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THE PRODUCTS ON σ-PARALINDELOF SPACES

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Abstract. In this paper, we mainly obtain two Tychonoff product theorems and one theorem of σ-product on σ-paralindelof spaces.

Since K.Nagami [1] researched the product of two paracompact spaces in 1969 a great advance has been got on with products of topological spaces characterized by covering properties. Particularly, in recent years, the papers [2,3,4,5,6] are published such that some properties of products of metacompact (submetacompact, metalindelof) spaces have been acquired. But, since locally finite, point finite, point countable differ on nature with locally countable, we have not got any better result on products of paralindelof spaces. In this paper, we first research Tychonoff products of σ-paralindelof of spaces. Next, we obtain one result of σ-product of σ-paralindelof spaces.

In this paper, $(\mathcal{U})_W, \mathcal{U}|_Y$ and N(K) denote, respectively, $\{U \in\mathcal{U}:W\cap U \neq\phi\}$, $\{\mathrm{U}\cap\mathrm{Y}:U \in\mathcal{U}\}$ and the open neighborhood system of a set K; $(\mathcal{U})_x$ and N(x) denote, respectively, $(\mathcal{U})_{\{x\}}$ and N($\{x\}$); N and ω are respectively the set of all natural numbers and the countable infinite cardinality. For a set A, its closur, interior and cardinality are respectively denoted by $\widetilde{A}$, IntA and $|A|$. And $\mathrm{A}^n=\{a : a \subset\mathrm{A} \text{ and } |a| = n\}$, $\mathrm{A}^{<\omega}=\bigcup\{A^n : n \in \omega\}$.

Definition 1 ([8]) A collection $\mathcal{U}$ of a topological space X is said to be locally countable if each x$\in$X there is some W$\in$N(x) such that $|(\mathcal{U})_W| \leq \omega$

X is said to be σ-paralindelof iff its every open cover $\mathcal{U}$ has an open refinement $\bigcup_{n\in N}\mathcal{V}_n$ such that $\mathcal{V}_n$ is locally countable for each $n \in$ N. Where we call $\bigcup_{n\in N}\mathcal{V}_n$ is a σ-locally countable open refiniment of $\mathcal{U}$.

Definition 2 ([9]) A tpoplogical space X is a P-space if for any index Ω and any collection $\{\mathrm{U}(\alpha_1, ..., \alpha_n)$: $(\alpha_1, ..., \alpha_n) \in \Omega^n\}$ by open sets in X such that for each $(\alpha_1, ..., \alpha_n, \alpha_{n+1}) \in \Omega^{n+1}$,

$$\mathrm{U}(\alpha_1, ..., \alpha_n)\subset\mathrm{U}(\alpha_1, ..., \alpha_n, \alpha_{n+1})$$

there exists a collection $\{\mathrm{F}(\alpha_1, ..., \alpha_n):(\alpha_1, ..., \alpha_n)\in \Omega^n\}$ by closed sets in X such that

(1) $\mathrm{F}(\alpha_1, ..., \alpha_n)\subset\mathrm{U}(\alpha_1, ..., \alpha_n)$ For each $(\alpha_1, ..., \alpha_n) \in \Omega^n$

(2) For $(\alpha_1, ..., \alpha_n, ...) \in \omega^\omega$, $\bigcup\{\mathrm{F}(\alpha_1, ..., \alpha_n) : n \in\mathrm{N}\}$=X if $\bigcup\{\mathrm{U}(\alpha_1, ..., \alpha_n) : n \in\mathrm{N}\}$=X.

Definition 3 ([1]) Let $\{\mathcal{F}_i : i \in \mathrm{N}\}$ be a sequence of locally finite closed coverings satisfying the following condition:

If $\mathrm{K}_1 \supset\mathrm{K}_2 \supset ...$ is a sequence of nonempty closed sets of X such that

$$\mathrm{K}_i \subset \bigcap\{\mathrm{F}\in \mathcal{F}_i\text{: x}\in\mathrm{F}\}$$

for some point x in X and for each $i \in$N, then $\bigcap\{\mathrm{K}_i : i \in \mathrm{N}\} \neq \phi$. We set

$$\mathrm{C(x)}=\bigcap\{\bigcap\{\mathrm{F}\in \mathcal{F}_i\text{:x}\in\mathrm{F}\}\text{: } i \in\mathrm{N}\}$$

then it is to be noted that every C(x) is closed and countable compact, then X is called a Σ-space. Particularly, if C(x) is compact for each x$\in$X, then X is called a strong Σ-space. And $\{\mathcal{F}_i\}_{i\in N}$ is respectively called a Σ-net or a strong Σ-net.

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Definition 4 ([5]) Let s=$(s_\alpha)_{\alpha\in A}$ be a fixed point in Tychonoff product $\prod\{X_\alpha\colon \alpha \in A\}$. For each x=$(x_\alpha)_{\alpha\in A}$ $\in\prod\{X_\alpha\colon \alpha \in A\}$, let Q(x)=$\{\alpha \in A\colon x_\alpha \neq s_\alpha\}$ and define $\sigma\{X_\alpha\colon \alpha \in A\}=\{x=(x_\alpha)_{\alpha\in A}\colon |Q(x)| <\omega\}$. We call $\sigma\{X_\alpha\colon \alpha \in A\}$ the σ-product of $\{X_\alpha\colon \alpha \in A\}$ and s the base point of it. And for every a$\in A^{<\omega}$, $\prod\{X_\alpha\colon \alpha \in a\} \times \{\{s_\alpha\}\colon \alpha \in A\text{-}a\}$ is called a finite subproduct of $\sigma\{X_\alpha\colon \alpha \in A\}$.

Definition 5 Let $\mathcal{A}^F=\{\bigcup\mathcal{B}\colon \mathcal{B} \in \mathcal{A}^{<\omega}\}$, the collection $\mathcal{A}$ is said to be directed if $\mathcal{A}^F$ refines $\mathcal{A}$.

Lemma 1 ([1]) If X is a strong Σ-space, then there exists a sequence $\{\mathcal{F}_i\}_{i\in N}$ by locally finite closed covers of X and index set Ω, satisfying:

(a) $\mathcal{F}_i=\{F(\alpha_1,...,\alpha_i)\colon (\alpha_1,...,\alpha_i) \in \Omega^i\}$

(b) For each $(\alpha_1,...,\alpha_i) \in \Omega^i$

$$F(\alpha_1,...,\alpha_i)=\bigcup\{F(\alpha_1,...,\alpha_i,\alpha_{i+1})\colon \alpha_{i+1} \in \Omega\}$$

(c) For each x$\in$ X there is $(\alpha_1,...,\alpha_i,...) \in \Omega^\omega$ such that

(i) x$\in \cap\{F(\alpha_1,...,\alpha_i)\colon i \in N\}$

(ii) C(x)=$\cap\{F\in \mathcal{F}_i\colon$ x$\in$ F and $i \in N\}$ is compact and if U is open in X, C(x)$\subset$ U, then there is $i \in$N such that C(x)$\subset F(\alpha_1,...,\alpha_i) \subset$U.

It is easy to prove the following lemma by Definition 1:

Lemma 2 A topological space X is σ-paralindelof iff every directed open cover has a σ-locally countable open refinement.

The followings are main results in this paper:

Lemma 3 Let $< \mathcal{F}_i=\{F_{i\alpha}\colon \alpha \in A_i\} >_{i\in N}$ is a strong Σ-net of a Σ-space X, if for each $i \in$ N there is a sequence by open collections of X:

$$< \mathcal{V}_{ni}=\{V_{ni\alpha}\colon \alpha \in A_i\} >_{n\in N}$$

such that the following two conditions hold:

(1) $\mathcal{V}_{ni}$ is locally countable for each $(n,i) \in$ N×N

(2) $F_{i\alpha} \subset \bigcup_{n\in N}V_{ni\alpha}$ for each $\alpha \in$A.

Then X is σ-paralindelof.

Proof Let $\mathcal{U}$ be a directed open cover of X. Without loss of generality, we assume that

$$\mathcal{F}_i \supset \{\bigcap\mathcal{F} : \mathcal{F} \in \mathcal{F}_i^{<\omega}\}$$

For each i$\in$ N, put

$$\mathcal{F}_i^* = \{F \in \mathcal{F}_i\colon \text{there is } U(F)\in\mathcal{U} \text{ such that } F\subset U(F)\}$$

then $\mathcal{F}^* = \bigcup_{i\in N}\mathcal{F}_i^*$ is a closed cover of X.

In fact, for each x$\in$ X, since there is U$\in\mathcal{U}$ such that C(x)=$\bigcap\{\bigcap(\mathcal{F}_i)_x\colon i \in N\} \subset$U, then there is i$\in$N such that $\bigcap(\mathcal{F}_i)_x \subset$U and x$\in \bigcap(\mathcal{F}_i^*)_x \in \mathcal{F}_i^*$. Hence $\mathcal{F}^*$ is a cover of X.

Let $\mathcal{F}_i^*=\{F_{i\alpha}\colon \alpha \in B_i\}$ for each $i \in$ N, where $B_i \subset A_i$ and $W_{ni\alpha}=V_{ni\alpha}\bigcap U(F_{i\alpha})$ for $\alpha \in B_i$, i$\in$ N. Then $\mathcal{W}_{ni}=\{W_{ni\alpha}\colon \alpha \in B_i\}$ is locally countable and partly refines $\mathcal{U}$. By (2), $F_{i\alpha} \subset \bigcup_{n\in N} W_{ni\alpha}$ for each $\alpha \in B_i$. Then $\bigcup\{\mathcal{W}_{ni}\colon (n,i) \in N\times N\}$ is a σ-locally countable open refinement of $\mathcal{U}$. □

Theorem 4 If $\{X_p\colon p \in N\}$ is a countable family of σ-paralindelof strong Σ-spaces, then $\prod_{p\in N}X_p$ is a σ-paralindelof strong Σ-space.

Proof Let $< \mathcal{F}_i^p=\{F_{i\alpha}^p\colon \alpha \in A_i^p\} >_{i\in N}$ be a strong Σ-net of X_p, sine X_p is σ-paralindelof and $\mathcal{F}_i^p$ is a closed cover of X_p for each i$\in N$, there is a sequence by open collections of X_p:

$$< \mathcal{V}_{ik}^p=\{V_{ik\alpha}^p\colon \alpha \in A_i^p\} >_{k\in N}$$

such that the following two conditions hold:

(1) Each $\mathcal{V}_{ik}^p$ is locally countable

(2) $F_{i\alpha}^p \subset \bigcup_{k\in N}V_{ik\alpha}^p$ for each $\alpha \in A_i^p$ and $i,p \in N$.

For each $p \in \mathrm{N}$ and each $(i_1, ..., i_p; k_1, ..., k_p) \in \mathrm{N}^p \times \mathrm{N}^p$, let

$\mathcal{V}(i_1, ..., i_p; k_1, ..., k_p) = \mathcal{V}^1_{i_1 k_1} \times \mathcal{V}^2_{i_2 k_2} \times ... \times \mathcal{V}^p_{i_p i_k} \times \prod_{q>p} \mathrm{X}_q$, then

(3) Each $\mathcal{V}(i_1, ..., i_p; k_1, ..., k_p)$ is locally countable in X

(4) $\mathrm{F}^1_{i_1 \alpha_1} \times ... \times \mathrm{F}^p_{i_p \alpha_p} \times \prod_{q>p} \mathrm{X}_q \subset \bigcup \{\mathrm{V}^1_{i_1 k_1 \alpha_1} \times ... \times \mathrm{V}^p_{i_p k_p \alpha_p} \times \prod_{q>p} \mathrm{X}_q\colon (k_1, ..., k_p) \in \mathrm{N}^p\}$.

Since $< \mathcal{F}^1_{i_1} \times \mathcal{F}^2_{i_2} \times ... \times \mathcal{F}^p_{i_p} \times \prod_{q>p} \mathrm{X}_q : (i_1, ..., i_p) \in \mathrm{N}^p$ and $p \in \mathrm{N} >$ is a strong Σ-net of $\prod_{p \in N} \mathrm{X}_p$, by Lemma 3, $\mathrm{X} = \prod_{p \in N} \mathrm{X}_p$ is a σ-paralindelof strong Σ-space. □

Theorem 5 Let X be a σ-paralindelof P-space. If Y is a paracompact Σ-space, then X×Y is a σ-paralindelof space.

Proof Since this proof is essentilly similar to those of [1,Theorem 4.1], we only give a brief statement.

Let $\mathcal{U}$ be a directed open cover of X and

$$< \mathcal{F}_i = \{\mathrm{F}(\alpha_1, ..., \alpha_i) : (\alpha_1, ..., \alpha_i) \in \Omega^i\} >_{i \in N}$$

is a Σ-net satisfying the above Lemma 1. For each $i \in \mathrm{N}$, since $\mathcal{F}_i$ is a locally finite closed cover of paracompact space Y, there is a locally finite open cover

$$\mathcal{H}_i = \{\mathrm{H}(\alpha_1, ..., \alpha_i) : (\alpha_1, ..., \alpha_i) \in \Omega^i\}$$

such that for each $(\alpha_1, ..., \alpha_i) \in \Omega^i$

$$\mathrm{F}(\alpha_1, ..., \alpha_i) \subset \mathrm{H}(\alpha_1, ..., \alpha_i)$$

For each $(\alpha_1, ..., \alpha_i) \in \Omega^i$, we can construct an open collection:

$$\xi(\alpha_1, ..., \alpha_i) = \{\mathrm{V}_\lambda \times \mathrm{W}_\lambda (\neq \phi)\colon \lambda \in \Lambda(\alpha_1, ..., \alpha_i)\}$$

such that the following two conditions hold:

(1) $\xi(\alpha_1, ..., \alpha_i)$ is a partial refinement of $\mathcal{U}$

(2) $\mathrm{F}(\alpha_1, ..., \alpha_i) \subset \mathrm{W}_\lambda \subset \mathrm{H}(\alpha_1, ..., \alpha_i)$ for each $\lambda \in \Lambda(\alpha_1, ..., \alpha_i)$.

Without loss of generality, we assume that $\xi(\alpha_1, ..., \alpha_i)$ is a maximal collection satisfying the conditions (1) and (2). Obviously, $\bigcup \{\xi(\alpha_1, ..., \alpha_i)\colon (\alpha_1, ..., \alpha_i) \in \Omega^i$ and $i \in \mathrm{N}\}$ is an open cover of X×Y. Let $\mathrm{V}(\alpha_1, ..., \alpha_i) = \bigcup \{\mathrm{V}_\lambda\colon \lambda \in \Lambda\, (\alpha_1, ..., \alpha_i)\}$ then

(3) For each $(\alpha_1, ..., \alpha_i, \alpha_{i+1}) \in \Omega^{i+1}$

$$\mathrm{V}(\alpha_1, ..., \alpha_i) \subset \mathrm{V}(\alpha_1, ..., \alpha_i, \alpha_{i+1}).$$

Since X is a P-space, it has a collection $\{\mathrm{C}(\alpha_1, ..., \alpha_i)\colon (\alpha_1, ..., \alpha_i) \in \Omega^i$ and $i \in \omega\}$ of closed sets such that

(4) $\mathrm{C}(\alpha_1, ..., \alpha_i) \subset \mathrm{V}(\alpha_1, ..., \alpha_i)$

(5) $\bigcup_{i \in N} \mathrm{C}(\alpha_1, ..., \alpha_i) = \mathrm{X}$ if $\bigcup_{i \in N} \mathrm{V}(\alpha_1, ..., \alpha_i) = \mathrm{X}$.

Let $\mathcal{V}(\alpha_1, ..., \alpha_i) = \{\mathrm{V}_\lambda\colon \lambda \in \Lambda\, (\alpha_1, ..., \alpha_i)\}$, then

$$\mathcal{V}(\alpha_1, ..., \alpha_i) \bigcup \{\mathrm{X}\text{-}\mathrm{C}(\alpha_1, ..., \alpha_i)\}$$

is an open cover of X and it has a σ-locally countable open refinement

$$\zeta(\alpha_1, ..., \alpha_i) = \bigcup_{n \in N} \zeta_n(\alpha_1, ..., \alpha_i)$$

where $\zeta_n(\alpha_1, ..., \alpha_i) = \{\mathrm{O}(\mathrm{n}, \lambda)\colon \lambda \in \Lambda(\alpha_1, ..., \alpha_i)\} \bigcup \{\mathrm{O}'_n\}$ is locally countable and satisfying:

(6) For each $n \in \mathrm{N}$ and each $\lambda \in \Lambda(\alpha_1, ..., \alpha_i)$

$$\mathrm{O}'_n \subset \mathrm{X}\text{-}\mathrm{C}(\alpha_1, ..., \alpha_i) \text{ and } \mathrm{O}(\mathrm{n}, \lambda) \subset \mathrm{V}_\lambda$$

Put

$$\mathcal{G}_n(\alpha_1, ..., \alpha_i) = \bigcup \{\mathrm{O}(\mathrm{n}, \lambda) \times [\mathrm{W}_\lambda \bigcap \mathrm{H}(\alpha_1, ..., \alpha_i)]\colon \lambda \in \Lambda(\alpha_1, ..., \alpha_i)\}$$

and $\mathcal{G}_{ni}$=$\bigcup\{\mathcal{G}_n(\alpha_1, ..., \alpha_i)$: $(\alpha_1, ..., \alpha_i)\in \Omega^i\}$. Then

(7) $\mathcal{G}_{ni}$ is locally countable.

It is easy to prove the following:

(8) $\bigcup\{\mathcal{G}_{ni}$: $(n, i) \in$ N×N} is an open cover of X×Y

Then $\bigcup\{\mathcal{G}_{ni}$: $(n, i) \in$ N×N} is a σ-locally countable open refinement of $\mathcal{U}$. So, X×Y is a σ-paralindelof space. □

Corollary If a space X is a σ-paralindelof P-space and Y is a meterizable space, then X×Y is a σ-paralindelof.

Now, we research σ-product of σ-paralindelof spaces.

Theorem 6 Let X=$\sigma\{X_\alpha : \alpha \in A\}$. If every finite subproduct of X is σ-paralindelof and X is normal, then X is σ-paralindelof.

Proof Let $\mathcal{U}$ be an open cover, we construct by induction a sequence $\{\mathcal{G}_{nm}$: $(n, m) \in \omega \times \omega\}$ of the collections of open subsets of X and a sequence $\{O_n$: $n \in \omega\}$ of open subsets such that

(1) Each $\mathcal{G}_{nm}$ is a locally countable partial refinement of $\mathcal{U}$

(2) $X_n \subset O_n \subset \widetilde{O}_n \subset \bigcup\{\bigcup \mathcal{G}_{im}$: $i \leq n, m \in \omega\}$, where X_n={x∈ X: $|Q(x)| \leq n\}$.

Put $U_0 \in \mathcal{U}$ such that s∈ U_0, then there is $O_0 \in N(s)$ such that

$$s\in O_0 \subset \widetilde{O}_0 \subset U_0$$

and let $\mathcal{G}_{0m}=\{U_0\}$. Then $\{\mathcal{G}_{0m}:m \in \omega\}$ and $\{O_0\}$ satisfy (1) and (2). Assume that we have constructed $\{\mathcal{G}_{im}:i \leq n$ and $m \in \omega\}$ and $\{O_i\}_{i\leq n}$ satisfying (1) and (2). For each $a \in A^{n+1}$, since Y_a is σ-paralindelof, $\mathcal{U}|Y_a$ has a σ-locally countable open refinement $\bigcup_{m\in\omega} \mathcal{H}'_{am}$

For each $m \in$ N, let $\mathcal{H}_{am}=\{H' - \widetilde{O}_n$: $H' \in \mathcal{H}'_{am}\}$, then

(a) $\mathcal{H}_{am}$ is a partial open refinement of $\mathcal{U}$

(b) Y_n- $\bigcup\{\bigcup \mathcal{G}_{im}$: $i \leq n$ and m∈ $\omega\} \subset \bigcup \mathcal{H}_{am} \subset Y_a$-$\widetilde{O}_n$

For each H∈ $\mathcal{H}_{am}$, there is U(H)∈ $\mathcal{U}$ such that H⊂U(H). Let

$$\mathcal{V}_{am}=\{p_a^{-1}(H)\bigcap U(H)\text{: } H\in \mathcal{H}_{am}\}$$

where p_a: $X\longmapsto Y_a$ as the following for each $x=(x_\alpha)_{\alpha\in A}$,

$$(p_a(x))_\alpha = \begin{cases} x_\alpha, & \alpha \in a \\ s_\alpha, & \alpha \in A - a \end{cases}$$

(c) Each $\mathcal{V}_{am}$ is a locally countable partial open refinement of $\mathcal{U}$

(d) Y_a-$\bigcup\{\bigcup \mathcal{G}_{im}:i \leq n$ and $m \in \omega\} \subset \bigcup_{m\in\omega}(\bigcup \mathcal{V}_{am}) \subset p_a^{-1}(Y_a$-$\widetilde{O}_n)$

Define $\mathcal{G}_{n+1,m}=\bigcup\{\mathcal{V}_{am}:a \in A^{n+1}\}$ for each $m \in$ N. Then

(e) X_{n+1}-$\bigcup\{\bigcup \mathcal{G}_{im}:i \leq n$ and $m \in \omega\} \subset \bigcup_{m\in\omega}(\bigcup \mathcal{G}_{n+1,m})$

The induction is completed if the following is proved

(f) Each $\mathcal{G}_{n+1,m}$ is a locally countable partial refinement of $\mathcal{U}$

In fact, by [5, Lemma 2], $\{p_a^{-1}(Y_a$-$\widetilde{O}_n)$: $a \in A^{n+1}\}$ is locally finite. Then for each x∈X there is $O_x \in N(x)$ such that

$$|\{a \in A^{n+1}\text{: } O_x \bigcap p_a^{-1}(Y_a\text{-}\widetilde{O}_n) \neq \phi\}| < \omega$$

Let

$$\{a \in A^{n+1}\text{: } O_x \bigcap p_a^{-1}(Y_a\text{-}\widetilde{O}_n) \neq \phi\}= \{a_0, a_1, ..., a_k\}$$

where $k \in \omega$.

For each i≤k and $m \in \omega$, by (c), there is $W(x, a_i, m) \in N(x)$ such that

$$|(\mathcal{V}_{a_i m})_{W(x,a_i,m)}| \leq \omega.$$

Put

$$\mathrm{W(x,m)=O}_x \bigcap [\bigcap \{\mathrm{W}(x, a_i, m)\text{: } i \leq k\}]$$

then $\mathrm{W}(x,m) \in \mathrm{N(x)}$ and $|(\mathcal{G}_{n+1,m})_{W(x,m)}| \leq \omega$, i.e., $\mathcal{G}_{n+1,m}$ is locally countable. And by (c) it is a partial refinement of $\mathcal{U}$.

Since X is normal, by (e), there is $\mathrm{O}_{n+1} \in \mathrm{N(X}_{n+1})$ such that

$$\mathrm{X}_{n+1} \subset \mathrm{O}_{n+1} \subset \widetilde{O}_{n+1} \subset \bigcup \{\bigcup \mathcal{G}_{im}\text{: } i \leq n+1, m \in \omega\}$$

The induction is completed.

By (b),

$$\mathrm{X}=\bigcup_{n\in\omega}\mathrm{X}_n \subset \bigcup\{\bigcup \mathcal{G}_{nm}\text{: } (n,m) \in \omega \times \omega\} \subset \mathrm{X}.$$

Hence, $\{\bigcup \mathcal{G}_{nm}\text{: } (n,m) \in \omega \times \omega\}$ is a σ-locally countable open refinement of $\mathcal{U}$. □

The following example shows that the above Theorem 5 doesn't hold if Y is not Σ-space.

Example There is separable metric space and Y a first countable separable lindelof space, but X×Y is not σ-paralindelof.

In [8, 6.11 Example], X is a separable metric space and Y a first countable separable lindelof space, but X×Y is not submetacompact.

New, we show that X×Y is not σ-paralindelof.

Since Y=R with the topology generated by $\tau \bigcup \rho$, where τ is the usual topology in R, then Y is a regular separable lindelof space. If X×Y is σ-paralindelof, then it is a metalindelof space. By the regular separable propety of both X and Y, X×Y is regular separable metalindelof space. Hence, X×Y is regular lindelof. This implies that X×Y is submetacompact, since a regular lindelof space is paracompact. This is a contradiction. □

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