Scientiae Mathematicae Japonicae

Log-majorization on spectral geometric mean and relative entropy

Dedicated to the memory of Professor Zirô Takeda with sincere respect

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Abstract. In this paper, we prove that the spectral geometric mean $A \operatorname{sp}_\alpha B$ for positive definite matrices A and B is log-majorized by $A^{\frac{1-\alpha}{2}} B^\alpha A^{\frac{1-\alpha}{2}}$ for $-1 \leq \alpha \leq 0$. Precisely, it is a new result for $-1 \leq \alpha \leq -\frac{1}{2}$, and we give an alternative simple proof for $-\frac{1}{2} \leq \alpha \leq 0$. Accordingly we discuss some applications to relative entropies of Tsallis type.