THE UP REVERSED HAZARD RATE STOCHASTIC ORDER

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Abstract. A stochastic order consisting of a shifted version of the well-known reversed hazard rate order is proposed. Namely, for two continuous random variables X and Y we say that X is smaller than Y in the up reversed hazard rate order, denoted as $X \le_{\mathrm{rh}\uparrow} Y$, if $X - x \le_{\mathrm{rh}} Y$ for each $x \ge 0$. Some properties of such order are presented, including the preservation under *(i)* transformations by strictly monotone convex functions, *(ii)* formation of coherent systems, *(iii)* Poisson shock models.

1 Introduction Recently Lillo *et al.* (2000) have studied in detail four shifted stochastic orders, namely the up likelihood ratio order, the down likelihood ratio order, the up hazard rate order, and the down hazard rate order, that have been obtained starting from the well-known likelihood ratio order and hazard rate order.

Along a similar line, in this paper we propose the 'up reversed hazard rate order', that is a shifted version of the reversed hazard rate order. This will be defined in Section 2 where some of its properties will also be discussed, including a preservation result concerning transformations of random variables by strictly monotone convex functions. The preservation of this order under the formation of coherent systems and under Poisson shock models is then presented in Section 3 and in Section 4, respectively.

Throughout this paper 'increasing' and 'decreasing' mean respectively 'nondecreasing' and 'nonincreasing'. Also, we use the convention $a/\infty = 0$ and $a/0 = \infty$ if $a > 0$. Moreover, '$=_{\mathrm{d}}$' denotes equality in law.

2 The up reversed hazard rate order Let X and Y be continuous random variables having respectively distribution function F_X and F_Y, survival function $\overline{F}_X$ and $\overline{F}_Y$, and support (l_X, u_X) and (l_Y, u_Y), with $-\infty \le l_X < u_X \le \infty$ and $-\infty \le l_Y < u_Y \le \infty$. When X and Y will be assumed absolutely continuous, their probability density functions will be denoted respectively by f_X and f_Y.

Let us preliminarly recall the definitions of some well-known stochastic orders (see, for instance, Shaked and Shanthikumar (1994)): X is said to be smaller than Y in the

- likelihood ratio order (denoted by $X \le_{\mathrm{lr}} Y$) if $f_X(t)/f_Y(t)$ is decreasing in $t \in (l_X, u_Y)$, provided that X and Y are absolutely continuous;
- hazard rate order (denoted by $X \le_{\mathrm{hr}} Y$) if $\overline{F}_X(t)/\overline{F}_Y(t)$ is decreasing in $t \in (-\infty, \max\{u_X, u_Y\})$;
- reversed hazard rate order (denoted by $X \le_{\mathrm{rh}} Y$) if

$$(1) \qquad \frac{F_X(t)}{F_Y(t)} \quad \text{is decreasing in } t \in (\min\{l_X, l_Y\}, \infty).$$

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Recently, Lillo *et al.* (2000) have analysed four stochastic orders obtained as shifted versions of $\leq_{\rm lr}$-order and $\leq_{\rm hr}$-order. Let us now recall two of such orders; we say that X is smaller than Y in the

- up likelihood ratio order (denoted by $X \leq_{\rm lr\uparrow} Y$) if $X - x \leq_{\rm lr} Y$ for each $x \geq 0$ or, equivalently, if

$$(2) \qquad \frac{f_X(t+x)}{f_Y(t)} \quad \text{is decreasing in } t \in (l_X - x, u_X - x) \cup (l_Y, u_Y) \quad \text{for all } x \geq 0,$$

  provided that X and Y are absolutely continuous;

- up hazard rate order (denoted by $X \leq_{\rm hr\uparrow} Y$) if $X - x \leq_{\rm hr} Y$ for each $x \geq 0$ or, equivalently, if

$$(3) \qquad \frac{\overline{F}_X(t+x)}{\overline{F}_Y(t)} \quad \text{is decreasing in } t \in (-\infty, u_Y) \quad \text{for all } x \geq 0.$$

Along a similar line, hereafter we define the 'up' shifted version of the reversed hazard rate order.

Definition 2.1 *Let X and Y be continuous random variables; X is said to be smaller than Y in the up reversed hazard rate order (denoted by $X \leq_{\rm rh\uparrow} Y$) if*

$$(4) \qquad X - x \leq_{\rm rh} Y \quad \textit{for each } x \geq 0.$$

Rewriting (4) by means of (1) one has the following result.

Theorem 2.1 *Let X and Y be continuous random variables; then $X \leq_{\rm rh\uparrow} Y$ if and only if*

$$(5) \qquad \frac{F_X(t+x)}{F_Y(t)} \quad \textit{is decreasing in } t \in (l_X, \infty) \quad \textit{for all } x \geq 0.$$

In Section 3 we shall make use of the following result, that is an immediate consequence of Theorem 2.1.

Corollary 2.1 *Let X and Y be continuous random variables; if $X \leq_{\rm rh\uparrow} Y$, then*

$$\overline{F}_X(t+x) \leq \overline{F}_Y(t) \quad \textit{for all } t \in (l_X, \infty) \textit{ and for all } x \geq 0.$$

In order to analyse some properties of $\leq_{\rm rh\uparrow}$-order let us recall that a distribution function F_X is said to be *logconcave* if $\{t \in \mathbf{R}: F_X(t) > 0\}$ is an interval of the form (l_X, ∞), with $-\infty \leq l_X < \infty$, and if $\log F_X$ is concave on (l_X, ∞); that is, if

$$(6) \qquad \frac{F_X(t+x)}{F_X(t)} \quad \text{is decreasing in } t \in (l_X, \infty) \quad \text{for all } x \geq 0.$$

Making use of Definition 2.1 and Theorem 2.1 it is not hard to prove the properties stated in the following

Proposition 2.1 *(i) (reflexivity) $X \leq_{\rm rh\uparrow} X$ if and only if F_X is logconcave;*

(ii) (transitivity) if $X \leq_{\rm rh\uparrow} Y$ and $Y \leq_{\rm rh\uparrow} Z$ then $X \leq_{\rm rh\uparrow} Z$;

(iii) (antisymmetry) if $X \leq_{\rm rh\uparrow} Y$ and $Y \leq_{\rm rh\uparrow} X$ then $X =_{\rm d} Y$ if and only if F_X is logconcave;

Hereafter we shall analyse some connections between $\leq_{\mathrm{rh}\uparrow}$-order and $\leq_{\mathrm{rh}}$-order. From (1) and (5) it easily follows that

$$X \leq_{\mathrm{rh}\uparrow} Y \quad \Longrightarrow \quad X \leq_{\mathrm{rh}} Y.$$

In the following theorem we give conditions under which the reciprocal implication holds. To this purpose, it is useful to recall that (1) gives

$$X \leq_{\mathrm{rh}} Y \quad \Longrightarrow \quad l_X \leq l_Y \ \text{ and } \ u_X \leq u_Y. \tag{7}$$

Theorem 2.2 *Let X and Y be continuous random variables. If X or Y have logconcave distribution function, and if $X \leq_{\mathrm{rh}} Y$, then $X \leq_{\mathrm{rh}\uparrow} Y$.*

Proof. Assume that F_X is logconcave. From (7) we have $l_X \leq l_Y$ and $u_X \leq u_Y$. For any fixed $x \geq 0$ and for $t \geq l_X$ it is

$$\frac{F_X(t+x)}{F_Y(t)} = \frac{F_X(t+x)}{F_X(t)}\,\frac{F_X(t)}{F_Y(t)}. \tag{8}$$

Due to (6) and (1), the given assumptions imply that the ratios at the right-hand-side of (8) are both decreasing in $t \in (l_X, \infty)$. Hence, recalling Theorem 2.1 we obtain $X \leq_{\mathrm{rh}\uparrow} Y$. When F_Y is logconcave the proof is similar. ∎

The following example shows that the logconcavity of F_X or F_Y is an essential assumption of Theorem 2.2.

Example 2.1 Assume that X has distribution function (see Block *et al.*, 1998)

$$F_X(t) = \exp\left\{-1-\frac{1}{t}\right\} \cdot \mathbf{1}_{(0,1]}(t) + \exp\left\{\frac{t^2-5}{2}\right\} \cdot \mathbf{1}_{(1,2]}(t) + \exp\left\{-\frac{1}{t}\right\} \cdot \mathbf{1}_{(2,\infty)}(t)$$

and that Y has distribution function

$$F_Y(t) = [F_X(t)]^{3/2}, \qquad t \in \mathbf{R},$$

where $\mathbf{1}_A(t)$ is the indicator function of the set A. From (1) it then follows $X \leq_{\mathrm{rh}} Y$, with X and Y having proportional reversed hazard rate functions (see Gupta *et al.*, 1998, or Di Crescenzo, 2000). However, Theorem 2.2 cannot be applied being F_X and F_Y not logconcave. Indeed, noting that $F_X(t+1)/F_Y(t)$ is not monotone for $t \in (0,\infty)$, Theorem 2.1 gives that $X \not\leq_{\mathrm{rh}\uparrow} Y$. Finally, being not hard to prove the stronger relation $X \leq_{\mathrm{lr}} Y$, this example also shows that

$$X \leq_{\mathrm{lr}} Y \quad \not\Longrightarrow \quad X \leq_{\mathrm{rh}\uparrow} Y.$$

Let us now give the following important preservation result.

Theorem 2.3 *Let X and Y be continuous random variables; if $X \leq_{\mathrm{rh}\uparrow} Y$ then*

(i) $\varphi(X) \leq_{\mathrm{rh}\uparrow} \varphi(Y)$ for any strictly increasing convex function φ;

(ii) $\varphi(Y) \leq_{\mathrm{hr}\uparrow} \varphi(X)$ for any strictly decreasing convex function φ.

Proof. Denote by φ^{-1} the inverse function of φ. In order to prove (i), note that from the strict monotonicity of φ we have

$$\frac{F_{\varphi(X)}(t+x)}{F_{\varphi(Y)}(t)} = \frac{F_X(\varphi^{-1}(t+x))}{F_Y(\varphi^{-1}(t))} = \frac{F_X(\varphi^{-1}(t)+\psi(t,x))}{F_Y(\varphi^{-1}(t))}, \tag{9}$$

where $\psi(t,x) := \varphi^{-1}(t+x) - \varphi^{-1}(t)$. Since φ^{-1} is increasing, due to Theorem 2.1 the ratio $F_X(\varphi^{-1}(t)+u)/F_Y(\varphi^{-1}(t))$ is decreasing in t for each $u \geq 0$. Also $\psi(t,x) \geq 0$ is decreasing in t due to the convexity of φ. Hence, the ratios in (9) are decreasing in t for all $x \geq 0$, so that $\varphi(X) \leq_{\mathrm{rh}\uparrow} \varphi(Y)$ by Theorem 2.1.

Let us prove (ii). Since φ is strictly decreasing, one has:

$$\frac{\overline{F}_{\varphi(Y)}(t+x)}{\overline{F}_{\varphi(X)}(t)} = \frac{F_Y(\varphi^{-1}(t+x))}{F_X(\varphi^{-1}(t))} = \frac{F_Y(\varphi^{-1}(t)+\psi(t,x))}{F_X(\varphi^{-1}(t))}, \tag{10}$$

with $\psi(t,x)$ defined as above. It is not hard to see that assumption $X \leq_{\mathrm{rh}\uparrow} Y$ is equivalent to the following condition:

$$\frac{F_Y(t+x)}{F_X(t)} \quad \text{is increasing in } t \in (l_X, \infty) \quad \text{for all } x \leq 0.$$

Since φ^{-1} has now been assumed to be decreasing, the ratio $F_Y(\varphi^{-1}(t)+u)/F_X(\varphi^{-1}(t))$ is then decreasing in t for each $u \leq 0$. Moreover, $\psi(t,x) \leq 0$ is still decreasing in t by the convexity of φ, so that the ratios (10) are decreasing in t for all $x \geq 0$. Due to (3), we finally have $\varphi(Y) \leq_{\mathrm{hr}\uparrow} \varphi(X)$. ■

It should be mentioned that the validity of Theorem 2.3 has been suggested by some interesting similar relations between $\leq_{\mathrm{hr}}$-order and $\leq_{\mathrm{rh}}$-order that have been given by Nanda and Shaked (2000). Other relations concerning monotone transformations and dual stochastic orders have also been given in Theorem 3.1 of Di Crescenzo and Ricciardi (1996).

Hereafter we shall obtain a characterization result about relation $X \leq_{\mathrm{rh}\uparrow} Y$ when X and Y are absolutely continuous. To this purpose, recall that the reversed hazard rate function of an absolutely continuous random variable Z is given by

$$\tau_Z(z) = \frac{d}{dz}\log F_Z(z) = \frac{f_Z(z)}{F_Z(z)}, \qquad z \in (l_Z, u_Z), \tag{11}$$

and that if X and Y are absolutely continuous then

$$X \leq_{\mathrm{rh}} Y \quad \Longleftrightarrow \quad \tau_X(t) \leq \tau_Y(t) \quad \text{for all } \ t \in (l_Y, u_X). \tag{12}$$

Note that the reversed hazard rate has been receiving increasing attention in the recent literature of reliability analysis and stochastic modeling (see, for instance, the papers by Block *et al.* (1998) and by Chandra and Roy (2001)).

Theorem 2.4 *Let X and Y be absolutely continuous random variables. Then $X \leq_{\mathrm{rh}\uparrow} Y$ if and only if there exists a random variable Z with a logconcave distribution function such that $X \leq_{\mathrm{rh}} Z \leq_{\mathrm{rh}} Y$.*

Proof. Let $X \leq_{\mathrm{rh}\uparrow} Y$. If $u_X \leq l_Y$ and Z is any random variable with a logconcave distribution function on $[u_X, l_Y]$, then $X \leq_{\mathrm{rh}} Z \leq_{\mathrm{rh}} Y$. Hence, let us assume that $l_Y < u_X$. Denoting by τ_X and τ_Y the reversed hazard rate functions of X and Y, respectively, it is not hard to see that condition (5) can be expressed as $\tau_Y(t) \geq \tau_X(t+x)$ for all $x \geq 0$ and $l_Y < t < u_X - x$; that is:

$$\tau_Y(v) \geq \tau_X(w) \qquad \text{for } l_Y < v \leq w < u_X. \tag{13}$$

Set now $\widetilde{\tau}(z) = \max_{w \geq z} \tau_X(w)$, $z \in (l_X, u_X)$. Then, $\widetilde{\tau}(z) \geq 0$ for $z \in (l_X, u_X)$ and $\int_{l_X}^{u_X} \widetilde{\tau}(z)\, dz = \infty$, due to relation

$$\widetilde{\tau}(z) \geq \tau_X(z) \quad \text{for } z \in (l_X, u_X). \tag{14}$$

Such conditions ensure that $\widetilde{\tau}$ is a reversed hazard rate function. Let us assume that Z has hazard rate function $\widetilde{\tau}$. As $\widetilde{\tau}$ is decreasing, due to (11) Z has a logconcave distribution function. Moreover, from (12) and (14) one has $X \leq_{\rm hr} Z$. From (13) it is $\tau_Y(v) \geq \max_{w \geq v} \tau_X(w) \equiv \widetilde{\tau}(v)$ so that Eq. (12) finally implies $Z \leq_{\rm hr} Y$.

In order to prove the sufficienty part of the theorem, assume that $X \leq_{\rm hr} Z \leq_{\rm hr} Y$, with Z having a logconcave distribution function. Then, Theorem 2.2 gives $X \leq_{\rm hr\uparrow} Z \leq_{\rm hr\uparrow} Y$. Thus, due to the transitivity property expressed by (iii) of Proposition 2.1, we have $X \leq_{\rm hr\uparrow} Y$. ■

Note that if X and Y are absolutely continuous random variables, due to (13) Theorem 2.4 yields the following equivalence:

$$X \leq_{\rm hr\uparrow} Y \iff \tau_Y(v) \geq \tau_X(w) \quad \text{for } l_Y < v \leq w < u_X, \tag{15}$$

use of which will be made in the proof of Theorem 3.1

Let us now give a characterization of relation $X \leq_{\rm rh\uparrow} Y$ for non-positive random variables. As usual, hereafter $[X - x \,|\, X < x]$ denotes the 'past life' of X, i.e. a random variable whose distribution is the same as the conditional distribution of $X - x$ given that $X < x$.

Theorem 2.5 *Let X and Y be non-positive continuous random variables; then $X \leq_{\rm rh\uparrow} Y$ if and only if*

$$[X - x \,|\, X < x] \leq_{\rm rh} Y \quad \textit{for all } x \in (l_X, u_X).$$

Proof. For brevity we limit the proof to the case when X and Y have the same support. Recalling that the distribution function of $[X - x \,|\, X < x]$ is given by $G(t) = F_X(x + t)/F_X(x)$ if $t < 0$ and by $G(t) = 1$ otherwise, we have that $G(t)/F_Y(t)$ is decreasing in $t \in (l_X, u_X)$ if and only if

$$\frac{F_X(t + x)}{F_Y(t)} \quad \text{is decreasing in } t \in (l_X, u_X).$$

The proof then follows as an immediate consequence of (1) and (5). ■

We conclude this section by pointing out the relations holding among the three shifted orders considered in this paper and the corresponding usual ones in the case of absolutely continuous random variables X and Y:

$$\begin{array}{ccccc} X \leq_{\rm hr\uparrow} Y & \Longleftarrow & X \leq_{\rm lr\uparrow} Y & \Longrightarrow & X \leq_{\rm rh\uparrow} Y \\ \Downarrow & & \Downarrow & & \Downarrow \\ X \leq_{\rm hr} Y & \Longleftarrow & X \leq_{\rm lr} Y & \Longrightarrow & X \leq_{\rm rh} Y \end{array}$$

3 Comparison of coherent systems Consider a coherent system consisting of n components, where the i-th component is characterized by an absolutely continuous random lifetime X_i, with survival function $\overline{F}_{X_i}(t)$. Following Barlow and Proschan (1965), let the system reliability function be given by $h(p_1, p_2, \ldots, p_n)$. The random lifetime of the system will be denoted by $h(\mathbf{X})$, and its reversed hazard rate function by $\tau_{h(\mathbf{X})}(t)$.

We aim to compare the above system with a system with identical structure, the random lifetimes of its components being now identically distributed to an absolutely continuous random lifetime Y. Thus, $h(Y)$ will denote the random lifetime of this system and $\tau_{h(Y)}(t)$ its reversed hazard rate function.

In the following theorem we compare the random lifetimes of the two systems according to the $\leq_{\rm rh\uparrow}$-order. This is an analogous of Theorem 3.1 of Nanda *et al.* (1998), where the same result is presented in the case of $\leq_{\rm rh}$-order.

Theorem 3.1 *If*

$$\sum_{i=1}^{n} \frac{(1-p_i)\,\partial h/\partial p_i}{1-h(\mathbf{p})} \quad \textit{is increasing in } p_i \textit{ for all } i=1,2,\ldots,n, \tag{16}$$

then $h(\mathbf{X}) \leq_{\mathrm{rh}\uparrow} h(Y)$ *whenever* $X_i \leq_{\mathrm{rh}\uparrow} Y$, $i=1,2,\ldots,n$.

Proof. Recalling the expression of the reversed hazard rate function of a coherent system (see (3.1) of Nanda *et al.* (1998), for instance), for $z>0$ and $t \geq 0$ we have:

$$\tau_{h(\mathbf{X})}(z+t) = \sum_{i=1}^{n} \tau_{X_i}(z+t)\left[1-\overline{F}_{X_i}(z+t)\right] \left.\frac{\partial h/\partial p_i}{1-h(\mathbf{p})}\right|_{p_i=\overline{F}_{X_i}(z+t)}.$$

Hence, due to relation (15), hypothesis $X_i \leq_{\mathrm{rh}\uparrow} Y$ gives

$$\tau_{h(\mathbf{X})}(z+t) \leq \tau_Y(z) \sum_{i=1}^{n} \left[1-\overline{F}_{X_i}(z+t)\right] \left.\frac{\partial h/\partial p_i}{1-h(\mathbf{p})}\right|_{p_i=\overline{F}_{X_i}(z+t)}.$$

Since the sum in the right-hand-side is increasing in p_i by assumption (16), recalling that Corollary 2.1 gives $\overline{F}_{X_i}(z+t) \leq \overline{F}_Y(z)$, we obtain

$$\tau_{h(\mathbf{X})}(z+t) \leq \tau_Y(z) \sum_{i=1}^{n} \left[1-\overline{F}_Y(z)\right] \left.\frac{\partial h/\partial p_i}{1-h(\mathbf{p})}\right|_{p_i=\overline{F}_Y(z)} \equiv \tau_{h(Y)}(z).$$

This concludes the proof by virtue of (15). ∎

As noted in Remark 3.2 of Nanda *et al.* (1998), the assumption (16) is satisfied by several coherent systems, including the k-out-of-n one (as proved in Theorem 3.2 of Nanda *et al.* (1998)).

4 Comparison of Poisson shock models Consider two devices, both subjected to shocks randomly occurring according to a Poisson process of intensity λ. Let P_k and Q_k denote respectively the probability that the first and the second device will not survive the first k shocks. Then, denoting by X and Y the random lifetimes of the two devices, their distribution functions are given by:

$$F_X(t) = \sum_{k=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!}\, P_k, \qquad F_Y(t) = \sum_{k=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!}\, Q_k, \qquad t \geq 0. \tag{17}$$

Numerous results on preservation of stochastic orderings under Poisson shock models have been presented in the literature (see, for instance, Singh and Jain (1989) and Kebir (1994)). Along the same line, in the following theorem we give conditions on P_k and Q_k such that lifetimes X and Y are ordered according to $\leq_{\mathrm{rh}\uparrow}$. The proof is based on classical results of total positivity (see Karlin (1968)).

Theorem 4.1 *If*

$$\frac{P_{k+j}}{Q_k} \textit{ is decreasing in } k \textit{ for all } j \geq 0, \tag{18}$$

then $X \leq_{\mathrm{rh}\uparrow} Y$.

Proof. From (17), for any real c, we have

$$F_X(t+s) - c\,F_Y(t) = \sum_{j=0}^{\infty} \frac{e^{-\lambda s}(\lambda s)^j}{j!} \sum_{k=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!} (P_{k+j} - c\,Q_j). \tag{19}$$

Assumption (18) implies that $P_{k+j} - c\,Q_j$ has at most one change of sign; if one change occurs, it occurs from + to −. Making use of the variation diminishing property of $e^{-\lambda t}(\lambda t)^k/k!$, due to (19) the same statement holds for $F_X(t+s) - c\,F_Y(t)$ as a function of t. This implies that $F_X(t+s)/F_Y(t)$ is decreasing in $t \geq 0$. The proof then follows from Theorem 2.1. ■

We note that, on the grounds of Theorem 2.1, assumption (18) can be viewed as a condition leading to the up reversed hazard rate ordering between integer-valued random variables.

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References

[1] Barlow, R.E. and Proschan, F. (1965), *Mathematical Theory of Reliability* (John Wiley & Sons, New York).

[2] Block, H.W., Savits, T.H. and Singh, H. (1998), The reversed hazard rate function, *Prob. Engin. Inf. Sci.* **12**, 69–90.

[3] Chandra, N.K. and Roy, D. (2001), Some results on reversed hazard rate, *Prob. Engin. Inf. Sci.* **15**, 95–102.

[4] Di Crescenzo, A. (2000), Some results on the proportional reversed hazards model, *Stat. Prob. Lett.* **50**, 313–321.

[5] Di Crescenzo, A. and Ricciardi, L.M. (1996), Comparing first-passage times for semi-Markov skip-free processes. *Stat. Prob. Lett.* **30**, 247–256.

[6] Gupta, R.C., Gupta, R.D. and Gupta, P.L. (1998), Modeling failure time data by Lehman alternatives, *Commun. Statist. Theory Meth.* **27**, 887–904.

[7] Karlin, S. (1968), *Total Positivity*, Vol. I, (Stanford University Press, Stanford).

[8] Kebir, Y. (1994), Order-preserving shock models, *Prob. Engin. Inf. Sci.* **8**, 125–134.

[9] Lillo, R.E., Nanda, A.K. and Shaked, M. (2001), Some shifted stochastic orders, In: *Recent Advances in Reliability Theory: Methodology, Practice and Inference*, (Limnios N. and Nikulin M., eds.), pp. 85–103, Birkhäuser, Boston.

[10] Nanda, A.K., Jain, K. and Singh, H. (1998), Preservation of some partial orderings under the formation of coherent systems, *Stat. Prob. Lett.* **39**, 123–131.

[11] Nanda, A.K. and Shaked, M. (2000), The hazard rate and the reversed hazard rate orders, with applications to order statistics. Preprint.

[12] Shaked, M. and Shanthikumar, J.G. (1994), *Stochastic Orders and Their Applications* (Academic Press, San Diego).

[13] Singh, H. and Jain, K. (1989), Preservation of some partial orderings under Poisson shock models, *Adv. Appl. Prob.* **21**, 713–716.

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