ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO RACETRACK MODEL IN SPATIAL ECONOMY

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ABSTRACT. We are concerned with the racetrack model which has been presented by Krugman et al.[9] in the study of spatial economics. They intended to describe the movement of labors and goods along a racetrack where dense economic regions are continuously distributed and to investigate geographical structures provoked automatically by the superposition of specific economic laws. Tabata et al.[26, 22, 23] have already studied similar but more general models mainly from the view point of mathematical analysis by constructing the global solutions, although they did not handle the very racetrack model.

The objective of the present paper is to explore asymptotic behavior of the global solutions to the racetrack model. After constructing global solutions, we shall show that, as $t \to \infty$, every solution tends to a stationary solution of the model. We shall then demonstrate in somewhat heuristic way that the stationary solution is given by either a homogeneous solution on the racetrack or a concentrated solution in which distribution of the manufacturing is a sum of Dirac delta functions. As the homogeneous stationary solution is proved to be always unstable, the asymptotic limit of the global solution is in general a concentrated solution, thereby there remains only a finite number of regions on the racetrack that possess the manufacturing sector.

These results may suggest that in the racetrack model the spatial economic state tends to a discretely concentrated state. According to numerical results, the number and the location of the spikes of concentration seem to have a certain freedom, evidently depending on the initial state. The maximum number of spikes is, however, clearly controlled by the transportation cost and the preference for variety of goods.