GEOMETRICAL EXPANSION OF AN OPERATOR EQUATION

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ABSTRACT. Lawson-Lim [9] had given a generalization of Karcher equation and the equations determining power means. We formulate these operator equations by simpler forms, which are geometrically meaningful.

Let $\mathbb{A}(A_1, \cdots, A_n)$ be positive operators and $\omega = \{\omega_1, \cdots, \omega_n\}$ be a weight. Then the operator equation

$$0 = \sum_{i=1}^{n} \omega_i T_r(X|A_i)$$

has a unique positive solution for each $r \in [-1,\ 1]$, where $T_r(X|A) = \dfrac{X \,\natural_r\, A - X}{r}$ is the Tsallis relative operator entropy and $A \,\natural_r\, B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^r A^{\frac{1}{2}}$ for $r \in \mathbf{R}$. We show the exact form to the unique solution of the above operator equation in the case $n = 2$.