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ON DUCCI MATRIX SEQUENCES II

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Abstract. In this paper, we shall consider various properties of Ducci matrix sequences. Those properties are analyzed from the viewpoint of measure theory and of (Baire) category. Considered sets include the set of all $\alpha \in (0,1) \setminus \mathbb{Q}$ whose Ducci matrix sequence expansion contains eventually periodic occurrences of a fixed block, which is of measure zero and meager, and the set of all $\alpha \in (0,1) \setminus \mathbb{Q}$ such that infinitely many i satisfy $j_\alpha(i) = \cdots = j_\alpha(i+l-1) = j_\alpha(i+l)$, which is of full measure and comeager.

It is known that the Ducci matrix sequence expansion is eventually periodic if and only if the continued fraction expansion is [J. Difference Equ. Appl. 22(3) (2016), pp. 411–427]. Inspired by this result, we prove that analogous statements are valid for positive Poisson stability and for the denseness of the orbit, while neither implication is valid for eventual abelian periodicity. For eventual almost periodicity, only one implication is valid.

Key words and phrases. Ducci map; Ducci matrix; Ducci matrix sequence; Continued fraction; Measure theory; Baire category; Abelian periodicity; Almost periodicity; Positive Poisson stability.