

**THE ORDER-PRESERVING PROPERTIES OF ESTIMATES IN
POLYTOMOUS ITEM RESPONSE THEORY MODELS WITH
APPROXIMATED LIKELIHOOD FUNCTIONS**

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ABSTRACT. In this study, we consider the ordering properties of the estimates of the rating scale model(RSM) and related polytomous item response theory (IRT) models. First, we propose a kind of approximation to the likelihood functions for these IRT models. The approximated likelihood functions are derived from the inequality of arithmetic and geometric means. We then evaluate upper limits of the functions based on the mathematical result of Specht(1960). Next, we derive the order-preserving statistics for these polytomous IRT models. All sets of statistics are derived by using the characteristics of arrangement increasing functions (Hollander *et al.*, 1977, Marshall *et al.*, 2011). We also carry out simulation study and confirm that our order-preserving statistics work well in typical educational testing. Finally, it is shown that the order-preserving statistics of the RSM in three major three estimation methods coincide.

Key words and phrases. Polytomous item response theory, Order-preserving property, Rating scale model, Arrangement increasing, Educational testing.