The order-preserving properties of the Rasch model and extended model in marginal maximum likelihood estimation

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Abstract In this study, we consider the order-preserving properties of Rasch model (Rasch, 1960) and linear logistic model(Fischer, 1994) in marginal maximum likelihood estimation (MMLE). More specially, we focus on the "manifest probability," as discussed by Cressie and Holland (1983) and derive the order-preserving statistics for the item parameters. We also derive order-preserving statistics for the ability parameters in maximum likelihood estimation under the condition that the estimates of the item parameters are already given. Both sets of statistics are derived using the characteristics of arrangement increasing functions (Hollander *et al.*, 1977, Marshall *et al.*, 2011). It is notable that the order-preserving statistics of the Rasch model in MMLE coincide with those of other estimation techniques, such as joint maximum likelihood estimation and conditional maximum likelihood estimation. However, while the marginal maximum likelihood estimates and the conditional maximum likelihood estimates are consistent, the joint maximum likelihood estimates are not. Here, we discuss the reasons for such coincidences, as well as the types of bias that occur in inconsistent estimates.