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MIXED SCHWARZ INEQUALITIES VIA THE MATRIX GEOMETRIC MEAN

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Abstract. In this paper, by using the Cauchy-Schwarz inequality for matrices via the matrix geometric mean due to J.I. Fujii, we show the following matrix version of a mixed Schwarz inequality for any square matrices: Let A be an n-square matrix. For any n-square matrices X, Y

$$|Y^*AX| \leq X^*|A|^{2\alpha}X \sharp U^*Y^*|A^*|^{2\beta}YU$$

holds for all $\alpha, \beta \in [0,1]$ with $\alpha + \beta = 1$, where U is a unitary matrix in a polar decomposition of $Y^*AX = U|Y^*AX|$. As applications, we show matrix Parseval's equation, Lin's type extensions for a weighted version of a mixed Schwarz inequality, and a weighted version of the Wielandt inequality for matrices.

Key words and phrases. weighted mixed Schwarz inequality, matrix geometric mean, Lin's type extension, Wielandt inequality.