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# COLORINGS FOR SET-VALUED MAPS

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Abstract. We know many results about colorability for single-valued maps. But we know a few results about colorability for set-valued maps. In this paper we generalize some results on colorability for single-valued maps to those for set-valued maps. Especially, our main result is a generalization of E. K. van Douwen's result, which insists that every fixed-point free continuous closed map $f : X \to X$ with $\sup\{|f^{-1}(x)| : x \in X\} < \infty$ on a finite-dimensional paracompact space $X$ is colorable. In fact, we prove the following: Let $X$ be a finite-dimensional paracompact space and $f : X \to \mathcal{F}_k(X)$ a fixed-point free upper semi-continuous map, where $\mathcal{F}_k(X)$ is the family of non-empty subsets of $X$ with at most $k$ elements. Suppose that $\sup\{|f^{-1}(x)| : x \in X\} < \infty$ and $\bigcup\{f(x) : x \in F\}$ is closed in $X$ for any closed subset $F$ of $X$. Then $f$ is colorable.

*Key words and phrases.* set-valued map, color, fixed-point free, extension.