More on decompositions of a fuzzy set in fuzzy topological spaces

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Received April 7, 2014

Abstract. Using new properties (Theorem B in Section 2) of the concept of fuzzy points in the sense of Pu Pao-Ming and Liu Ying-Ming (Definition 2.1), we first prove that every fuzzy set $\lambda \neq 0$ is decomposed by two fuzzy sets $\lambda_{\mathcal{O}(X,\sigma^f)}$ and $\lambda^*_{\mathcal{PC}(X,\sigma^f)}$ (Theorem A;cf. Theorem 2.5(ii)), where (X,σ^f) is a specified Chang's fuzzy space (Definition 1.2, Remarks 1.3,1.4). Namely, $\lambda = \lambda_{\mathcal{O}(X,\sigma^f)} \vee \lambda^*_{\mathcal{PC}(X,\sigma^f)}$ and $\lambda_{\mathcal{O}(X,\sigma^f)} \wedge \lambda^*_{\mathcal{PC}(X,\sigma^f)} = 0$ hold, and the fuzzy set $\lambda_{\mathcal{O}(X,\sigma^f)}$ is fuzzy open in (X,σ^f) (Theorem 2.5(iii)). Finally, these results are applied to the case where $X = \mathbb{Z}^n (n > 0)$ and $\sigma^f = (\kappa^n)^f$ (Theorem 3.3 and Theorem 3.5), where the topological space (X,σ) is the digital n-space $(\mathbb{Z}^n,\kappa^n)$ (cf. Section3).