NOTE ON PROPER BCI–ALGEBRAS WITH ORDERS FIVE AND SIX

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ABSTRACT. It is showed that I_{5-3-10} and I_{5-3-11} given in [1] are isomorphic. Some problems appeared in [2] are also discussed.

The first author has determined all proper BCI-algebras of order $n \leq 5$ in [1]. But owing to a neglect in the computation, I_{5-3-10} and I_{5-3-11} given in [1] are in fact isomorphic. Therefore I_{5-3-11} should be deleted from the classification table. The multiplication table of I_{5-3-10} and I_{5-3-11} are showed here by Table 1 and Table 2 respectively.

$*$	0	1	2	3	4
0	0	0	0	3	3
1	1	0	1	4	3
2	2	2	0	3	3
3	3	3	3	0	0
4	4	3	4	1	0

T*able* 1

I_{5-3-10}

$*$	0	1	2	3	4
0	0	0	0	3	3
1	1	0	1	3	3
2	2	2	0	4	3
3	3	3	3	0	0
4	4	4	3	2	0

T*able* 2

I_{5-3-11}

If we rewrite Table 2 by changing the order of the figures 1 and 2, then we obtain Table 3.

$*$	0	2	1	3	4
0	0	0	0	3	3
2	2	0	2	4	3
1	1	1	0	3	3
3	3	3	3	0	0
4	4	3	4	2	0

Table 3

Now, compare Table 3 with Table 1, it is easy to see that I_{5-3-11} is isomorphic to I_{5-3-10}. We examine carefully the classification tables given in [1] and find that all other proper BCI-algebras given in [1] are indeed not isomorphic to each other. J.Meng, Y.B.Jun and E.H.Roh have determined all proper BCI-algebras of order 6 in [2]. Let X be a proper BCI-algebra of order 6. By $B(X)$ we denote its BCK-part, and by $P(X)$ its p-semisimple part which is composed of all minimal elements of X. If $X = B(X) \cup_L P(X)$, i.e.X is the Li Xin union algebra of $B(X)$ and $P(X)$ (see [1]), then we call its construction to be simple, otherwise its construction to be complicated. In [2] the Cayley tables and Hasse diagrams of all 69 complicated proper BCI-algebras of order 6 are given. By I_{6-r} we mean the algebra given by the rth Cayley table in [2].

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We find that Table 37 and Table 38 given in [2] are just the same. There must be some mistakes, since I_{6-37} and I_{6-38} are not isomorphic. Here we show a proper BCI-algebra of order six, its Cayley table is showed by Table 4. We denote this algebra by X. The Cayley table of I_{6-37} is showed by Table 5.

$*$	0	1	2	3	4	5
0	0	0	0	3	3	3
1	1	0	0	3	3	3
2	2	1	0	4	3	3
3	3	3	3	0	0	0
4	4	3	3	1	0	0
5	5	3	3	1	1	0

T*able* 4 (X)

$*$	0	1	2	3	4	5
0	0	0	0	3	3	3
1	1	0	0	3	3	3
2	2	1	0	3	3	3
3	3	3	3	0	0	0
4	4	3	3	1	0	0
5	5	3	3	1	1	0

T*able* 5 (I_{6-37})

These two tables are different only on one point: In Table 4, $2 * 3 = 4$, but in Table 5, we have $2 * 3 = 3$. By routine calculation we can find that X contains two subalgebras of order five, i.e.,{0,1,2,3,4} and {0,1,2,3,5}, which are of the type I_{5-3-4}. But I_{6-37} contains two subalgebras of order five, i.e., {0,1,2,3,4} and {0,1,3,4,5}, which are of type I_{5-3-5}.(For the meaning of the symbols I_{5-3-4} and I_{5-3-5}, the reader is referred to [1].) So X is not isomorphic to I_{6-37}. We belive that X is the I_{6-38} in [2]. There is a misprint in its Cayley table given in [2] on the point $2 * 3$.

Table 62 and Table 63 given in [2] are also the same. By similar calculation, we find that there is a misprint in Table 63 on the point $1 * 2$, it should equal 2 instead of 5. The correct Cayley table of I_{6-63} is showed by Table 6.

$*$	0	1	2	3	4	5
0	0	0	2	3	4	2
1	1	0	2	3	4	2
2	2	2	0	4	3	0
3	3	3	4	0	2	4
4	4	4	3	2	0	3
5	5	2	1	4	3	0

Table 6

The Cayley tables of I_{6-62} and I_{6-63} are different only on one point: In I_{6-62} we have $1 * 2 = 5$, but in I_{6-63} we have $1 * 2 = 2$. These two algebras are indeed not isomorphic, since I_{6-63} contains a subalgebra $\mathrm{I}_{5-2-1} =$ {0,1,2,3,4}, but I_{6-62} contains no subalgebra of order five.

References

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