EQUIVALENCE RELATIONS AMONG FURUTA-TYPE INEQUALITIES WITH NEGATIVE POWERS

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Abstract. We show equivalence relations among Furuta-type inequalities in the complementary domain. And also we give an expression which interpolates them.

1. Introduction. A capital letter means a bounded linear operator on a complex Hilbert space H. An operator T is said to be positive (denoted by $T \geq 0$) if $(Tx,x) \geq 0$ for all $x \in H$ and also an operator T is said to be strictly positive (denoted by $T > 0$) if T is positive and invertible. For ordered positive invertible operators, the following results have been given by some authors.

Theorem A ([2][11][13][14]). *If $A \geq B \geq 0$ with $A > 0$, then the following inequalities hold:*

(I) $A^{1-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{1-t}{p-t}}$ *for* $1 \geq p > t \geq 0$ *with* $p \geq \frac{1}{2}$.

(II) $A^{-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{-t}{p-t}}$ *for* $1 \geq t > p \geq 0$ *with* $\frac{1}{2} \geq p$.

(III) $A^{2p-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{2p-t}{p-t}}$ *for* $\frac{1}{2} \geq p > t \geq 0$.

(IV) $A^{2p-1-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{2p-1-t}{p-t}}$ *for* $1 \geq t > p \geq \frac{1}{2}$.

Yoshino [14] solved a part of (I). Afterwards, the domain given by him was enlarged to (I) by Fujii, Kamei and Furuta [2]. Kamei [11] gave simplified proofs of (I) and (III). Tanahashi [13] showed all the inequalities in Theorem A and also proved that the outside exponents of (I),(II) and (IV) are best possible. Extensions of Theorem A are shown in [3][4][8] and [9].

We recall the background of Theorem A. The following Theorem F is an extension of the celebrated Löwner-Heinz theorem: $A \geq B \geq 0$ *ensures* $A^{\alpha} \geq B^{\alpha}$ *for any* $\alpha \in [0,1]$.

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Theorem F (Furuta inequality [5]).

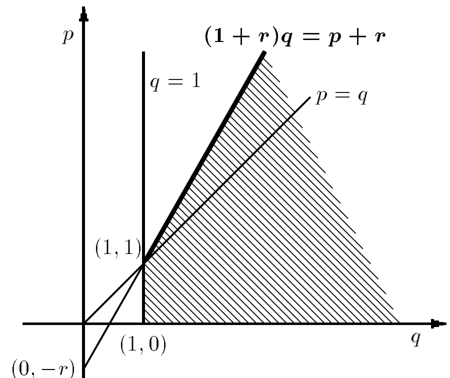
If $A \ge B \ge 0$, then for each $r \ge 0$,

(i) $\qquad (B^{\frac{r}{2}}A^pB^{\frac{r}{2}})^{\frac{1}{q}} \ge (B^{\frac{r}{2}}B^pB^{\frac{r}{2}})^{\frac{1}{q}}$

and

(ii) $\qquad (A^{\frac{r}{2}}A^pA^{\frac{r}{2}})^{\frac{1}{q}} \ge (A^{\frac{r}{2}}B^pA^{\frac{r}{2}})^{\frac{1}{q}}$

hold for $p \ge 0$ and $q \ge 1$ with $(1+r)q \ge p+r$.



Figure

We remark that Theorem F yields Löwner-Heinz theorem when we put $r = 0$ in (i) or (ii) stated above. Alternative proofs of Theorem F are given in [1][10] and also an elementary one-page proof in [6]. It is shown in [12] that the domain drawn for p, q and r in the Figure is best possible one for Theorem F.

In Theorem F, the invertibility of A or B is not assumed. Theorem A can be understood to state that the inequalities in Theorem F remain valid for some negative numbers p,q and r in case A and B are invertible. In fact, the inequalities in Theorem A have the same forms as (ii) in Theorem F by replacing $r = -t \le 0$, and especially, (I) just corresponds to the following inequality:

$$A \ge B \ge 0 \textit{ ensures } A^{1+r} \ge (A^{\frac{r}{2}}B^pA^{\frac{r}{2}})^{\frac{1+r}{p+r}} \textit{ for } p \ge 1 \textit{ and } r \ge 0,$$

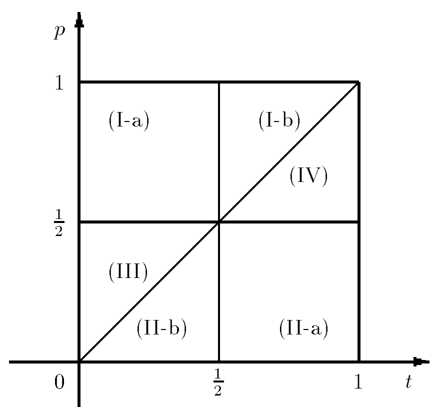
which is the essential part of Theorem F, so that we call (I)–(IV) in Theorem A *Furuta-type* inequalities.

In this paper, we discuss some equivalence relations among the inequalities in Theorem A. And also we give an expression into which the inequalities are unified.

2. Results.

We divide the domain (I) (resp. (II)) in Theorem A into (I-a) (resp. (II-a)) and (I-b) (resp. (II-b)) as follows. Now there are four *triangular* domains and two *square* domains in the Figure.

Firstly, we show a relation among the inequalities which hold in the four *triangular* domains.



Figure

Theorem 1. *The following assertions hold and follow from each other.*

(I-b) *If $A \ge B \ge 0$ with $A > 0$, then $A^{1-t} \ge (A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{1-t}{p-t}}$ for $1 \ge p > t \ge \frac{1}{2}$.*

(II-b) *If $A \ge B \ge 0$ with $A > 0$, then $A^{-t} \ge (A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{-t}{p-t}}$ for $\frac{1}{2} \ge t > p \ge 0$.*

(III) *If $A \ge B \ge 0$ with $A > 0$, then $A^{2p-t} \ge (A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{2p-t}{p-t}}$ for $\frac{1}{2} \ge p > t \ge 0$.*

(IV) *If $A \ge B \ge 0$ with $A > 0$, then $A^{2p-1-t} \ge (A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{2p-1-t}{p-t}}$ for $1 \ge t > p \ge \frac{1}{2}$.*

Secondly, we show a relation between the inequalities which hold in the two *square* domains.

Theorem 2. *The following assertions hold and follow from each other.*

(I-a) *If* $A \geq B \geq 0$ *with* $A > 0$*, then* $A^{1-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{1-t}{p-t}}$ *for* $1 \geq p \geq \frac{1}{2} \geq t \geq 0$ *with* $p > t$*.*

(II-a) *If* $A \geq B \geq 0$ *with* $A > 0$*, then* $A^{-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{-t}{p-t}}$ *for* $1 \geq t \geq \frac{1}{2} \geq p \geq 0$ *with* $t > p$*.*

We remark that each of the assertions in Theorem 1 and Theorem 2 is shown in Theorem A.

Lastly, we give the following interpolating expression.

Theorem 3. *If* $A \geq B \geq 0$ *with* $A > 0$*, then*

$$A^{q-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{q-t}{p-t}} \tag{2.1}$$

holds under the condition (i) *or* (ii)*:*

(i) $2p \geq q \geq p > t \geq 0$ *and* $1 \geq q > 0$*,*

(ii) $1 \geq t > p \geq q \geq 2p - 1$ *and* $1 > q \geq 0$*.*

(I) and (III) in Theorem A are obtained by putting $q = 1$ and $q = 2p$ in (i) of Theorem 3 respectively. And (II) and (IV) are also obtained by putting $q = 0$ and $q = 2p - 1$ in (ii) respectively. We remark that Theorem F is transformed into the form of (2.1) for $p \geq 0$, $t \leq 0$ and $\min\{1, p\} \geq q \geq t$.

3. Proofs of results. We need the following lemmas to give proofs of the results in the previous section.

Lemma 1 ([7])**.** *Let* $A > 0$ *and* B *be an invertible operator. Then*

$$(BAB^*)^{\lambda} = BA^{\frac{1}{2}}(A^{\frac{1}{2}} B^* B A^{\frac{1}{2}})^{\lambda - 1} A^{\frac{1}{2}} B^*$$

holds for any real number λ*.*

Lemma 2. *If* $A \geq B \geq 0$ *with* $A > 0$*, then*

$$A^{-(1-p)s} \geq (A^{\frac{-1}{2}} B^p A^{\frac{-1}{2}})^s$$

holds for $2 \geq s \geq 1$ *and* $1 \geq (1-p)s \geq 0$*.*

Proof of Lemma 2. We may assume that B is also invertible. By Lemma 1 and Löwner-Heinz theorem,

$$\begin{aligned}
(A^{\frac{-1}{2}} B^p A^{\frac{-1}{2}})^s &= A^{\frac{-1}{2}} B^{\frac{p}{2}} (B^{\frac{p}{2}} A^{-1} B^{\frac{p}{2}})^{s-1} B^{\frac{p}{2}} A^{\frac{-1}{2}} \\
&\leq A^{\frac{-1}{2}} B^{\frac{p}{2}} (B^{\frac{p}{2}} B^{-1} B^{\frac{p}{2}})^{s-1} B^{\frac{p}{2}} A^{\frac{-1}{2}} \\
&= A^{\frac{-1}{2}} B^{1-(1-p)s} A^{\frac{-1}{2}} \\
&\leq A^{\frac{-1}{2}} A^{1-(1-p)s} A^{\frac{-1}{2}} \\
&= A^{-(1-p)s}.
\end{aligned}$$

Hence the proof of Lemma 2 is complete. □

Proof of Theorem 1. We may assume that B is also invertible. (I-b),(II-b),(III) and (IV) have been already proved in [2][11][13] and [14] so that we have only to prove the equivalence relation among them. We show (III)$\Longrightarrow$(IV)$\Longrightarrow$(I-b)$\Longrightarrow$(II-b)$\Longrightarrow$(III) as follows.
(III)$\Longrightarrow$(IV). Assume (III). Let $1 \geq t > p \geq \frac{1}{2}$. By putting $s = 2$ in Lemma 2, we have

$$A^{-2(1-p)} \geq (A^{\frac{-1}{2}} B^p A^{\frac{-1}{2}})^2. \tag{3.1}$$

Applying (III) to (3.1), we have

$$\left(A^{-2(1-p)}\right)^{2p_1-t_1} \geq \left\{\left(A^{-2(1-p)}\right)^{\frac{-t_1}{2}} (A^{\frac{-1}{2}} B^p A^{\frac{-1}{2}})^{2p_1} \left(A^{-2(1-p)}\right)^{\frac{-t_1}{2}}\right\}^{\frac{2p_1-t_1}{p_1-t_1}} \quad \text{for } \frac{1}{2} \geq p_1 > t_1 \geq 0. \tag{3.2}$$

Putting $p_1 = \frac{1}{2}$ in (3.2), we have

$$A^{-2(1-p)(1-t_1)} \geq (A^{\frac{2(1-p)t_1-1}{2}} B^p A^{\frac{2(1-p)t_1-1}{2}})^{\frac{1-t_1}{\frac{1}{2}-t_1}} \quad \text{for } \frac{1}{2} > t_1 \geq 0. \tag{3.3}$$

We can put $t_1 = \frac{1-t}{2(1-p)}$ in (3.3) since $1-p > 1-t \geq 0$, so that $\frac{1}{2} > \frac{1-t}{2(1-p)} \geq 0$. Then we have

$$A^{2p-1-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{2p-1-t}{p-t}}. \quad \text{for } 1 \geq t > p \geq \frac{1}{2}.$$

Hence the proof of (III)$\Longrightarrow$(IV) is complete.
(IV)$\Longrightarrow$(I-b). Assume (IV). Let $1 \geq p > t \geq \frac{1}{2}$. (IV) is equivalent to that the hypothesis $A \geq B > 0$ implies the following:

$$(B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{2t-1-p}{t-p}} \geq B^{2t-1-p}. \tag{3.4}$$

(3.4) yields the following (3.5) by Löwner-Heinz theorem and then taking inverses since $-2t+1+p > 1-p \geq 0$, so that $\frac{1-p}{2t-1-p} \in [-1, 0]$:

$$B^{1-p} \geq (B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{1-p}{t-p}}. \tag{3.5}$$

Then we have

$$\begin{aligned}
(A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{1-t}{p-t}} &= A^{\frac{-t}{2}} B^{\frac{p}{2}} (B^{\frac{p}{2}} A^{-t} B^{\frac{p}{2}})^{\frac{1-p}{p-t}} B^{\frac{p}{2}} A^{\frac{-t}{2}} && \text{by Lemma 1} \\
&\leq A^{\frac{-t}{2}} B^{\frac{p}{2}} B^{1-p} B^{\frac{p}{2}} A^{\frac{-t}{2}} && \text{by (3.5)} \\
&= A^{\frac{-t}{2}} B A^{\frac{-t}{2}} \\
&\leq A^{1-t}.
\end{aligned}$$

Hence the proof of (IV)$\Longrightarrow$(I-b) is complete.
(I-b)$\Longrightarrow$(II-b). Assume (I-b). Let $\frac{1}{2} \geq t > p \geq 0$. By putting $s = \frac{1}{1-p}$ in Lemma 2, we have

$$A^{-1} \geq (A^{\frac{-1}{2}} B^p A^{\frac{-1}{2}})^{\frac{1}{1-p}}. \tag{3.6}$$

Applying (I-b) to (3.6), we have

(3.7)

$$(A^{-1})^{1-t_1} \geq \left\{(A^{-1})^{\frac{-t_1}{2}} (A^{\frac{-1}{2}} B^p A^{\frac{-1}{2}})^{\frac{p_1}{1-p}} (A^{-1})^{\frac{-t_1}{2}}\right\}^{\frac{1-t_1}{p_1-t_1}} \quad \text{for } 1 \geq p_1 > t_1 \geq \frac{1}{2}.$$

We can put $p_1 = 1-p$ and $t_1 = 1-t$ in (3.7) since $1 \geq 1-p > 1-t \geq \frac{1}{2}$. Then we have

$$A^{-t} \geq (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{-t}{p-t}}. \quad \text{for } \frac{1}{2} \geq t > p \geq 0.$$

Hence the proof of (I-b)$\Longrightarrow$(II-b) is complete.
(II-b)$\Longrightarrow$(III). Assume (II-b). Let $\frac{1}{2} \geq p > t \geq 0$. (II-b) is equivalent to that the hypothesis $A \geq B > 0$ implies the following:

$$(B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{-p}{t-p}} \geq B^{-p}. \tag{3.8}$$

Then we have

$$\begin{aligned}
(A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{2p-t}{p-t}} &= A^{\frac{-t}{2}} B^{\frac{p}{2}} (B^{\frac{p}{2}} A^{-t} B^{\frac{p}{2}})^{\frac{p}{p-t}} B^{\frac{p}{2}} A^{\frac{-t}{2}} && \text{by Lemma 1} \\
&\leq A^{\frac{-t}{2}} B^{\frac{p}{2}} B^p B^{\frac{p}{2}} A^{\frac{-t}{2}} \\
&= A^{\frac{-t}{2}} B^{2p} A^{\frac{-t}{2}} \\
&\leq A^{2p-t}.
\end{aligned}$$

The first inequality holds by (3.8) and then taking inverses, and the last inequality holds by Löwner-Heinz theorem since $2p \in [0,1]$. Hence the proof of (II-b)$\Longrightarrow$(III) is complete. Consequently, the proof of Theorem 1 is complete. □

Proof of Theorem 2. We may assume that B is also invertible. (I-a) and (II-a) have been already proved in [2][11][13] and [14] so that we have only to prove the equivalence relation between them.
(I-a)$\Longrightarrow$(II-a). Assume (I-a). Let $1 \geq t \geq \frac{1}{2} \geq p \geq 0$ and $t > p$. (I-a) is equivalent to that the hypothesis $A \geq B > 0$ implies the following:

$$(B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{1-p}{t-p}} \geq B^{1-p}. \tag{3.9}$$

(3.9) yields the following (3.10) by Löwner-Heinz theorem and then taking inverses since $1-p \geq p \geq 0$ and $1-p > 0$, so that $\frac{-p}{1-p} \in [-1,0]$:

$$B^{-p} \geq (B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{-p}{t-p}}. \tag{3.10}$$

Then we have

$$\begin{aligned}
(A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^{\frac{-t}{p-t}} &= A^{\frac{-t}{2}} B^{\frac{p}{2}} (B^{\frac{p}{2}} A^{-t} B^{\frac{p}{2}})^{\frac{-p}{p-t}} B^{\frac{p}{2}} A^{\frac{-t}{2}} && \text{by Lemma 1} \\
&\leq A^{\frac{-t}{2}} B^{\frac{p}{2}} B^{-p} B^{\frac{p}{2}} A^{\frac{-t}{2}} && \text{by (3.10)} \\
&= A^{-t}.
\end{aligned}$$

Hence the proof of (I-a)$\Longrightarrow$(II-a) is complete.
(II-a)$\Longrightarrow$(I-a). Assume (II-a). Let $1 \geq p \geq \frac{1}{2} \geq t \geq 0$ and $p > t$. (II-a) is equivalent to that the hypothesis $A \geq B > 0$ implies the following:

$$(B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{-p}{t-p}} \geq B^{-p}. \tag{3.11}$$

(3.11) yields the follwing (3.12) by Löwner-Heinz theorem and then taking inverses since $p \geq 1-p \geq 0$ and $p > 0$, so that $\frac{1-p}{-p} \in [-1,0]$:

$$B^{1-p} \geq (B^{\frac{-p}{2}} A^t B^{\frac{-p}{2}})^{\frac{1-p}{t-p}}. \tag{3.12}$$

Then we have

$$\begin{aligned}(A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{1-t}{p-t}} &= A^{\frac{-t}{2}}B^{\frac{p}{2}}(B^{\frac{p}{2}}A^{-t}B^{\frac{p}{2}})^{\frac{1-p}{p-t}}B^{\frac{p}{2}}A^{\frac{-t}{2}} && \text{by Lemma 1}\\ &\le A^{\frac{-t}{2}}B^{\frac{p}{2}}B^{1-p}B^{\frac{p}{2}}A^{\frac{-t}{2}} && \text{by (3.12)}\\ &= A^{\frac{-t}{2}}BA^{\frac{-t}{2}}\\ &\le A^{1-t}.\end{aligned}$$

Hence the proof of (II-a)$\Longrightarrow$(I-a) is complete.
Consequently, the proof of Theorem 2 is complete. □

Proof of Theorem 3. We may assume that B is also invertible.
Case (i). Let $2p \ge q \ge p > t \ge 0$ and $1 \ge q > 0$. The hypothesis $A \ge B > 0$ implies $A^q \ge B^q > 0$ by Löwner-Heinz theorem. Then applying (I) in Theorem A, we have

$$(A^q)^{1-t_1} \ge \left\{(A^q)^{\frac{-t_1}{2}}(B^q)^{p_1}(A^q)^{\frac{-t_1}{2}}\right\}^{\frac{1-t_1}{p_1-t_1}} \quad \text{for } 2p_1 \ge 1 \ge p_1 > t_1 \ge 0. \tag{3.13}$$

We can put $p_1 = \frac{p}{q}$ and $t_1 = \frac{t}{q}$ in (3.13) since $\frac{2p}{q} \ge 1 \ge \frac{p}{q} > \frac{t}{q} \ge 0$. Then we have

$$A^{q-t} \ge (A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{q-t}{p-t}}.$$

Hence the proof of case (i) is complete.
Case (ii). Let $1 \ge t > p \ge q \ge 2p-1$ and $1 > q \ge 0$. We can put $s = \frac{1-q}{1-p}$ in Lemma 2 since $2(1-p) \ge 1-q \ge 1-p$, so that $2 \ge \frac{1-q}{1-p} \ge 1$ and $1 \ge 1-q > 0$. Then we have

$$A^{-(1-q)} \ge (A^{\frac{-1}{2}}B^pA^{\frac{-1}{2}})^{\frac{1-q}{1-p}}. \tag{3.14}$$

Applying (I) in Theorem A to (3.14), we have

$$\left(A^{-(1-q)}\right)^{1-t_1} \ge \left\{\left(A^{-(1-q)}\right)^{\frac{-t_1}{2}}\left(A^{\frac{-1}{2}}B^pA^{\frac{-1}{2}}\right)^{\frac{(1-q)p_1}{1-p}}\left(A^{-(1-q)}\right)^{\frac{-t_1}{2}}\right\}^{\frac{1-t_1}{p_1-t_1}} \quad \text{for } 2p_1 \ge 1 \ge p_1 > t_1 \ge 0. \tag{3.15}$$

We can put $p_1 = \frac{1-p}{1-q}$ and $t_1 = \frac{1-t}{1-q}$ in (3.15) since $2(1-p) \ge 1-q \ge 1-p > 1-t \ge 0$, so that $2p_1 \ge 1 \ge p_1 > t_1 \ge 0$. Then we have

$$A^{q-t} \ge (A^{\frac{-t}{2}}B^pA^{\frac{-t}{2}})^{\frac{q-t}{p-t}}.$$

Hence the proof of case (ii) is complete.
Consequently, the proof of Theorem 3 is complete. □

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