VELOCITY AND ACCELERATION ON THE PATHS $A \natural_t B$ AND $A \sharp_{t,r} B$

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Abstract. Let A and B be strictly positive linear operators on a Hilbert space. The derivative of the path $A \natural_t B$ ($t \in \mathbf{R}$) gives the relative operator entropy, that is, $\frac{d}{dt} A \natural_t B = S_t(A|B)$, which we can regard as the velocity function along $A \natural_t B$. The derivative of velocity function is the acceleration function, so we define the acceleration by $\mathcal{A}_t(A|B) = \frac{d}{dt} S_t(A|B)$. In this paper, we discuss properties of $S_t(A|B)$ and $\mathcal{A}_t(A|B)$. Firstly, we interpret some properties of $S_t(A|B)$ concerning interpolational property and the noncommutative ratio from the viewpoint of velocity. Secondly, we show the properties of $\mathcal{A}_t(A|B)$ similar to those of $S_t(A|B)$.

Key words and phrases. velocity, acceleration, path, relative operator entropy.