

**SUPPLEMENT OF THE PAPER
“SOME IDEALS IN BCI-ALGEBRAS”**

EUN HWAN ROH[†], YOUNG BAE JUN[†] AND SHIMING WEI[‡]

Received November 21, 1996

ABSTRACT. The aim of this paper is the supplement and generalization of [1]. First, we consider the special subset $B_A(a)$ of a BCI-algebra, and to investigate its property. Next, we show that every obstinate ideal is a closed ideal.

In this note, we shall use symbols and definitions of [1].

Theorem 1. *Let A be an ideal of a BCI-algebra X . Then A is a $(*)$ -ideal of X if and only if for all $a \in X - A$, $0 * a \in A$.*

Proof. Suppose that $0 * x \in A$ for all $a \in X - A$. For all $x \in A$ and all $y \in x - a$, since $0 * y \in A$ and $(x * y) * (0 * y) \leq x \in A$, we have $(x * y) * (0 * y) \in A$, and hence $x * y \in A$. Therefore A is a $(*)$ -ideal of X .

The converse is obvious. \square

Theorem 2. *Let A be a $(*)$ -ideal of an associative BCI-algebra X , then for any $a \in X - A$, $B_A(a) = X$.*

Proof. For any $x \in X$, if $x \in A$ then $x * a \in A$. If $x \in X - A$, then by Theorem 1 we have $0 * x \in A$ and so $(0 * x) * x \in A$. Thus we obtain

$$(x * a) * (0 * x) = (x * (0 * x)) * a = ((0 * x) * x) * a \in A.$$

Therefore $x * a \in A$, and hence $X = B_A(a)$. \square

Remark. By Theorem 2, Roh et al. [1] proved following theorems are trivial, that is, if A is a $(*)$ -ideal (resp. a subalgebra and $(*)$ -ideal) of an associative BCI-algebra X , then for any $a \in X - A$, $B_A(a)$ is an ideal (resp. subalgebra) of X .

Now, we discuss obstinate ideals in BCI-algebras and their relation with closed ideals.

Theorem 3. *If A is an obstinate ideal of a BCI-algebra X , then A is a closed ideal of X , and hence a subalgebra of X .*

Proof. For any $x \in A$, assume $0 * x \in X - A$, then $(0 * x) * x \in X - A$. Since A is an obstinate ideal of X , we have $0 * x = ((0 * x) * x) * (0 * x) \in A$. A contradiction. Hence $0 * x \in A$. Therefore A is closed. \square

1991 Mathematics Subject Classification. 03G25, 06F35.

Key words and phrases. $(*)$ -ideal, obstinate ideal.

[†] Supported by the Basic Science Research Institute Program, Ministry of Education, 1996, Project No. BSRI-96-1406.

Theorem 4. *Let X and X' be BCI-algebras. If A is an obstinate $(*)$ -ideal in X , then there exists a homomorphism $f : X \rightarrow X'_+$ such that $\text{Ker}(f) = A$.*

Proof. For a nonzero element c in X' , define $f : X \rightarrow X'_+$ by

$$f(x) = \begin{cases} 0 & \text{if } x \in A, \\ c & \text{otherwise.} \end{cases}$$

Obviously, $\text{Ker}(f) = A$. Let $x, y \in X$ be arbitrary elements. We show that $f(x * y) = f(x) * f(y)$. We consider the following four cases:

Case i. Let $x, y \in A$. Since A is an obstinate ideal, A is a subalgebra by Theorem 3. Thus $x * y \in A$ and hence $f(x * y) = 0 = 0 * 0 = f(x) * f(y)$.

Case ii. If $x \in A$ and $y \notin A$, then $x * y \in A$ because A is a $(*)$ -ideal. Thus $f(x * y) = 0 = 0 * c = f(x) * f(y)$.

Case iii. If $x \notin A$ and $y \in A$, then $x * y \notin A$. Thus $f(x * y) = c = c * 0 = f(x) * f(y)$.

Case iv. If $x, y \notin A$, then $x * y \in A$ because A is obstinate. Thus $f(x * y) = 0 = c * c = f(x) * f(y)$.

Hence, in all cases, we have $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. \square

Remark. Theorem 4 is the generalization of Theorem 7 in [1].

REFERENCES

1. E. H. Roh, Y. B. Jun and S. M. Wei, *Some ideals in BCI-algebras*, Math. Japon. **43** (1996), 47-50.

† DEPARTMENT OF MATHEMATICS EDUCATION, GYEONGSANG NATIONAL UNIVERSITY, CHINJU 660-701, KOREA.

‡ DEPARTMENT OF MATHEMATICS, HUAIBEI COAL MINING NORMAL COLLEGE, HUAIBEI 235000, P. R. CHINA.