

**ON RELATIONS BETWEEN OPERATOR VALUED α -DIVERGENCE
AND RELATIVE OPERATOR ENTROPIES**

HIROSHI ISA⁽¹⁾, MASATOSHI ITO⁽²⁾, EIZABURO KAMEI⁽³⁾,
HIROAKI TOHYAMA⁽⁴⁾ AND MASAYUKI WATANABE⁽⁵⁾

Received March 5, 2014 ; revised June 20, 2014

ABSTRACT. Let A and B be two strictly positive operators, and $\alpha \in (0, 1)$. The operator valued α -divergence is defined by

$$D_\alpha(A|B) \equiv \frac{1}{\alpha(1-\alpha)} (A \nabla_\alpha B - A \sharp_\alpha B),$$

where $A \nabla_\alpha B = (1-\alpha)A + \alpha B$ and $A \sharp_\alpha B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^\alpha A^{\frac{1}{2}}$. In this paper, firstly, we show some fundamental relations between operator valued α -divergence and relative operator entropies (relative operator entropy, Tsallis relative operator entropy etc.). Next, we introduce noncommutative ratio $(A \natural_{u+v} B)(A \natural_u B)^{-1}$ on the path $A \natural_w B$, and we discuss noncommutative ratio translation. Moreover, we discuss α -divergence for operator distributions.

Key words and phrases. operator divergence, operator valued α -divergence, relative operator entropy, Tsallis relative operator entropy, operator mean.