

## EXAMPLE OF CUBE SLICES THAT ARE NOT ZONOIDS

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**To the memories of Som Nainpally and Joe Diestel**

### Abstract

Let  $Q$  be the unit cube in  $R^n$  centered at the Origin  $O$  and  $H$  a hyperplane through  $O$ . The intersection is called a central Cube slice and its study was initiated by Hadwiger, Hensley and Vaaler, continued by Ball and others. A zonoid is the range of a non atomic vector measure into  $R^n$ . In this paper, when  $n = 4$  we give examples of non-zonoid cube slices. Let  $H: x + y + z + t = 0$ ; the slice has triangle faces and is not a zonoid. This contrasts with a result in  $R^3$ , where it follows from a classical Theorem due to Herz and Lindenstrauss that every central cube slice is a zonoid (zonotope). We also give nontrivial examples in which the slice is a zonoid. For ex. let  $H: ax + y + z + t = 0$  with  $a > 1$ . If  $a \geq 3$ , the slice is a zonotope. Otherwise it has faces that are trapeziums or pentagons and is not a zonoid. We also give other examples of the like nature.