)$ where both binary operations are associative, commutative, idempotent and satisfy the absorption laws $x \wedge (y \vee x) = x = x \vee (y \wedge x)$ or as partially ordered sets $(L; \leq)$where infimum and supremum with respect to $\leq$ for any two-element subset exist. A lattice is called complete if arbitrary infima and suprema exist. In "Lattices and their applications", AAA Darmstadt 1991, pp. 7-25, Res. Exp. Math. 23, Heldermann-Verlag, Lemgo 1995, Garrett Birkhoff gave a nice survey on the field. Good texts in lattice theory and partially ordered sets are [6], [33], [15] and [24].

Lattice theory is applied to many areas. We want to mention here formal concept analysis, a method which is well-known in knowledge representation and knowledge acquisition. A triple consisting of a set of objects together with a set of predicates (properties of the objects) and a relation between these sets is called a formal context. A concept of this context is a pair consisting of a set of objects and a set of properties. By "object g has property m" a relation between the set of objects and the set of predicates is defined. This relation creates a Galois connection and two complete lattices which are dually isomorphic. Usually one combines both lattices into one. The corresponding partial order relation is defined by

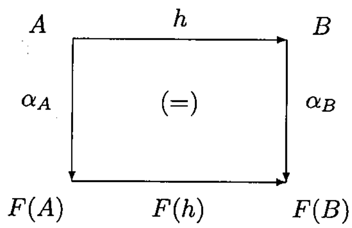
$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 (B_2 \subseteq B_1).$$

This lattice allows to order knowledge under a conceptual point of view and can be used in many practical situations. A standard reference is [28]. For more background on the concept of a Galois connection and applications of Galois connections in different areas of mathematics see [19].

The interest in partially ordered algebraic structures began with P. Conrad's work on valuations and his subsequent papers on partially ordered groups and rings. Other threads were Philip Hall's course at the University of Cambridge on orderable groups, Helmut Wielandt's results in the study of infinite permutation groups, Garrett Birkhoff's work on lattices and L. Fuch's seminal book "Partially ordered algebraic systems". From these starting points on the subject mushroomed to such an extent that a detailed survey is unmanageable. Today there are many classes of ordered algebraic structures which are intensively studied, under them algebras which are already mentioned as BCK-algebras, Boolean algebras or Boolean algebras with operators. The following important classes should be mentioned: cylindric algebras, de Morgan algebras, Heyting algebras, hoops, Kleene algebras, Łukasiewicz algebras, Post algebras, Stone algebras, semilattices, etc. A good reference for partially ordered groups is [31].

8 State-based Systems and Coalgebras In the sixtieth of the last century the following concept of a coalgebra of type τ had already been studied. Let A be a non-empty set and let $f_i^A : A \to \{1, \ldots, n\} \times A$ be an n-ary cooperation, i.e. a mapping from A to the nth copower of A. Then the pair $(A; (f_i^A)_{i \in I})$ where f_i^A are n_i-ary cooperations is called a coalgebra of type τ. The field of coalgebras really took off after it was realized that coalgebras can be conceived as a general and uniform theory of dynamic systems, taken in a broad sense. Many structures in mathematics and theoretical computer science can naturally be represented as coalgebras. The first example was provided by Aczel ([1]), who modeled transition systems and non-well-founded sets as coalgebras. Barwise and Moss ([3]) discussed circularity and self-reference with applications ranging from theoretical economics to the semantics of natural languages using coalgebras. Rutten ([74]) showed that deterministic automata can be regarded as coalgebras. Further important examples include the representation of infinite data structures, and the formal modeling of objects and classes in object oriented programming ([72], [43]).

What are coalgebras ? A state-based system simply consists of a set S (states) endowed with some kind of transition, formally modeled as some map σ from S to another set $F(S)$. Here F is some functor constituting the type or signature of the coalgebra at stake. The transition map provides some kind of structure on S, but whereas algebraic operations are ways to construct complex objects out of simple ones, coalgebraic operations, going out of the carrier set, should be seen as ways to unfold or observe objects. This explains the central role of behavior in the theory of coalgebras. More generally, given an endofunctor F on the category **Set** and consider for any given set A the map $\alpha_A : A \to F(A)$. Then the pair $(A; \alpha_A)$ is called an$F$-coalgebra. If $(B; \alpha_B)$ is another F-coalgebra, then the mapping $h : A \to B$ is a coalgebra homomorphism if the diagram



commutes, i.e. if $\alpha_B(h(a)) = F(h)(\alpha_A(a))$ for all $a \in A$. It is easy to see that the identity map id_A on A is a homomorphism and that the composition of two homomorphisms is a homomorphism. Therefore the class of all F-coalgebras forms a (concrete) category $\mathbf{Set}_F$. From this starting point the whole theory of F-coalgebras can be developed step by step. This definition can be generalized if we choose an arbitrary concrete category **C** instead of **Set**. Coalgebras of type τ are F-coalgebras for the functor $F : \mathbf{Set} \to \mathbf{Set}$ which maps sets X to sets $\prod_{j \in J} X^{\sqcup n_j}$, and takes mappings $f : X \to Y$ to mappings $F(f) : \prod_{i \in I} X^{\sqcup n_i} \to \prod_{i \in I} Y^{\sqcup n_i}$ defined by $(k_i, a)_{i \in I} \mapsto (k_i, f_i^A(a))_{i \in I}$, where $k_i \in \{1, \ldots, n_i\}$. Dually one can define

F-algebras as pairs $(A;\beta_A)$ where $\beta_A : F(A) \to A$ for an set-endofunctor F. For an algebra of type τ the functor $F : \mathbf{Set} \to \mathbf{Set}$ is defined as follows. For sets X, we use the disjoint sum of powers of X, letting $F(X) = \sum_{i\in I} X^{n_i}$. For any mapping $f : X \to Y$, we have the morphism $F(f) : \sum_{i\in I} X^{n_i} \to \sum_{i\in I} Y^{n_i}$ defined by $F(f)(i,(x_1,...,x_{n_i})) = (i,(f(x_1),...,f(x_{n_i})))$ for all $x_1,...,x_{n_i} \in X$.

If β_A is a partial mapping, we obtain partial algebras. Another instances of F-coalgebras are Kripke systems which connect coalgebras with modal logic. For references on the theory of coalgebras see [74].

9 Conclusions Our leading question and the title of this article was: What is general algebra ? The author is unable to give a clear definition. Instead of this some main streams and trends were described, influenced by the author's own (wide) interests, his (restricted) knowledge and his experiences. Each year European algebraists are organizing two workshops on general algebra (the AAA's). These workshops provide excellent opportunities for academic exchange and discussion. Especially "young algebraists", graduate and Ph. D. students feel attracted to join these meetings because of the open academic atmosphere and the helpful discussion. (There is (almost) no conference fee and sometimes financial support for travelling and accomodation can be provided.) There is a series of conference volumes, the "Contributions to General Algebra", volumes 1-17. Each of the conferences has its scientific highlights: new results, new directions, solutions of old problems, new problems and counterexamples arem presented. General algebra is a living and growing field of mathematics with many applications and connections to other areas.

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Communications:

Ⅰ. Conferences for Young Algebraists

Klaus Denecke*

The following list gives some information on the future conferences of young algebraists:

1. AAA Tampere (Finland), June 7-10, 2007
2. CYA (75. AAA), Darmstadt (Germany), November 1-4, 2007
3. AAA Linz (Austria), May 22-25, 2008
4. CYA (77. AAA), Potsdam, Februar 2009
   (dedicated to K. Denecke)
5. AAA Bern (Switzerland), June 11-14, 2009
6. CYA (79. AAA), Olomouc (Czech Republic), February 2010

See also pages 10 and 11 of Notices from the ISMS, January 2007.

Ⅱ. Announcement of Meetings in Topology

Communicated by Gerhard Preuss**

July 2007

Summer Conference on Topology, and its Applications 2007 Unibversitat Jaume I, Castellón, Spain.
Organizers: J. Font, J.Galindo, S.Hernandez, S.Macario, M.Sanchis, A.Rodenas
See: www.sumtop07.uji.es

August 6-August 12, 2007:

International Conference on Quantum Topology, Hanoi Institute of Mathematics
Hanoi,Vietnam
Description: The conference aims to educate young researchers in Quantum Topology,

Discuss new developments in the area, and bring together different points of view of Geometry, Topology, Algebra, and Quantum Field Theory.
Organizers: Do Ngoc Diep and Nguyen Viet Dung (Institute of Mathematics, Hanoi, Vietnam), Stavros Garoufalidis and Thang Le (Georgia Tech).
Scientific Committee: C.Gordon, University of Texas, Austin, V.Jones, University of California, Berkeley, V.Trraev, University of Strasbourg.
See: http://www.math.gatech.edu/~stavros/vietnam.html

Ⅲ. Announcement of QTNA 2007

Communicated by Wuyi Yue*

The Second Asia-Pasific Symposium on Queuering Theory and Network Applications (QTNA2007)

August 1-August 4, 2007 International Conference Center, Kobe, Japan

Webpage of QTNA2007 is: http://www.iict.konan-u.ac.jp/QTNA2007/.
See also page 6 of Notices from the ISMS, November 2006.

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*Klaus Denecke is a professor of University of Potsdam, Institute of Mathematics, an Editor of SCMJ, and an Editor of Notices from the ISMS.

**Gerhard Preuss is a professor of Freie Universität Berlin, FB Mathematik, an Editor of SCMJ, and an Editor of Notices from the ISMS.

Ⅳ. BIOCOMP2007

Communicated by L.M. Ricciardi **

Stimulated by some friends, and on the grounds of the successful experience of *BIOCOMP2002 and "BIOCOMP2005:* Conferences, Prof. L.M. Ricciardi has now been induced to plan another Conference to be held in the same location (Vietri sul Mare, Italy), September 24-28, 2007.
The title is ***BIOCOMP2007 - Collective Dynamics: Topics on Competition and Cooperation in the Biosciences.***

The title is motivated by the nature of our sponsors and supporting grants, but the main purpose of this Conference is to bring together a limited number of well-known specialists in the fields of applied mathematics, physics and theoretical biology for an in-depth discussion of model building and computational strategies in some selected areas of the life sciences with special emphasis on theoretical neurobiology, molecular motors and quantitative problems in ecology and population dynamics.
This will be implemented through a program of plenary talks, parallel sessions and a poster session.
The interdisciplinary nature of the conference will allow cross-fertilization of recent advances in applied nonlinear mathematics and computational approaches. Several invited lectures on different topics of biomathematical interest will also be given, especially tailored on the needs of graduate students and young researches.

V. The 7th International Conference on Optimization (ICOTA 7)

Communicates by Wuyi Yue

The 7th International Conference on Optimization(ICOTA 7) : Techniques and applications

December 12-December 15, 2007, International Conference Center, Kobe, Japan

ICOTA webpage is: http://www.iict.konan-u.ac.jp/ICOTA7/

See also pages 8 of Notices from the ISMS, November 2006.

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* Wuyi Yue is a professor of Konan University, an Editor of SCMJ, and an Editor of Notices from the ISMS.

** L.M.Ricciardi is a professor of Dipartimento di Matematica e Applicazioni, Universita di Napoli Federico II, and an International Advisor of SCMJ.

The ISMS

(I) RESULTS OF CONFIDENCE VOTES FOR SECRETARIES (Officers)

The confidence votes for secretaries were conducted based on the Bylaws 2007 with the deadline of March 31, 2007. We would like to announce that all of the seventeen candidates won the confidence of the ISMS members. The following is the new administration including the above results.

Board of Officers

1. President......**Kiyoshi Iseki** (Term of office: Until Dec. 31, 2007)
2. President Elect .....**Hisao Nagao** (Until Dec. 31, 2007)
3. Immediate Past President ......Vacant
4. Treasurer ....**Toshio Nishida** (Until Dec. 31, 2007)
5. Secretaries
   (1) Secretaries in charge of publishing (9 eats)
      (a) SCMJ (2 seats) **Tadashige Ishihara** (Until Dec. 31, 2007)
      Shunsuke Sato (Until June 30, 2009)
      (b) Notices (5 seats)
      W.W. Comfort (Until June 30, 2009), **K. Denecke** (Until June 30, 2009)
      S.S. Kutateledze (Until June 30, 2009), **I.A. Rus** (Until June 30, 2009)
      D. P. Rolewicz (Until June 30, 2009)
      (c) Kaiho (1 seat) **Masatoshi Fujii** (Until June 30, 2009)
      (d) WWW (1 seat) **Shintaro Mohri** (Until June 30, 2009)
   (2) Secretaries in charge of Meetings (12 seats)
      (a) Assembly type meetings (1 seat) **Nobuo Inagaki** (Until Dec. 31, 2007)
      (b) IVMS (4 seats)
      Atsushi Yagi (Until Dec. 31, 2007), **K. Szajowski** (Until June 30, 2009)
      Juniti Nagata (Until June 30, 2009), **Shizu Nakanishi** (Until June 30, 2009)
      (c) Distance Symposium (2 seats)
      Masaru Nagisa (Until June 30, 2009), **Shinji Kuriki** (Until June 30, 2009)
      (d) International co-sponsored meetings (5 seats)
      Wuyi Yue (Until June 30, 2009), **Wataru Takahashi** (Until June 30, 2009)
      A.Favini (Until June 30, 2009), **G. Preuss** (Until June 30, 2009)
      L.M. Ricciardi (Until June 30, 2009)
   (3) Secretaries in charge of Business Administration (5 seats)
      (a) PR for institutional members
      Yoshinobu Teraoka (Until June 30, 2009), **A.V. Arhangel'skii** (Until June 30, 2009)
      Ashis SenGupta (Until June 30, 2009)
      (b)PR for Individual Members
      (1 seat is vacant)
      (c) Journal Exchange (1 seat) **Akira Tsutsumi** (Until Dec. 31, 2007)
   (4) Secretaries in charge of Prizes (4 seats)
      (a) JAMS & ISMS Prizes **Masako Sato** (Until Dec. 31, 2007)
      (b) Kunugui Prize **J.B. Conway** (Until June 30, 2009)
      (c) Kitagawa Prize **A. Salomaa** (Until June 30, 2009), **P.K. Sen** (Until June 30, 2009)

(Ⅱ)BYLAWS 2007 (July)

The following is a draft of Bylaws 2007 (July) of the ISMS which is recommended by the Board of Officers and will be delivered to the full membership.
The changes to be made are: (1) Adding "contributing member" as a new membership category in Article VI, (2) The term of office of the president becomes one and a half years, and not three years, in Article V, (3) Adding "Free access to the online version of Notices from the ISMS" as a privilege of an individual member in Section 2, Article VII, (3) The effective date becomes July 1, 2007.
The bylaws will be enacted with the approval of the two-thirds of the voting members (a) through the voting system of our website, and (b) by the mail ballot to pgp7j@jams.jp by June 25, 2007.

BYLAWS 2007 (July)(Draft)
International Society for Mathematical Sciences

Article I – Board of Officers
Article II – Board of Trustees
Article III – Committees
Article IV – Council
Article V – Election of Officers and Term of Office
Article VI – Members
Article VII – Dues and Privileges of Members
Article VIII – IVMS
Article IX – Annual Meeting
Article X – Business Meeting
Article XI – Publications
Article XII – Amendments
Addendum – Enforcement

Article I – Board of Officers
There shall be a president, a president elect, an immediate past president, thirty secretaries and a treasurer.

Article II – Board of Trustees
Section 1. There shall be a Board of Trustees consisting of the treasurer, the auditor, the associate treasurer, the immediate past associate treasurer.
Section 2. The function of the Board of Trustees shall be to administer the funds of the Society, to make a budget for the fiscal year, and to produce the statement of account.
Section 3. The auditor and the associate treasurer shall be nominated by the Board of Officers with a confidence vote by the membership.

Article III – Committees
There shall be committees, including the following, which assist the practices of the secretaries. The committee members shall be appointed by the Board of Officers.
(1) Board of SCMJ Managing Editors, (2) Editorial Committee of "Notices from the ISMS" (3) Board of ISMS and JAMS Prize Nominators (4) International Joint Meeting Committee (5) Board of Business Administrators.

Article IV – Council
Section 1. The Council shall consist of fifty two members: thirty four officers, ten foreign members and eight domestic members.
Section 2. The function of the Council shall be to discuss the financial budget, the statement of account and the activities of the Society.
Section 3. The members of the Council shall be nominated by the Board of Officers with a

confidence vote by the membership.

Article V – Election of Officers and Term of Office
Section 1. The term of office shall be one and a half year in the case of the president, the president elect and the immediate past president; three years in the case of the secretaries, the treasurer.
Section 2. The president elect, the secretaries, and the treasurer shall be elected by ballot of the members.

Article VI – Members
There shall be four classes of members: individual, institutional, associate, corporate and contributing.

Article VII – Dues and Privileges of Members
Section 1. The annual dues of an individual member of the Society shall be established by the Board of Officers.
Section 2. The privileges of an individual member are: (1) Free access to the online version of SCMJ, (2) Free access to the online version of Notices from the ISMS (3) Discounted prices for the printed version of SCMJ, (4) Discounted page charges.
Section 3. The privileges of an institutional member are: (1) Discounted subscription price (2) Designation of two persons who belong to the institution as associate members
Section 4: The privileges of an associate member are: (1) No membership dues (2) The same privileges as those of individual members except (1) of the above Section 2.

Article VIII – IVMS
Section 1. International Videoconferences of Mathematical Sciences (IVMS) shall be held from time to time on request basis of the members. Every member has the right to organize or participate in the IVMS.
Section 2. The IVMS shall be held through the videoconferencing system. Those members who wish to hold an IVMS shall inform the head office of the ISMS of such intention, and the ISMS shall make an announcement of necessary arrangement among (joining) institutions.

Article IX – Annual Meeting
Section 1. The annual meeting by IVMS (Annual IVMS) shall be held between the first of July and the thirty first of August.
Section 2. The annual assembly meeting of the Society shall be held between the first of July and the fifteenth of October every year.

Article X—Business Meeting
Section 1. There shall be a business meeting of the Society in online meetings or assembly meetings. The agenda for the business meeting shall be determined by the Board of Officers. A business meeting of the Society can take action only on items notified to the full membership of the Society in the call for the meeting. A majority of members present and voting is required for passage of such an item with a quorum of two percent of the full membership. Members simultaneously connected online with the meeting site shall be deemed present at the business meeting and shall have the right to vote.

Article XI – Publications
The Society shall publish Scientiae Mathematicae Japonicae (SCMJ), Notices from the ISMS, and Kaiho (Newsletter in Japanese).

Article XII– Amendments
These bylaws may be amended or suspended on recommendation of the Board of Officers and with the approval of the membership of the Society, the approval consisting of an affirmative vote by two-thirds of the total number of the members present at the business meeting and the members who vote in the mail ballot in advance.

Addendum

(1) ISMS and JAMS PRIZES

The ISMS Prize shall include (a) the JAMS PRIZE, (b) the JAMS T. SHIMIZU PRIZE, (c) the ISMS PRIZE, (d) the ISMS T. KITAGAWA PRIZE in Applied Mathematics, and (e) the ISMS K. KUNIGI PRIZE in Pure Mathematics. The prizes (b), (d) and (e) are established in order to foster young researchers. The upper age limits are forty-five years old for (b), forty years old for (d) and (e).

(2) Board of Business Administrators

The Board of Business Administrators shall include (a) Publishing Committee, (b) Membership Committee, and (c) Accounting Committee.

(3) The Half Watch Rule

Half of the secretaries and the Council members shall be elected every one and a half years.

(4) Enforcement

The Bylaws become effective on July 1, 2007.

(Ⅲ) Online version of SCMJ

The full texts of the accepted papers will be located on the online version of SCMJ in the following two manners from Vol.66, No. 1 (July 2007).

(1) A list of papears in the order of the accepted date.

(2) A list of accepted papers organized by filed of specialization with a link to (1). The field of specialization of the accepted papers will be chosen by the authors in the following fields of f-1 - f-14.

f-1. Mathematical logic, Set theory, Relative systems, Algebra systems
f-2. Classical algebra, Number theory, Combinatorics, Cryptology
f-3. Topology, Geometry, Imaging
f-4. Real analysis, Complex analysis
f-5. Functional analysis, Operator theory
f-6. Differential equations, Integral equations, Functional equation, Numerical analysis
f-7. Infinite dimensional dynamical systems, Inverse problems
f-8. Fluid dynamics, Atmospheric research, Rheology, Computer aided design, Control theory, Nanoscience
f-9. Probability theory, Statistics, Experimental Design, Quality control
f-10. Operations Research, Decision theory, Queuing theory, Scheduling, Mathematical finance, Mathematical economics
f-11. Informatics, Pattern recognition, Imaging, Computer science, Computer simulation
f-12. Biomathematics, Proteomics, Bio informatics, Imaging, System biology, Bioscience
f-13. Mathematical education, History of mathematics
f-14. Over several fields (Ex. Fixed point theory)

(IV) Call for ISMS Members

Call for Academic and Institutional Members

Discounted subscription price: When organizations become the Academic and Institutioanl Members of the ISMS, they can subscribe our journal Scientiae Mathematicae Japonicae at the yearly price of US$300. At this price, they can add the subscription of the online version upon their request.

Invitation of two associate members: We would like to invite two persons from the organizations to the associate members with no membership fees. The two persons will enjoy almost the same privileges as the individual members do including the discount of the page charge. Although the associate members cannot have their own ID Name and Password to read the online version of SCMJ, they can read the online version of SCMJ at their organization.

To apply for the Academic and Institutional Member of ISMS, please use the following application form.

Application for Academic and Institutional Member of ISMS

Subscription of SCMJ Check one of the two.	☐Print (US$300)	☐Print + Online (US$300)
University (Institution)		
Department		
Postal Address where SCMJ should be sent		
E-mail address		
Person in charge	Name: Signature:	
Payment Check one of the two.	☐Bank transfer	☐Credit Card (Visa, Master)
Name of Associate Membership	1.	
	2.	

Call for regular Members
ISMS Membership Dues from 2007

A new category "life member" has been established and can be applied for from 2005. An eligible member may become a life member by making a one-time payment of dues. A member who has been an ISMS member for ten years or more is eligible for a life member. The amounts of dues are : \70,000 for the domestic members, US$ 600 (€480)for the foreign members, and US$ 500 (€400)for the members in developing countries.

We have reduced the ISMS memership dues since 2001 and copies of the printed journal have not been distributed to the members, free of charge. Instead, we give User Name and Password to each member so that he/she can view or print out the full text of the papers published in SCMJ except papers in the international plaza from our Web site (http://www.jams.or.jp).

The Membership Dues for each category is as follows. Applications for the 3-year members can be made only in 2005 and in every three years.

Membership Dues for 2007

Membership	JAPAN	S-JAPAN	Foreign	S-Foreign	Developing
1-year	A1 ¥7,000	SA1 ¥3,500	F1 US$50 €40	SF1 US$30 €24	D1 US$30 €24
3-year	A3 ¥18,000	SA3 ¥9,000	F3 US$120 €96	SF3 US$60 €48	D3 US$70 €56
Life Member	Life ¥70,000	Life ¥70,000	FL US$600 €480	FL US$600 €480	DL US$500 €400

Category D is for those who reside in the countries of Eastern Europe, CIS or developing countries. Category S is for students and for the aged (older than 70). The figure 1 and 3 means a year and 3 years respectively.

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Payment Instructions

Payment can be made through a post office or a bank, or by credit card. Members may choose the most convenient way of remittance. Please note that we do not accept payment by bank drafts (checks). For more information, please refer to an invoice.

Methods of Overseas Payment:

Payment can be made through (1) a post office, (2) a bank, (3) by credit card, or (4) UNESCO Coupons.

Authors or members may choose the most convenient way of remittance as are shown below. Please note that **we do not accept payment by bank drafts (checks).**

(1) Remittance through a post office to our giro account No. 00930-1-11872 or send International Postal Money Order to our postal address (2) Remittance through a bank to our account No. 94103518 at Shinsaibashi Branch of CITIBANK (3) **Payment by credit cards** (AMEX, VISA, MASTER or NICOS), or (4) Payment by UNESCO Coupons.

Methods of Domestic Payment:

Make remittance

(1) to our Post Office Transfer Account - 00930-3-73982 or

(2) to our account No.1565679 at SUMITOMO BANK, Sakai, Osaka, Japan.

All the correspondences concerning subscriptions, back numbers, individual and institutional memberships, should be addressed to the Publications Department, International Society for Mathematical Sciences.

Membership Application Form (from 2007)

To determine what membership category you are eligible for, read "Join ISMS" on the inside of the back cover.

1. Name: Family Name, First Name, Middle Name (in this order)
2. Home Address
3. Name of Firm or Institution affiliation
4. Postal address to which correspondence should be sent
5. e-mail address
6. Telephone Number, Fax Number
7. Membership Category
8. Panel (Please choose one out of the following 12panels and write the panel number. You could choose one or more.)

(e-1)Mathematical Logic, Set Theory, Lattice Theory, Ordered Systems.
(e-2)Algebra, Algebraic Geometry, Number Theory, Combinatorics, Cryptology.
(e-3) Topology, Geometry, Imaging.
(e-4) Real Analysis, Functional Analysis, Complex Functions.
(e-5) Differenation Equations, Integral Equations, Functional Equations.
(e-6) Fluid Dynamics, Rheology, Imaging and other Applied Analysis, Control Theory, Numerical Analysis、Simulation.
(e-7) Probability, Statistics, Data Mining, Decision theory. Quality Control.
(e-8) Game, Finance, Operations Research, Mathematical Economics. Ecology
(e-9) Informatics, Computer Sciences.
(e-10) Biomathematics, Neuroinformatics, Genome Sciences, Nanoscience.
(e-11) Mathematical Education, History of Mathematics.
(e-12) Over several fields.(Ex. Fixed Point Theory, Semi-group)

9. Would you like to buy the printed copies of SCMJ, whose prices a year are US$60(6,000yen) for 1-year-members(A1, D1, S-A1, S-D1)and US$55(5,500yen) for 4-year-members(A4, D4, S-A4, S-D4) ? Type YES or NO.
10. If you apply for an aged member (70 years old or over), please type the year of your birth.
11. If you wish to be a student member, please verify.
12. Is your university (institution) an Academic or Institutional Member of the ISMS? Yes or No.
13. If the answer of 12 is Yes, please answer the following. Are you designated associate member by your university (institution)?
14. Date
15. Signature

For Japanese Applicants, please send two application forms, one in English and the other in Japanese.

I wish to enroll as a member of ISMS and will pay to International Society for Mathematical Sciences the annual dues upon presentation of an invoice. Copies of Mathematica Japonica, Scientiae Mathematicae and Scientiae Mathematicae Japonicae received as an ISMS member will be for my personal use and shall not be placed in institutional, university or other libraries or organizations, nor can membership subscriptions be used for library purposes.

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Join ISMS !

ISMS Publications: We published **Mathematica Japonica (M.J.)**, which enjoyed an international reputation, for about sixty years in print and its offshoot **Scientiae Mathematicae (SCM)** both online and in print. In January 2001, the two publications were unified and changed to **Scientiae Mathematicae Japonicae (SCMJ)**, which is the "21st Century New Unified Series of Mathematica Japonica and Scientiae Mathematicae" and published both online and in print. Ahead of this, the online version of SCMJ was first published in September 2000. The number of the annual total pages of the print version has been from 900 to 1,200 pages in six issues since January 1978. The whole number of SCMJ exceeds 240, which is the largest amount in the publications of mathematical sciences in Japan. The features of SCMJ are:

1) About 90 eminent professors and researchers of not only Japan but also 20 foreign countries join the Editorial Board. The submitted papers are received directly by the editors and are refereed quickly. The accepted papers are published online with no lead time after compiling or proofreading. SCMJ is reviewed by Mathematical Review and Zentralblatt from cover to cover.
2) SCMJ is distributed to many libraries of the world. The papers in SCMJ are introduced to the relevant research groups for the positive exchanges between researchers.
3) The original papers and surveys of distinguished mathematical scientist appear in every issue of SCMJ. The section called "International Plaza" of SCMJ has very interesting expository papers written by the eminent mathematical scientist of the world. Presentations of recent research frontier including award lectures by the winners of the ISMS Prize or Shimizu Prize are made.
4) **ISMS Annual Meeting:** Many researchers of ISMS members and non-members gather and take time to make presentations and discussions in their research groups every year.
5) The ISMS holds inter-regional videoconferences called **International Videoconference of Mathematical Sciences (IVMS)** via internet. There is no need for the participants to travel abroad.

Privileges to ISMS Members: (1) Free access (**including printing out**) to the online version of SCMJ, (2)Discounted price for the printed version of SCMJ (See **Table 1**), (3) Discounted page charges (See **Table 2**).

Privileges to Institutional Members: (1) Two associate members can be registered, free of charge, from an institution. (2) The discounted page charges (Table 2) are applied to the associate members.

Table 1: Subscription Price (from 2007)

	Individual 1-year mem.	Individual 3-year mem.	Institutional member	List Price
Print / year	¥6,000 US$60, €48	¥5,500 * US$55, €44	¥33,000 US$300, €240	¥45,000 US$400, €320
Online/year	Free	Free	—	—
Online+Print / year	¥6,000 US$60, €48	¥5,500 * US$55, €44	¥33,000 US$300, €240	¥45,000 US$400, €320

Postal charge is US$2 (€1.6) per issue. *In case three-year members make the payment at a time in advance, the price for 3 years is ¥15,000 (US$150, €120). The authors can buy a copy of the print version at a price of ¥1,200 (US$12) per issue including postage.

Table 2: Page Charge per printed page

	Individual/Associate Member	Non Member
Paper : P	¥3,850 (US$35, €28)	¥4,450 (US$43、€35)
TeX: T	¥2,200 (US$18, €14)	¥2,800 (US$26, €21)
ISMS style: Js	¥1,100 (US$8, €7)	¥1,700 (US$16, €14)

The above page charges include 20 offprints.

Table 3: Membership Dues for this year

Categories	Domestic	Overseas	Developing countries
1-year member (1A)	A1: ¥7,000	F1: US$50 , €40	D1: US$30, €24
3-year member (3A)	A3: ¥18,000	F3: US$120, €96	D3: US$70, €56
1-year students or aged (1S)	SA1: ¥3,500	SF1: US$30, €24	SD1: US$20, €16
3-year students or aged (3S)	SA3: ¥9,000	SF3: US$70, €56	SD3: US$50, €40
Life member* (L)	AL: ¥70,000	FL: US$600, €480	DL: US$500, €400

*The members who have been the ISMS members for more than 10 years are eligible for this category.The categories 1S and 3S are for students or persons over 70 years old.