ON KEHAYOPULU'S THEOREMS IN *po*–SEMIGROUPS

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ABSTRACT. In this paper, we give an improvement of Kehayopulu's Theorems in *po*-semigroups ([4]).

The concept of interior ideal of a semigroup S has been introduced by Lajos([1]). Szasz studied the interior ideals of semigroups in [10,11]. Kehayopulu introduced the concepts of interior ideals in *po*-semigroups and the interior ideal elements in *poe*-semigroups in [3,4]. In [4], Kehayopulu showed that the ideals and the interior ideals coincide in regular(resp. interior regular) *po*-semigroups, and the ideal elements and the interior ideal elements coincide in *poe*-semigroups.

In this paper, we show that the ideals and the interior ideals coincide in g-regular *po*-semigroups, and the ideal elements and the interior ideal elements coincide in g-regular *poe*-semigroups.

A *po*-semigroup(: ordered semigroup) is an ordered set $(S, \leq)$ at the same time a semigroup such that $a \leq b \Longrightarrow ca \leq cb$ and $ac \leq bc$ for all $c \in S$. A *poe*-semigroup is a *po*-semigroup having a greatest element e.

If S is a *po*-semigroup, then for $H \subseteq S$, we denote $(H] := \{t \in S | t \leq h \text{ for some } h \in H\}$.

A *po*-semigroup S is called *regular* if for every $a \in S$, there exists $x \in S$ such that $a \leq axa$. Equivalent Definition: $A \subseteq (ASA]$ for all subset A of S ([6]). A *po*-semigroup S is called *left* (resp. *right*) *regular* if if for every $a \in S$, there exists $x \in S$ such that $a \leq xa^2$ (resp. $a \leq a^2x$). Equivalent Definition: $A \subseteq (SA^2]$ (resp. $A \subseteq (A^2S]$) for all subset A of S. A *po*-semigroup S is called *strongly regular* if for every $a \in S$, there exists $x \in S$ such that $a \leq axa$ and $ax = xa$ ([5]). A *po*-semigroup S is called *intra-regular* if for every $a \in S$, there exists $x \in S$ such that $a \leq xa^2x$ ([4,7]). Equivalent Definition: $A \subseteq (SA^2S]$ for all subset A of S. A non-empty subset A of a *po*-semigroup S is called a *left* (resp. *right*) *ideal* of S if 1) $SA \subseteq A$ (resp. $AS \subseteq A$). 2) $a \in A,\ b \leq a$ for $b \in S$ imply $b \in A$ ([6]). The non-empty subset A of a *po*-semigroup S is an *ideal* of S if it is both a left and a right ideal of S.

A *poe*-semigroup S is *regular* if $a \leq aea$ for all $a \in S$. An *poe*-semigroup S is called *left* (resp. *right*) *regular* if $a \leq ea^2$ (resp. $a \leq a^2e$) for all $a \in S$. An *poe*-semigroup S is called *strongly regular* if $a \in S$, $a \leq aea$ and $ae = ea$ for all $a \in S$. An *poe*-semigroup S is called *intra-regular* if $a \leq ea^2e$ for all $a \in S$. An element a of a *po*-groupoid S is called a *left* (resp. *right*) *ideal element* of S if $xa \leq a$ (resp. $ax \leq a$) for all $x \in S$. An element a is an *ideal element* if it is both a left and a right ideal element. In particular, if the *po*-groupoid S is

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a *poe*-semigroup, then the element a of S is a left (resp. right) ideal element of S if $ea \leq a$ (resp. $ae \leq a$).

For the necessary definitions we also refer to [7,8].

Definition 1 ([3,4]). Let S be a *po*-semigroup, $\emptyset \neq I \subseteq S$. The set I is called an *interior ideal* of S if 1) $SIS \subseteq I$. 2) $a \in I, b \leq a$ for $b \in S$ imply $b \in I$.

Remark 1([4]). Every ideal of a *po*-semigroup S is an interior ideal of S. In fact: Since $I^2 \subseteq IS \subseteq I$ for ideal I, I is a subsemigroup of S. Moreover $S(IS) \subseteq SI \subseteq I$.

The author introduced a quasi-completely regular *po*-semigroup in [2].

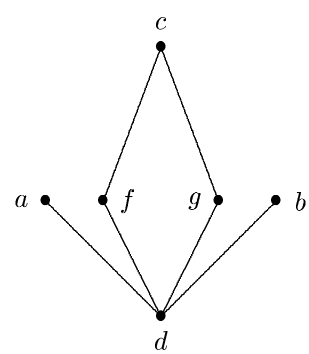
Definition 2. A *po*-semigroup S is called *quasi-completely regular* if for every $a \in S$, there exist $x, y, x', y' \in S$ such that $a \leq xaya$ or $a \leq ax'ay'$.

Now we give new concept.

Definition 3. An *po*-semigroup S is called *g-regular* if for every $a \in S$,there exist $x, y \in S$ such that $a \leq xay$.

Example([9]). Let $S := \{a, b, c, d, f, g\}$ be a *po*-semigroup with Cayley table and Hasse diagram as follows:

$\cdot$	a	b	c	d	f	g
a	b	b	a	d	a	a
b	b	b	b	d	b	b
c	a	b	c	d	c	c
d	d	d	d	d	d	d
f	a	b	c	d	c	c
g	a	b	c	d	f	g



Then S is g-regular. In fact: For $a, f \in S$, we have $cac = ac = a \geq a$ and $fff = cf = c \geq f$. For other elemenets, it is trivial.

But S is not quasi completely regular and not intra-regular.

We can show the following lemma easily.

Lemma. 1) *If a po-semigroup S is regular, then it is quasi-completely regular.*
2) *If a po-semigroup S is left(right) regular, then it is quasi-completely regular.*
3) *If a po-semigroup S is intra regular, then it is g-regular.*
4) *If a po-semigroup S is quasi-completely regular, then it is g-regular.*

Now we show that the intra ideals and ideals coincide in a g-regular *po*-semigroup. These are an improvement of Kehayopulu's Theorems(Proposition 1, and 2 in [4]) in *po*-semigroups.

Proposition 1. *In g-regular po-semigroups, the ideals and the interior ideals coincide.*

Proof. Let S be a g-regular *po*-semigroup and I an interior ideal of S. Since S is g-regular, we get $I \subseteq (SIS]$. Then we have

$$IS \subseteq (SIS]S = (SIS](S] \subseteq (SIS^2] \subseteq (SIS] \subseteq (I] = I$$

and

$$SI \subseteq S(SIS] = (S](SIS] \subseteq (S^2IS] \subseteq (SIS] \subseteq (I] = I.$$

Hence I is an ideal of S.

Using the Lemma, we get the following colollaries.

Corollary 1([4]). *In regular (intra-regular) ordered semigroups, the ideals and the interior ideals coincide.*

Corollary 2. *In quasi-completely (left, right, strongly) regular ordered semigroups, the ideals and the interior ideals coincide.*

Definition 4. An *poe*-semigroup S is called *quasi-completely regular* if for every $a \in S$, $a \leq eaea$ or $a \leq aeae$. An *poe*-semigroup S is called *g-regular* if for every $a \in S$, $a \leq eae$.

Remark 2. Every ideal element in *poe*-semigroups is an interior ideal element.

Now we show that in a g-regular *poe*-semigroup, the interior ideal elements and ideal elements coincide. This is an improvementof Kehayopulu's Theorems(Proposition 3, and 4 in [4]) in *poe*-semigroups.

Proposition 2. *In g-regular poe-semigroups, the ideal elements and the interior ideal elements coincide.*

Proof. Let S be a g-regular *poe*-semigroup and a an interior ideal element of S. Since S is g-regular, we get $a \leq eae$. Then we have

$$ae \leq (eae)e \leq eae^2 \leq eae \leq a \qquad \text{and} \qquad ea \leq e(eae) \leq e^2ae \leq eae \leq a.$$

Hence a is an ideal element of S.

Using the Lemma, we get the following corollaries.

Corollary 3([4]). *In regular (intra-regular) poe-semigroups, the ideal elements and the interior ideal elements coincide.*

Corollary 4. *In quasi-completely (left, right, strongly) regular poe-semigroups, the ideal elements and the interior ideal elements coincide.*

References

1. S. Lajos, *(m, k, n)-ideals in semigroups*, In: Notes on Semigroups, Karl Marx Univ. Econ., Dept. Math. Budapest, **No. 1** (1976), 12-19.
2. Sang Keun Lee and Young In Kwon, *On completely regular and quasi-completely regular ordered semigroups*, Math. Japon., **47(2)** (1998), 247-251.
3. N. Kehayopulu, *Interior ideals and interior ideal elements in ordered semigroups*, PU. M. A., **40(3)** (1999), 323-329.
4. N. Kehayopulu, *Note on interior ideals, ideal elements in ordered semigroups*, Scincetiae Mathematicae, **2(3)** (1999), 407-409.
5. N. Kehayopulu, *A note on strongly regular ordered semigroups*, Scincetiae Mathematicae, **1(1)** (1998), 33-36.
6. N. Kehayopulu, *On regular ordered semigroups*, Math. Japon., **45(3)** (1997), 549-553.
7. N. Kehayopulu, *On intra-regular ordered semigroups*, Semigroup Forum, **46** (1993), 271-278.
8. N. Kehayopulu, *On intra-regular Ve-semigroups*, Semigroup Forum, **19** (1980), 111-121.
9. N. Kehayopulu and M. Tsingelis, *Remark on ordered semigroups*, Sovremennaja Algebra, St. Petersburg Gos. Ped. Herzen Inst., [In: Decompositions and Homomorphic Mappings of Semigroups.] (1992), 50-55.
10. G. Szasz, *Interior ideals in semigroups*, In: Notes on Semigroups, Karl Marx Univ. Econ., Dept. Math. Budapest, **No 5** (1977), 1-7.
11. G. Szasz, *Remark on interior ideals of semigroups*, Studia Scient. Math. Hung., **16** (1981), 61-63.

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