ORTHOGONAL TRACES ON SEMI–PRIME GAMMA RINGS

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Abstract. We deal with some conditions in order that traces would be orthogonal on semi-prime Γ-rings.

1. Introduction

In [8], J. Vukman proved some results related with symmetric bi-derivations on prime and semiprime rings, and then M. A. Öztürk et al. [5] applied the Vukman's idea to prime and semi-prime Γ-rings. In this paper, we consider (orthogonal) traces of symmetric bi-derivations on semi-prime Γ-rings, and we provide some conditions in order that traces would be orthogonal on semi-prime Γ-rings.

2. Preliminaries

Let M and Γ be two abelian groups. If for all $x, y, z \in M$ and all α, $\beta \in \Gamma$ the conditions
(i) $x\alpha y \in M$,
(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha+\beta)z = x\alpha z + x\beta z$, $x\alpha(y+z) = x\alpha y + x\alpha z$,
(iii) $(x\alpha y)\beta z = x\alpha(y\beta z)$
are satisfied, then we call M a Γ-*ring*. By a *right* (resp. *left*) *ideal* of a Γ-ring M we mean an additive subgroup U of M such that $U\Gamma M \subseteq U$ (resp. $M\Gamma U \subseteq U$). If U is both a right and a left ideal, then we say that U is an *ideal* of M. For each a of a Γ-ring M the smallest right ideal containing a is called the *principal right ideal generated* by a and is denoted by $\langle a\rangle_r$. Similarly we define $\langle a\rangle_l$ (resp. $\langle a\rangle$), the *principal left* (resp. *two sided*) *ideal generated* by a. An ideal P of a Γ-ring M is said to be *prime* if for any ideals A and B of M, $A\Gamma B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$. An ideal Q of a Γ-ring M is said to be *semi-prime* if for any ideal U of M, $U\Gamma U \subseteq Q$ implies $U \subseteq Q$. A Γ-ring M is said to be *prime* (resp. *semi-prime*) if the zero ideal is prime (resp. semi-prime).

Theorem 2.1 ([3, Theorem 4]). *If M is a Γ-ring, the following conditions are equivalent:*
(i) *M is a prime Γ-ring.*
(ii) *If $a, b \in M$ and $a\Gamma M\Gamma b = \langle 0\rangle$, then $a = 0$ or $b = 0$.*
(iii) *If $\langle a\rangle$ and $\langle b\rangle$ are principal ideals in M such that $\langle a\rangle\Gamma\langle b\rangle = \langle 0\rangle$, then $a = 0$ or $b = 0$.*
(iv) *If A and B are right ideals in M such that $A\Gamma B = \langle 0\rangle$, then $A = \langle 0\rangle$ or $B = \langle 0\rangle$.*
(v) *If A and B are left ideals in M such that $A\Gamma B = \langle 0\rangle$, then $A = \langle 0\rangle$ or $B = \langle 0\rangle$.*

Let M be a Γ-ring. If there exists a least positive integer n such that $nx = 0$ for all $x \in M$, then M is said to have *characteristic* n, denoted by *char*M. A Γ-ring M is said to

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be *n-torsion free* if $nx = 0$ implies $x = 0$ for any $x \in M$ and a positive integer n. If M is a prime Γ-ring with $char M \neq n$, that is, there exists $0 \neq b \in M$ such that $nb \neq 0$, then M is n-torsion free. Because if $na = 0$, then by $0 = na\Gamma M \Gamma nb$, we have $a = 0$ which means that M is n-torsion free.

Lemma 2.2 ([3, Corollary 1]). *A Γ-ring M is semi-prime if and only if $a\Gamma M \Gamma a = 0$ implies $a = 0$.*

A mapping $D(\cdot,\cdot) : M \times M \to M$ is said to be *symmetric bi-additive* if it is additive in both arguments and $D(x, y) = D(y, x)$ for all $x, y \in M$. By the *trace* of $D(\cdot,\cdot)$ we mean a map $d : M \to M$ defined by $d(x) = D(x, x)$ for all $x \in M$. A symmetric bi-additive map is called a *symmetric bi-derivation* if $D(x\beta z, y) = D(x, y)\beta z + x\beta D(z, y)$ for all $x, y, z \in M$ and $\beta \in \Gamma$. Since a map $D(\cdot,\cdot)$ is symmetric bi-additive, the trace of $D(\cdot,\cdot)$ satisfies the relation $d(x + y) = d(x) + d(y) + 2D(x, y)$ for all $x, y \in M$, and is an even function.

Definition 2.3 ([6, Definition 2.1]). Let M be a semi-prime Γ-ring, and let $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$ be symmetric bi-derivations of M. If the traces d_1 and d_2 of $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$, respectively, satisfy $d_1(x)\Gamma M \Gamma d_2(y) = 0 = d_2(y)\Gamma M \Gamma d_1(x)$ for all $x, y \in M$, then d_1 and d_2 are called *orthogonal traces*.

Note that, the trace of a non-zero symmetric bi-derivation in a semi-prime Γ-ring isn't orthogonal with itself. Let M be a Γ-ring. For a subset S of M, $l(S) = \{a \in M \mid a\Gamma S = 0\}$ is called the *left annihilator* of S. A *right annihilator* $r(S)$ can be defined similarly.

Lemma 2.4 ([5, Lemma 3]). *Let M be a 2-torsion free semi-prime Γ-ring, U a non-zero ideal of M and $a, b \in M$. Then the following are equivalent.*

(i) $a\alpha x\beta b = 0$ *for all* $x \in U$ *and* $\alpha, \beta \in \Gamma$,

(ii) $b\alpha x\beta a = 0$ *for all* $x \in U$ *and* $\alpha, \beta \in \Gamma$,

(iii) $a\alpha x\beta b + b\alpha x\beta a = 0$ *for all* $x \in U$ *and* $\alpha, \beta \in \Gamma$.

If one of the conditions is fulfilled and $l(U) = 0$, then $a\alpha b = 0 = b\alpha a$ for all $\alpha \in \Gamma$. Moreover if M is a prime Γ-ring, then $a = 0$ or $b = 0$.

Lemma 2.5 ([5, Lemma 4]). *Let M be a 2,3-torsion free semi-prime Γ- ring and U a non-zero ideal of M. Let $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$ be symmetric bi-derivations of M, and let d_1 and d_2 be the traces of $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$ respectively. Then*

(i) *If* $d_1(U)\Gamma U \Gamma d_2(U) = 0$, *then* $d_1(M)\Gamma U \Gamma d_2(M) = 0$,

(ii) *If* $l(U) = 0$ *and* $d_1(M)\Gamma U \Gamma d_2(M) = 0$, *then* $d_1(M)\Gamma M \Gamma d_2(M) = 0$.

3. Main results

Theorem 3.1. *Let M be a 2,3-torsion free semi-prime Γ-ring, U a non-zero ideal of M and $l(U) = 0$. Let $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$ be symmetric bi-derivations of M, and let d_1 and d_2 be the traces of $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$ respectively. Then d_1 and d_2 are orthogonal if and only if $d_1(u)\Gamma d_2(v) + d_2(u)\Gamma d_1(v) = 0$ for all $u, v \in U$.*

Proof. If d_1 and d_2 are orthogonal, then $d_1(x)\Gamma M \Gamma d_2(y) = 0 = d_2(y)\Gamma M \Gamma d_1(x)$ for all $x, y \in M$. So we have $d_1(u)\Gamma d_2(v) = 0 = d_2(v)\Gamma d_1(u)$ by Lemma 2.4, and hence

$$d_1(u)\Gamma d_2(v) + d_2(u)\Gamma d_1(v) = 0$$

for all $u, v \in U$. Conversely, assume that $d_1(u)\Gamma d_2(v) + d_2(u)\Gamma d_1(v) = 0$ for all $u, v \in U$. Then

$$d_1(u)\gamma d_2(v) + d_2(u)\gamma d_1(v) = 0 \tag{1}$$

for all $u, v \in U$ and $\gamma \in \Gamma$. Replacing v by $v+w$ in (1) where $w \in U$ and using the fact that M is 2-torsion free, we get

$$d_1(u)\gamma D_2(v,w) + d_2(u)\gamma D_1(v,w) = 0 \tag{2}$$

for all $u, v, w \in U$ and $\gamma \in \Gamma$. Substituting $u+v$ for u in (2) we have

$$D_1(u,v)\gamma D_2(v,w) + D_2(u,v)\gamma D_1(v,w) = 0 \tag{3}$$

for all $u, v, w \in U$ and $\gamma \in \Gamma$. Now replacing w by $w\beta u$ in (3) where $\beta \in \Gamma$ and using (3), we obtain

$$D_1(u,v)\gamma w\beta D_2(v,u) + D_2(u,v)\gamma w\beta D_1(v,u) = 0 \tag{4}$$

for all $u, v, w \in U$ and $\gamma, \beta \in \Gamma$. Substituting u for v in (4), we get

$$d_1(u)\gamma w\beta d_2(u) + d_2(u)\gamma w\beta d_1(u) = 0 \tag{5}$$

for all $u, w \in U$ and $\gamma, \beta \in \Gamma$. It follows from (5) and Lemma 2.4 that $d_1(u)\Gamma U\Gamma d_2(u) = 0$ for all $u \in U$. In a similar way, we get $d_2(u)\Gamma U\Gamma d_1(u) = 0$ for all $u \in U$. This shows that d_1 is orthogonal with d_2 by Lemma 2.5. $\square$

Theorem 3.2. *Let M be a 2,3-torsion free semi-prime Γ-ring, U a non-zero ideal of M and $l(U) = 0$. Let $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$ be symmetric bi-derivations of M such that $d_2(U) \subset U$ and d_1 and d_2 the traces of $D_1(\cdot,\cdot)$ and $D_1(\cdot,\cdot)$ respectively. Then the following are equivalent:*
- (i) *d_1 and d_2 are orthogonal,*
- (ii) *$d_1 d_2 = 0$,*
- (iii) *There exists $a, b \in M$ and $\gamma, \beta \in \Gamma$ such that $(d_1 d_2)(u) = a\beta u + u\gamma b$ for all $u \in U$,*
- (iv) *$d_1 d_2 = f$, where f is the trace of a symmetric bi-additive mapping $F(\cdot,\cdot)$ of M.*

Proof. (i) $\Rightarrow$ (ii), (i) $\Rightarrow$ (iii) and (i) $\Rightarrow$ (iv) are given in [6, Theorem 2.7].

(ii) $\Rightarrow$ (i): Assume that $d_1 d_2 = 0$. Then

$$(d_1 d_2)(u) = 0 \quad \text{for all} \quad u \in U. \tag{6}$$

Since M is 2-torsion free, by linearizing (6) we obtain

$$\begin{aligned} &D_1(d_2(u), d_2(v)) + 2D_1(D_2(u,v), d_2(v)) \\ &\quad + 2D_1(D_2(u,v), d_2(u)) + 4D_1(D_2(u,v), D_2(u,v)) = 0 \end{aligned} \tag{7}$$

for all $u, v \in U$. Substituting $-u$ for u in (7), we have

$$\begin{aligned} &D_1(d_2(u), d_2(v)) - 2D_1(D_2(u,v), d_2(v)) \\ &\quad - 2D_1(D_2(u,v), d_2(u)) + 4D_1(D_2(u,v), D_2(u,v)) = 0 \end{aligned} \tag{8}$$

for all $u, v \in U$. Adding (7) and (8) and using the fact that M is 2-torsion free, we obtain

$$D_1(d_2(u), d_2(v)) + 2D_1(D_2(u,v), D_2(u,v)) = 0 \tag{9}$$

for all $u, v \in U$. Substituting $u+w$ for u in (9) where $w \in U$ and using the fact that M is 2-torsion free, we have

$$D_1(D_2(u,w), d_2(v)) + 2D_1(D_2(u,v), D_2(w,v)) = 0 \tag{10}$$

for all $u, v, w \in U$. Replacing u by $u\gamma k$ in (10) where $k \in U$ and $\gamma \in \Gamma$ and using (10) again, we have

$$\begin{aligned} D_2(u,w)\gamma D_1(k,d_2(v)) &+ D_1(u,d_2(v))\gamma D_2(k,w) \\ &+ 2D_1(u,D_2(w,v)\gamma D_2(k,v)) \\ &+ 2D_2(u,v)\gamma D_1(k,D_2(w,v)) = 0 \end{aligned} \tag{11}$$

for all $u, v, w, k \in U$ and $\gamma \in \Gamma$. Since M is 3-torsion free, by substituting v for w in (11) we get

$$D_2(u,v)\gamma D_1(k,d_2(v)) + D_1(u,d_2(v))\gamma D_2(k,v) = 0 \tag{12}$$

for all $u, v, k \in U$ and $\gamma \in \Gamma$. Using $k\beta u$ for k in (12) where $\beta \in \Gamma$, we get

$$D_2(u,v)\gamma k\beta D_1(u,d_2(v)) + D_1(u,d_2(v))\gamma k\beta D_2(u,v) = 0 \tag{13}$$

for all $u, v, k \in U$ and $\gamma, \beta \in \Gamma$. It follows from Lemma 2.4 that

$$D_2(u,v)\gamma k\beta D_1(u,d_2(v)) = 0 \tag{14}$$

for all $u, v, k \in U$ and $\gamma, \beta \in \Gamma$. Writing $v + w$ for v in (14) and by using (14), we get

$$\begin{aligned} D_2(u,v)\gamma k\beta D_1(u,d_2(w)) &+ D_2(u,w)\gamma k\beta D_1(u,d_2(v)) \\ &+ 2D_2(u,v)\gamma k\beta D_1(u,D_2(v,w)) \\ &+ 2D_2(u,w)\gamma k\beta D_1(u,D_2(v,w)) = 0 \end{aligned} \tag{15}$$

for all $u, v, k, w \in U$ and $\gamma, \beta \in \Gamma$. Replacing w by $-w$ in (15), we have

$$\begin{aligned} -D_2(u,v)\gamma k\beta D_1(u,d_2(w)) &- D_2(u,w)\gamma k\beta D_1(u,d_2(v)) \\ &- 2D_2(u,v)\gamma k\beta D_1(u,D_2(v,w)) \\ &+ 2D_2(u,w)\gamma k\beta D_1(u,D_2(v,w)) = 0 \end{aligned} \tag{16}$$

for all $u, v, k, w \in U$ and $\gamma, \beta \in \Gamma$. Adding up (15) and (16) and using the fact that M is 2-torsion free, we obtain

$$D_2(u,v)\gamma k\beta D_1(u,d_2(w)) + 2D_2(u,w)\gamma k\beta D_1(u,D_2(v,w)) = 0 \tag{17}$$

for all $u, v, k, w \in U$ and $\gamma, \beta \in \Gamma$. Replacing k by $k\beta D_1(d_2(w),u)\beta' m\gamma' D_2(u,v)\gamma k$ in (17) where $m \in M$ and $\gamma', \beta' \in \Gamma$ and using (15) and the fact that M is a semi-prime Γ-ring, we get

$$D_2(u,v)\gamma k\beta D_1(d_2(w),u) = 0 \tag{18}$$

for all $u, v, k, w \in U$ and $\gamma, \beta \in \Gamma$. Substituting $w + p$ for w in (18) where $p \in U$ and using the fact that M is 2-torsion free, we have

$$D_2(u,v)\gamma k\beta D_1(D_2(w,p),u) = 0 \tag{19}$$

for all $u, v, k, w, p \in U$ and $\gamma, \beta \in \Gamma$. Writing $k\gamma' t$ for k in (19) where $t \in U$ and $\gamma' \in \Gamma$, we get

$$D_2(u,v)\gamma k\gamma' t\beta D_1(D_2(w,p),u) = 0 \tag{20}$$

for all $u, v, k, w, p, t \in U$ and $\gamma, \beta, \gamma' \in \Gamma$. In the similar manner, writing $t\gamma' w$ for w in (20) and using (20), we have

$$D_2(u,v)\gamma k\beta D_2(t,p)\gamma' D_1(w,u) + D_2(u,v)\gamma k\beta D_1(t,u)\gamma' D_2(w,p) = 0 \tag{21}$$

for all u, v, k, w, p, $t \in U$ and γ, β, $\gamma' \in \Gamma$. Writing $d_2(t)$ for t in (21), it follows from (18) that

$$D_2(u,v)\gamma k\beta D_2(d_2(t),p)\gamma' D_1(w,u) = 0 \tag{22}$$

for all $u, v, k, w, p, t \in U$ and $\gamma, \beta, \gamma' \in \Gamma$. Writing $t + q$ for t in (22) where $q \in U$ and using the fact that M is 2-torsion free, we get

$$D_2(u,v)\gamma k\beta D_2(D_2(t,q),p)\gamma' D_1(w,u) = 0 \tag{23}$$

for all $u, v, k, w, p, t, q \in U$ and $\gamma, \beta, \gamma' \in \Gamma$. It follows by sustituting $k\beta' r$ for k in (23), where $r \in U$ and $\beta' \in \Gamma$, that

$$D_2(u,v)\gamma k\beta' r\beta D_2(D_2(t,q),p)\gamma' D_1(w,u) = 0 \tag{24}$$

for all $u, v, k, w, p, t, q, r \in U$ and $\gamma, \beta, \gamma', \beta' \in \Gamma$. Again, writing $r\beta' w$ for w in (23), we have

$$D_2(u,v)\gamma k\beta' r\beta D_2(D_2(t,q),p)\gamma' r\beta' D_1(w,u) = 0 \tag{25}$$

for all $u, v, k, w, p, t, q, r \in U$ and $\gamma, \beta, \gamma', \beta' \in \Gamma$. Substituting $r\beta' t$ for t in (23) and using (24)and (25), we have

$$\begin{aligned} &D_2(u,v)\gamma k\beta D_2(r,p)\beta' D_2(t,q)\gamma' D_1(w,u) \\ &\qquad + D_2(u,v)\gamma k\beta D_2(r,q)\beta' D_2(t,p)\gamma' D_1(w,u) = 0 \end{aligned} \tag{26}$$

for all u, v, k, w, p, t, q, $r \in U$ and γ, β, γ', $\beta' \in \Gamma$. Since M is 2-torsion free, it follows by replacing q by p in (26) that

$$D_2(u,v)\gamma k\beta D_2(r,p)\beta' D_2(t,p)\gamma' D_1(w,u) = 0 \tag{27}$$

for all $u, v, k, w, p, t \in U$ and $\gamma, \beta, \gamma', \beta' \in \Gamma$. Replacing k by $D_2(t,p)\gamma' D_1(w,u)\alpha k\alpha' m$ in (27) where $m \in M$ and α, $\alpha' \in \Gamma$, we have

$$D_2(u,v)\gamma D_2(t,p)\gamma' D_1(w,u)\alpha k\alpha' m\beta D_2(r,p)\beta' D_2(t,p)\gamma' D_1(w,u) = 0.$$

Taking β' for γ and u, v for r, p respectively in the previous equation and using $l(U) = 0$, we have

$$D_2(u,v)\gamma D_2(t,v)\beta D_1(w,u) = 0 \tag{28}$$

for all $u, v, w, t \in U$ and $\gamma, \beta \in \Gamma$. Replacing w by $k\gamma' w$ in (28), we get

$$D_2(u,v)\gamma D_2(t,v)\beta k\gamma' D_1(w,u) = 0 \tag{29}$$

for all $u, v, k, w, t \in U$ and $\gamma, \beta, \gamma' \in \Gamma$. Again, replacing t by $t\gamma' k$ in (28) and using (29), we get

$$D_2(u,v)\gamma t\gamma' D_2(k,v)\beta D_1(w,u) = 0 \tag{30}$$

for all $u, v, k, w, t \in U$ and $\gamma, \beta, \gamma' \in \Gamma$. Replacing t by $D_1(w,u)\alpha t\beta' m$ in (30) where $m \in M$, we have

$$D_2(u,v)\gamma D_1(w,u)\alpha t\beta' m\gamma' D_2(k,v)\beta D_1(w,u) = 0 \tag{31}$$

for all $u, v, w, k, t \in U$ and $\gamma, \beta', \gamma' \in \Gamma$ and $m \in M$. Writing β for γ and writing u for k in (31), it follows from $l(U) = 0$ that

$$D_2(u,v)\beta D_1(w,u) = 0 \tag{32}$$

for all $u, v, w \in U$ and $\beta \in \Gamma$. Now, writing $w\gamma v$ for w in (32), we get

$$D_2(u,v)\beta w\gamma D_1(u,v) = 0$$

for all $u, v, w \in U$ and $\gamma, \beta \in \Gamma$, and so by taking u for v in the previous equation, we get $d_2(x)\Gamma M\Gamma d_2(y) = 0$ for all $x, y \in M$ by Lemma 2.5. Similarly, we get $d_1(y)\Gamma M\Gamma d_2(x) = 0$ for all $x, y \in M$.

(iii) $\Rightarrow$ (i): Assume that there exists $a, b \in M$ and $\gamma, \beta \in \Gamma$ such that $(d_1 d_2)(u) = a\beta u + u\gamma b$ for all $u \in U$. Then by linearizing and using the fact that M is a 2-torsion free, we get

$$\begin{aligned} &D_1(d_2(u), d_2(v)) + 2D_1(D_2(u,v), d_2(u)) \\ &\quad + 2D_1(D_2(u,v), d_2(v)) + 2D_1(D_2(u,v), D_2(u,v)) = 0 \end{aligned} \tag{33}$$

for all $u, v \in U$. Applying all steps which start from (7), we get the result.

(iv) $\Rightarrow$ (i): Let $(d_1 d_2)(u) = f(u)$, where f is the trace of a symmetric bi-additive mapping $F(\cdot,\cdot)$ of M. By linearizing this expression and by using the fact that M is 2-torsion free, we get

$$\begin{aligned} &D_1(d_2(u), d_2(v)) + 2D_1(D_2(u,v), d_2(u)) \\ &\quad + 2D_1(D_2(u,v), d_2(v)) + 2D_1(D_2(u,v), D_2(u,v)) = F(u,v) \end{aligned} \tag{34}$$

for all $u, v \in U$. Writing $-u$ for u in (34), we get

$$\begin{aligned} &D_1(d_2(u), d_2(v)) - 2D_1(D_2(u,v), d_2(u)) \\ &\quad - 2D_1(D_2(u,v), d_2(v)) + 2D_1(D_2(u,v), D_2(u,v)) = -F(u,v) \end{aligned} \tag{35}$$

for all $u, v \in U$. Adding (34) and (35) and using the fact that M is 2-torsion free, we get

$$D_1(d_2(u), d_2(v)) + 2D_1(D_2(u,v), D_2(u,v)) = 0 \tag{36}$$

for all $u, v \in U$. Thus, applying all steps which start from (8) we get the result. Hence the proof of the theorm is completed. □

Corollary 3.3. *Let M be a 2,3-torsion free prime Γ-ring and U a non-zero ideal of M. Let $D_1(\cdot,\cdot)$, $D_2(\cdot,\cdot)$ be symmetric bi-derivations of M such that $d_2(u) \subset U$ and d_1 and d_2 the traces of $D_1(\cdot,\cdot)$ and $D_2(\cdot,\cdot)$, respectively. If one of the equivalent conditions in Theorem 3.2 is valid, then $D_1 = 0$ or $D_2 = 0$.*

Corollary 3.4. *Let M be a 2,3-torsion free semi-prime Γ-ring and U a non-zero ideal of M. Let $D(\cdot,\cdot)$ be a symmetric bi-derivation of M such that $d(u) \subset D(\cdot,\cdot)$ and d the traces of $D(\cdot,\cdot)$. If one of the equivalent conditions in Theorem 3.2 is valid, then $D = 0$.*

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