## SOME INTEGRALS BETWEEN THE LEBESGUE INTEGRAL AND THE DENJOY INTEGRAL

## Toshiharu Kawasaki

Received December 16, 2015; revised March 11, 2016

ABSTRACT. B. Bongiorno, Di Piazza and Preiss gave a minimal constructive integration process of Riemann type, called the C-integral, which contains the Lebesgue integral and the Newton integral. D. Bongiorno gave a minimal constructive integration process of Riemann type, called the  $\tilde{C}$ -integral, which contains the Lebesgue integral and the improper Newton integral. On the other hand, Nakanishi gave criteria for the restricted Denjoy integrability. Motivated by the results of Nakanishi, Kawasaki and Suzuki gave criteria for the C-integrability, and Kawasaki gave criteria for the  $\tilde{C}$ -integrability. In this paper, motivated by the results above, we give new integrals between the Lebesgue integral and the restricted Denjoy integral. Moreover we give criteria for the integrability of one of them in the style of Nakanishi.

 $Key\ words\ and\ phrases.$  C-integral,  $\tilde{C}$ -integral, Lebesgue integral, Improper Lebesgue integral, Denjoy integral, McShane integral, Henstock-Kurzweil integral.