NORM INEQUALITIES RELATED TO THE MATRIX GEOMETRIC MEAN OF NEGATIVE POWER

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Abstract. In this paper, we show norm inequalities related to the matrix geometric mean of negative power for positive definite matrices: For positive definite matrices A and B,

$$\left\| e^{(1-\beta)\log A+\beta \log B} \right\| \le \| A \natural_\beta B \| \le \left\| A^{1-\beta} B^{\beta} \right\|$$

for every unitarily invariant norm and $-1 \le \beta \le -\frac{1}{2}$, where the β-quasi geometric mean $A \natural_\beta B$ is defined by $A \natural_\beta B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\beta}A^{\frac{1}{2}}$. For our purposes, we show the Ando-Hiai log-majorization of negative power.

Key words and phrases. Ando-Hiai inequality, matrix geometric mean, unitarily invariant norm, positive definite matrix.