ON BCI–ALGEBRAS WITH CONDITION (S)

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ABSTRACT. In the present paper we prove that, for a BCI-algebra X with condition (S), if the p-semisimple part of X is a subalgebra of X then X is isomorphic to direct product of a BCK-algebra with condition (S) and a p-semisimple BCI-algebra, and obtain other results on such algebras. Moreover we show that the concepts of regular ideals, closed p-ideals and stong ideals coincide.

1. INTRODUCTION

The study of BCK/BCI-algebras was initiated by Iséki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Since then a great deal of literature has been produced on the theory of BCK/BCI-algebras, in particular, emphasis seems to have been put on the ideal theory of BCK/BCI-algebras. For the general development of BCK/BCI-algebras the ideal theory plays an important role. In the present paper we first give a structure theorem of BCI-algebras, more precisely, we prove that, for a BCI-algebra X with condition (S), $L(X)$ is a subalgebra of X if and only if there are a BCK-algebra Y with condition (S) and a p-semisimple BCI-algebra Z such that $X \cong Y \times Z$. Next we discuss the relation of several ideals in BCI-algebras, in particular, we show that the concepts of regular ideals, closed p-ideals and strong ideals coincide. Finally we investigate the BCK-part and the p-semisimple part in quotient algebras of BCI-algebras via ideals.

2. PRELIMINARIES

We review some definitions and properties that will be useful in our results.

By a *BCI-algebra* we mean an algebra $(X, *, 0)$ of type (2,0) satisfying the following conditions:

(I) $((x * y) * (x * z)) * (z * y) = 0$,
(II) $(x * (x * y)) * y = 0$,
(III) $x * x = 0$,
(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$.

A BCI-algebra X satisfying $0 \leq x$ for all $x \in X$ is called a *BCK-algebra*. In any BCI-algebra X one can define a partial order $\leq$ by putting $x \leq y$ if and only if $x * y = 0$.

A BCI-algebra X has the following properties for any $x, y, z \in X$:

(1) $x * 0 = x$,
(2) $(x * y) * z = (x * z) * y$,
(3) $x \leq y$ implies that $x * z \leq y * z$ and $z * y \leq z * x$,

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(4) $(x*z)*(y*z) \leq x*y$,
(5) $x*(x*(x*y)) = x*y$,
(6) $0*(x*y) = (0*x)*(0*y)$.

A nonempty subset I of X is called an *ideal* of X if it satisfies
(i) $0 \in I$,
(ii) $x*y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X$.

Any ideal I has the property: $y \in I$ and $x \leq y$ imply $x \in I$.

In general, an ideal I of X need not be a subalgebra. If I is also a subalgebra of X, we say that I is a *closed ideal*, equivalently, an ideal I is closed if and only if $0*x \in I$ whenever $x \in I$.

Meng and Xin [13] systematically investigated the theory of atoms and branchs of BCI-algebras. An element a of X is called an *atom* if, for all $x \in X$, $x*a = 0$ implies $x = a$, that is, a is a minimal element of $(X, \leq)$. Obviously, 0 is an atom of X. The sets $B(X) := \{x \in X | 0 \leq x\}$ and $L(X) := \{a \in X | \ a$ is an atom of $X\}$ are called *BCK-part* and *p-semisimple part* of X, respectively. We know that $(B(X), *, 0)$ is a BCK-algebra, and $(L(X), *, 0)$ is a p-semisimple BCI-algebra. For any $a \in L(X)$, the set $V(a) := \{x \in X | a \leq x\}$ is called a *branch* of X. It is clear that $V(0) = B(X)$. For short, we denote $a_x = 0*(0*x)$. With respect to atoms and branchs of X we have:
(7) a is an atom of X if and only if $0*(0*a) = a$, i.e., $a = a_a$,
(8) a is an atom of X if and only if, for all $x \in X$, $a = x*(x*a)$,
(9) if a and b are atoms of X, then for all $x \in V(b)$ we have $a*x = a*b$, in particular, $0*x = 0*a_x$,
(10) if $x \in V(a)$ and $y \in V(b)$, then $x*y \in V(a*b)$,
(11) x and y belong to the same branch if and only if $x*y \in B(X)$.

3. BCI-algebras with condition (S)

In 1980, Iséki generalized the notion of BCK-algebras with condition (S) to BCI-algebras. This is a class of important algebraic systems. We first recall some definitions and results.

Definition 3.1 (Iséki [6])**.** A BCK/BCI-algebra X is said to be *with condition* (S) if the set

$$A(a,b) := \{x \in X | x*a \leq b\}$$

has a greatest element, written $a \circ b$, for all $a, b \in X$

Note that $(X, \circ, 0)$ is a commutative semigroup in which 0 is a unit. We know that a BCI-algebra with condition (S) have two binary operations "$*$" and "$\circ$", hence it has three classes of subalgebras and homomorphisms (isomorphisms).

Definition 3.2. Let X be a BCI-algebra with condition (S) and A a nonempty subset of X. A is a $*$-*subalgebra* (resp. $\circ$-*subalgebra*) of X if $x*y \in A$ (resp. $x \circ y \in A$) whenever $x, y \in A$. If A is both a $*$-subalgebra and a $\circ$-subalgebra of X, we say that A is a *subalgebra* of X.

In general, a $*$-subalgebra may not be a $\circ$-subalgebra as shown in the following example.

Example 3.3. Let $X = \{0, a, b\}$ in which $*$ and $\circ$ tables are given as follows:

$*$	0	a	b
0	0	a	0
a	a	0	a
b	b	a	0

$\circ$	0	a	b
0	0	a	b
a	a	b	a
b	b	a	b

Then $(X, *, 0)$ is a BCI-algebra with condition (S). Observe that $L(X) = \{0, a\}$, and we know that $L(X)$ is a $*$-subalgebra of X. But $L(X)$ is not a $\circ$-subalgebra of X, since $a \circ a = b \notin L(X)$.

In the sequel we will see that if $L(X)$ is a subalgebra of a BCI-algebra X with condition (S), then X have a valuable structure. To establish this important result we need some preparations.

Definition 3.4. Suppose X and Y are BCI-algebras with condition (S). A mapping $f : X \to Y$ is called a *$*$-homomorphism* (resp. *$\circ$-homomorphism)* if $f(x * y) = f(x) * f(y)$ (resp. $f(x \circ y) = f(x) \circ f(y)$) for all $x, y \in X$. If f is both a $*$-homomorphism and a $\circ$-homomorphism, we say that f is a *homomorphism*. Likely, we can define $*$-isomorphism, $\circ$-isomorphism and isomorphism.

Note that every $*$-homomorphism $f : X \to Y$ is order presevering, i.e., $x \leq y$ implies $f(x) \leq f(y)$ for all $x, y \in X$.

Proposition 3.5. *Suppose X and Y are BCI-algebras with condition (S) and $f : X \to Y$ is a $*$-homomorphism. Then $f(x \circ y) \leq f(x) \circ f(y)$ for all $x, y \in X$.*

Proof. Let $x, y \in X$. Note that $(x \circ y) * x \leq y$ and so

$$f(x \circ y) * f(x) = f((x \circ y) * x) \leq f(y).$$

Hence $f(x \circ y) \leq f(x) \circ f(y)$, ending the proof. □

The following result shows that a $*$-isomorphism of BCI-algebras with condition (S) must be an isomorphism.

Proposition 3.6. *Let X and Y be BCI-algebras with condition (S). If $f : X \to Y$ is a $*$-isomorphism, then $f(x \circ y) = f(x) \circ f(y)$ for all $x, y \in X$.*

Proof. By Proposition 3.5, we have $f(x \circ y) \leq f(x) \circ f(y)$ for all $x, y \in X$. On the other hand, f^{-1} is a $*$-homomorphism from Y onto X, and so

$$f^{-1}(f(x) \circ f(y)) \leq f^{-1}(f(x)) \circ f^{-1}(f(y)) = x \circ y.$$

Thus $f(x) \circ f(y) = f(f^{-1}(f(x) \circ f(y))) \leq f(x \circ y)$. ¿From the above two inequalities, we conclude that $f(x) \circ f(y) = f(x \circ y)$. This completes the proof. □

Note that, in a BCI-algebra X with condition (S), the identity $(x * y) * z = x * (y \circ z)$ holds.

Proposition 3.7 (Liu [7]). *Each p-semisimple BCI-algebra X is with condition (S) and $x \circ y = x * (0 * y)$ for all $x, y \in X$.*

Proposition 3.8. *Suppose X is a BCI-algebra with condition (S) and let $L(X)$ is a subalgebra of X. Then $a_{x \circ y} = a_x * (0 * a_y) = a_x \circ a_y$ for all $x, y \in X$.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned} a_{x \circ y} &= 0 * (0 * (x \circ y)) \\ &= 0 * ((0 * x) * y) \\ &= (0 * (0 * x)) * (0 * y) \\ &= a_x * (0 * a_y) \\ &= a_x \circ a_y. \end{aligned}$$

This completes the proof. □

Remark. In Proposition 3.8, we can not remove the condition "$L(X)$ is a subalgebra of X". In fact, the condition guarantees the last equality in the proof of Proposition 3.8, i.e., if $L(X)$ is not a subalgebra of X then the last equality in the proof of Proposition 3.8 may not hold. For example, the set $L(X) = \{0, a\}$ in Example 3.3 is not a subalgebra and $a \circ a = b \neq a * (0 * a) = a * a = 0$.

Proposition 3.9. *Let X be a BCI-algebra with condition (S). Then $L(X)$ is a subalgebra of X if and only if $x = (x * a) * (0 * a)$ for all $x \in X$ and $a \in L(X)$.*

Proof. Suppose $L(X)$ is a subalgebra of X. Then

$$\begin{aligned}(x * a) * (0 * a) &= x * (a \circ (0 * a)) \\ &= x * (a * (0 * (0 * a))) \qquad \text{[by Proposition 3.8]} \\ &= x * (a * a) = x.\end{aligned}$$

Conversely, assume that $x = (x * a) * (0 * a)$ for all $x \in X$ and $a \in L(X)$ and let $a, b \in L(X)$. Then $a \circ b = ((a \circ b) * b) * (0 * b)$. Since $(a \circ b) * b \leq a$, it follows that $a \circ b \leq a * (0 * b)$. Note that $a * (0 * b) \in L(X)$, and so $a \circ b = a * (0 * b) \in L(X)$. Therefore $L(X)$ is a subalgebra of X. This completes the proof. $\square$

Proposition 3.10. *Let X be a BCI-algebra with condition (S). Then $L(X)$ is a subalgebra of X if and only if $(x * a) * (y * b) = (x * y) * (a * b)$ for all $x, y \in X$ and $a, b \in L(X)$.*

Proof. Suppose $L(X)$ is a subalgebra of X. Since

$$\begin{aligned}&(((x * y) * (a * b)) * ((x * a) * (y * b))) * a \\ &= (((x * a) * ((x * a) * (y * b))) * y) * (a * b) \\ &\leq ((y * b) * y) * (a * b) \\ &= (0 * b) * (a * b) \\ &\leq 0 * a,\end{aligned}$$

it follows from Proposition 3.7 that

$$((x * y) * (a * b)) * ((x * a) * (y * b)) \leq a \circ (0 * a) = a * (0 * (0 * a)) = a * a = 0.$$

Hence

$$((x * y) * (a * b)) * ((x * a) * (y * b)) = 0$$

or

$$(x * y) * (a * b) \leq (x * a) * (y * b).$$

On the other hand, since

$$\begin{aligned}&(((x * a) * (y * b)) * ((x * y) * (a * b))) * (a * b) \\ &= (((x * (a * b)) * ((x * y) * (a * b))) * (y * b)) * a \\ &\leq ((x * (x * y)) * (y * b)) * a \\ &\leq (y * (y * b)) * a \\ &\leq b * a = 0 * (a * b),\end{aligned}$$

we obtain

$$\begin{aligned}
&((x*a)*(y*b))*((x*y)*(a*b))\\
&\le (a*b)\circ(0*(a*b))\\
&=(a*b)*(0*(0*(a*b)))\\
&=(a*b)*(a*b)\\
&=0.
\end{aligned}$$

Thus $((x*a)*(y*b))*((x*y)*(a*b))=0$ or $(x*a)*(y*b)\le(x*y)*(a*b)$. Consequently we have $(x*a)*(y*b)=(x*y)*(a*b)$.

Conversely, suppose $(x*a)*(y*b)=(x*y)*(a*b)$ for all $x,y\in X$ and $a,b\in L(X)$. Then

$$(x*a)*(0*a)=(x*0)*(a*a)=x.$$

It follows from Proposition 3.9 that $L(X)$ is a subalgebra of X. This completes the proof. □

As usual, we can define direct product of two BCI-algebras with condition (S), and such direct product is also a BCI-algebra with condition (S). By Proposition 3.7 we know that each p-semisimple BCI-algebra is with condition (S), so if X is a BCK-algebra with condition (S) and Y is a p-semisimple BCI-algebra, then $X\times Y$ is with condition (S) where $(x,y)\circ(x',y')=(x\circ x',y\circ y')$ and $(x,y)*(x',y')=(x*x',y*y')$ for all $x,x'\in X$ and $y,y'\in Y$. In the sequel we say that $X\times Y$ is a $K\times L$ product BCI-algebra with condition (S). Now we give one of main results in this paper.

Theorem 3.11. *Suppose that X is a BCI-algebra with condition (S) and $L(X)$ is a subalgebra of X. Then there are a BCK-algebra Y with condition (S) and a p-semisimple BCI-algebra Z such that $X\cong Y\times Z$.*

Proof. As it is well known, $B(X)$ is a BCK-algebra with condition (S) and $L(X)$ is a p-semisimple BCI-algebra. Hence it is enough to prove $X\cong B(X)\times L(X)$. Let $g:X\to B(X)\times L(X)$ be such that $g(x):=(x*a_x,a_x)$ for all $x\in X$. Let $y\in B(X)$ and $z\in L(X)$. Putting $x=y*(0*z)$, then

$$a_x=a_{y*(0*z)}=a_y*(a_0*a_z)=0*(0*z)=z$$

and

$$\begin{aligned}
g(x)&=(x*a_x,a_x)\\
&=((y*(0*z))*z,z)\\
&=(y*((0*z)\circ z),z)\\
&=(y*((0*z)*(0*z)),z) &&\text{[by Proposition 3.7]}\\
&=(y,z),
\end{aligned}$$

which shows that g is surjective. For any $x,y\in X$ we have

$$\begin{aligned}
g(x*y)&=((x*y)*a_{x*y},a_{x*y})\\
&=((x*y)*(a_x*a_y),a_x*a_y)\\
&=((x*a_x)*(y*a_y),a_x*a_y) &&\text{[by Proposition 3.10]}\\
&=(x*a_x,a_x)*(y*a_y,a_y)\\
&=g(x)*g(y).
\end{aligned}$$

Hence g is a $*$-homomorphism. Moreover if $g(x) = g(y)$ then $(x * a_x, a_x) = (y * a_y, a_y)$, i.e., $x * a_x = y * a_y$ and $a_x = a_y$. From Proposition 3.9 it follows that

$$x = (x * a_x) * (0 * a_x) = (y * a_y) * (0 * a_y) = y.$$

Therefore g is one to one. Summarizing the above facts we have proved that g is a $*$-isomorphism from X to $B(X) \times L(X)$. It follows from Proposition 3.6 that $g(x \circ y) = g(x) \circ g(y)$ for all $x, y \in X$. Therefore $g : X \to B(X) \times L(X)$ is an isomorphism. Hence $X \cong B(X) \times L(X)$, ending the proof. □

The converse of Theorem 3.11 is also true as state in the following.

Theorem 3.12. *Suppose X is a BCI-algebra with condition (S). If there are a BCK-algebra Y with condition (S) and a p-semisimple BCI-algebra Z such that $X \cong Y \times Z$, then $L(X)$ is a subalgebra of X.*

Proof. Note that $L(Y \times Z) = \{(0, z) | z \in Z\}$ is a subalgebra of $Y \times Z$ and $L(Y \times Z) \cong L(X)$. Hence $L(X)$ is a subalgebra of X. This completes the proof. □

4. p-IDEALS AND REGULAR IDEALS IN BCI-ALGEBRAS

The notion of ideals in BCI-algebras was introduced by Iséki([4]). Since then several special cases of ideals are studied. In this section the relation between p-ideals and regular ideals will be clarified.

An ideal I of a BCI-algebra X is said to be *regular* (Meng [8]) if $x * y \in I$ and $x \in I$ imply $y \in I$. A subset I of a BCI-algebra X is called a *p-ideal* (Zhang et al. [15]) if it satisfies

(i) $0 \in I$,

(ii) $(x * z) * (y * z) \in I$ and $y \in I$ imply $x \in I$ for all $x, y, z \in X$.

Proposition 4.1 (Meng et al. [11]). *An ideal I of a BCI-algebra X is a p-ideal if and only if $B(X) \subseteq I$.*

Proposition 4.2. *Every regular ideal of BCI-algebras must be closed.*

Proof. Straightforward. □

In what follows we give the main result of this section.

Theorem 4.3. *Let I be an ideal of a BCI-algebra X. Then I is regular if and only if I is a closed p-ideal.*

Proof. Suppose I is regular. It is sufficient to show by Proposition 4.2 that I is a p-ideal. Let $x \in B(X)$. Then $0 * x = 0 \in I$. Since I is regular, it follows that $x \in I$. This proves that $B(X) \subseteq I$, and hence I is a p-ideal by Proposition 4.1.

Conversely, let I be a closed p-ideal. Assume $x * y \in I$ and $x \in I$. Then $0 * y = (x * y) * x \in I$, and so $0 * (0 * y) \in I$ by closeness of I. Since y and $0 * (0 * y)$ belong to the same branch, therefore $y * (0 * (0 * y)) \in B(X) \subseteq I$. It follows from $0 * (0 * y) \in I$ that $y \in I$. Therefore I is a regular ideal. This completes the proof. □

Since every ideal of a p-semisimple BCI-algebra is a p-ideal, we have the following result as a special case of the theorem above.

Corollary 4.4. *Every closed ideal I of a p-semisimple BCI-algebra X is regular.*

Proof. Straightforward. □

An ideal I of a BCI-algebra X is said to be *strong* (Bhatti [1]) if $x \in I$ and $a \in X \setminus I$ imply $x * a \in X \setminus I$. In [3, Theorem 11] Hong, Jun and Meng obtained that an ideal of a BCI-algebra is strong if and only if it is a closed p-ideal. Combining Theorem 4.3, we have the following result.

Theorem 4.5. *In BCI-algebras, the concept of regular ideals, strong ideals and closed p-ideals coincide.*

5. Atoms in quotient algebras

Meng and Xin [13] systematically investigated the theory of atoms and branchs in BCI-algebras, which is a useful tool for studying BCI-algebras. In this section we will discuss the problem which is relative to atoms in quotient algebras. For an ideal I of a BCI-algebra X, denote $L(I) := \{0 * (0 * x) | x \in I\}$. Meng and Wei [12] gave that $L(I) = L(X) \cap I$, and I is closed if and only if $L(I)$ is a closed ideal of the p-semisimple BCI-algebra $L(X)$.

Let I be a closed ideal of a BCI-algebra X. A binary relation $\sim$ on X defined by $x \sim y$ if and only if $x*y \in I$ and $y*x \in I$ is a congruence relation on X. Denote $C_x := \{y \in X | x \sim y\}$ and define $C_x * C_y = C_{x*y}$ for all $x, y \in X$. Then $C_0 = I$ and $(X/I, *, I)$ is a BCI-algebra where $X/I := \{C_x | x \in X\}$.

Theorem 5.1. *Let I be a closed ideal of a BCI-algebra X. Then*
(12) *$C_x \in B(X/I)$ if and only if $a_x \in L(I)$,*
(13) *$C_x \in L(X/I)$ and $C_x \neq I$ imply $a_x \in L(X) \setminus L(I)$,*
(14) *$a \in L(X) \setminus L(I)$ implies $C_a \in L(X/I)$ and $C_a \neq I$.*

Proof. (12) Assume that $C_x \in B(X/I)$. Then $C_0 \leq C_x$, and so $C_{0*x} = C_0 * C_x = C_0 = I$, i.e., $0 * x \in I$. Since I is a closed ideal, it follows that $a_x = 0 * (0 * x) \in I$. Note that $a_x \in L(X)$ and $L(I) = L(X) \cap I$, so that $a_x \in L(I)$.

Conversely let $a_x \in L(I)$. Since x and a_x belong to the same branch, therefore $x * a_x \in B(X)$. Thus $x * a_x \geq 0$, and so $C_x * C_{a_x} \geq C_0$. From $a_x \in I$ we have $C_{a_x} = C_0$, hence $C_x = C_x * C_0 = C_x * C_{a_x} \geq C_0$. This shows that $C_x \in B(X/I)$. Therefore (12) holds.

(13) Let $I \neq C_x \in L(X/I)$, i.e., C_x is a nonzero atom in X/I. Then $C_x \neq C_0$, and hence it needs to consider only two cases: $0*x \in I$ and $0*x \notin I$ for every $x \in X$. If $0*x \notin I$, then $0 * a_x \notin I$ since $0 * x = 0 * a_x$. Since I is a closed ideal, it follows that $a_x = 0 * (0 * a_x) \notin I$. Combining $a_x \in L(X)$, we obtain $a_x \in L(X) \setminus L(I)$. If $0 * x \in I$, then $a_x = 0 * (0 * x) \in I$ because I is closed. Since a_x is an atom of X, we get $a_x \in L(I)$. It follows from (12) that $C_x \in B(X/I)$, i.e., $C_x = C_0$, a contradiction.

(14) Let $a \in L(X) \setminus L(I)$ and assume $C_x * C_a = C_0$. Then $C_{x*a} = C_0$, and so $x * a \in I$. Since I is a closed ideal, we have

$$\begin{aligned} a * x &= (0 * (0 * a)) * x && \text{[by (7)]} \\ &= (0 * x) * (0 * a) && \text{[by (2)]} \\ &= 0 * (x * a) \in I. && \text{[by (6)]} \end{aligned}$$

Hence $a \sim x$, and so $C_x = C_a$. This shows that C_a is an atom of X/I. Obviously, $C_a \neq I$. This completes the proof. □

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