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# Classification of idempotents and square roots in the upper triangular matrix Banach algebras and their inductive limit algebras

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Abstract. We study idempotents and square roots in the upper triangular matrix Banach algebras over real or complex numbers. We compute explicitly and determine algebraically the idempotents and the square roots in the cases of size: two by two, three by three, and four by four. We also consider their equivalence classes by homotopy and classify topologically the upper triangular matrix algebras in those cases and in general by the groups generated by the homotopy classes. Moreover, we consider some infinite dimensional, Banach algebras obtained as inductive limits of the upper triangular matrix algebras and obtain several topological classification results for the inductive limits.

*Key words and phrases.* Idempotent, square root, matrix algebra, upper triangular matrix, non self-adjoint algebra, Banach algebra, inductive limit.