

**VERMA MODULES OVER A  $\mathbb{Z}_2 \otimes \mathbb{Z}_2$  GRADED SUPERALGEBRA AND  
INVARIANT DIFFERENTIAL EQUATIONS**

NARUHIKO AIZAWA

Received January 11, 2018 ; revised February 7, 2018

**ABSTRACT.** Lowest weight representations of the  $\mathbb{Z}_2 \otimes \mathbb{Z}_2$  graded superalgebra introduced by Rittenberg and Wyler are investigated. We give an explicit construction of Verma modules over the  $\mathbb{Z}_2 \otimes \mathbb{Z}_2$  graded superalgebra and show their reducibility by using singular vectors. The explicit formula of singular vectors are given and are used to derive partial differential equations invariant under the color supergroup generated by the  $\mathbb{Z}_2 \otimes \mathbb{Z}_2$  graded superalgebra.

*Key words and phrases.*  $\mathbb{Z}_2 \otimes \mathbb{Z}_2$  graded superalgebra, Verma modules, singular vectors, invariant PDEs .