GENERALIZED d-DERIVATIONS OF RINGS WITHOUT UNIT ELEMENTS

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Abstract. We show the difference between the set of Brešar generalized derivations and the set of generalized derivations as K-modules over a commutative ring K. We also refer to the extendability of Brešar generalized derivations. Moreover, we apply the results to generalized Jordan derivations.

1 Introduction The notion of derivations has been generalized in several forms. One of them is defined by M. Brešar[1], which we call *Brešar generalized derivations* or *generalized d-derivations*. We denote by $\mathrm{BDer}(A,\ M)$ the set of Brešar generalized derivations from a K-algebra A to an A/K bimodule M over a commutative ring K. A number of authors have studied these derivations (e.g. [2] and [3]). In [3], A. Nakajima defined a notion of derivations generalized in another form, which we call *generalized derivations*. We denote the set of generalized derivations from A to M by $\mathrm{gDer}(A,\ M)$. If A has a unit element, then $\mathrm{gDer}(A,\ M)$ is in one-to-one correspondence with $\mathrm{BDer}(A,\ M)$; however, this relation does not hold when A does not have a unit element.

In this paper, we consider the difference between $\mathrm{gDer}(A,\ M)$ and $\mathrm{BDer}(A,\ M)$, and give a necessary and sufficient condition for $\mathrm{gDer}(A,\ M)$ to be isomorphic to $\mathrm{BDer}(A,\ M)$ as K-modules. Moreover, we apply the results to generalized Jordan derivations.

We also refer to the extendability of these generalized derivations to a ring having a unit element.

2 Preliminaries Let K be a commutative ring with a unit element, and A a K-algebra. Let M be an A/K-bimodule, that is, M is an A-bimodule and is a unitary K-bimodule such that, for any $a \in A$, $\alpha \in K$ and $m \in M$, $\alpha(am) = (\alpha a)m = a(\alpha m)$, $(ma)\alpha = (m\alpha)a = m(a\alpha)$ and $\alpha m = m\alpha$. A K-homomorphism $d : A \longrightarrow M$ is called a *K-derivation* if $d(ab) = d(a)b + ad(b)$ for all $a,\ b \in A$. We denote the set of K-derivations from A to M by $\mathrm{Der}(A,\ M)$.

Let $d : A \longrightarrow M$ be a K-derivation, and $f : A \longrightarrow M$ a K-homomorphism. Then a pair $(f,\ d)$ is said to be a *Brešar generalized derivation* or a *generalized d-derivation* if $f(ab) = f(a)b + ad(b)$ for all $a,\ b \in A$. Two Brešar generalized derivations $(f_1,\ d_1)$ and $(f_2,\ d_2)$ are equal if $f_1 = f_2$ and $d_1 = d_2$. We denote by $\mathrm{BDer}(A,\ M)$ the set of Brešar generalized derivations from A to M. If $(f_1,\ d_1)$ and $(f_2,\ d_2)$ are Brešar generalized derivations and $\alpha \in K$, then $(f_1 + f_2,\ d_1 + d_2)$ and $(\alpha f_1,\ \alpha d_1)$ are also Brešar generalized derivations and hence, $\mathrm{BDer}(A,\ M)$ is a K-module.

Take $m \in M$, and let $f : A \longrightarrow M$ be a K-homomorphism. Then a pair $(f,\ m)$ is said to be a *generalized derivation* if $f(ab) = f(a)b + af(b) + amb$ for all $a,\ b \in A$. Two generalized derivations $(f_1,\ m_1)$ and $(f_2,\ m_2)$ are equal if $f_1 = f_2$ and $m_1 = m_2$. We denote by $\mathrm{gDer}(A,\ M)$ the set of generalized derivations from A to M. If $(f_1,\ m_1)$ and $(f_2,\ m_2)$

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are generalized derivations and $\alpha \in K$, then $(f_1 + f_2,\ m_1 + m_2)$ and $(\alpha f_1,\ \alpha m_1)$ are also generalized derivations and hence, $\mathrm{gDer}(A,\ M)$ is also a K-module.

The following result is proved in [3]:

Proposition 2.1. *(1) If $(f,\ m) : A \longrightarrow M$ is a generalized derivation, then $f + m_\ell : A \longrightarrow M$ is a K-derivation, where $m_\ell : A \to M$ is a left multiplication, i.e. $m_\ell(a) = ma$.*

(2) If $d : A \longrightarrow M$ is a K-derivation, then $(d + m_\ell,\ -m) : A \longrightarrow M$ is a generalized derivation for all $m \in M$.

(3) If $(f,\ m) : A \longrightarrow M$ is a generalized derivation, then $(f,\ f + m_\ell) : A \longrightarrow M$ is a Brešar generalized derivation.

(4) If A has a unit element and $(f,\ d) : A \longrightarrow M$ is a Brešar generalized derivation, then $(f,\ -f(1)) : A \longrightarrow M$ is a generalized derivation.

From Proposition 2.1 (1), (2), we have the following split exact sequence of K-modules:

$$0 \longrightarrow M \xrightarrow{\varphi_1} \mathrm{gDer}(A,\ M) \xrightarrow{\varphi_2} \mathrm{Der}(A,\ M) \longrightarrow 0,$$

where $\varphi_1(m) = (m_\ell,\ -m)$ and $\varphi_2(f,\ m) = f + m_\ell$. And from Proposition 2.1 (3), (4), we can handle generalized derivations as Brešar generalized derivations, and if A has a unit element, we can also handle Brešar generalized derivations as generalized derivations. Hence, our aim is to show the difference between generalized derivations and Brešar generalized derivations when A does not have a unit element.

In the following, K will denote a commutative ring with a unit element, A a K-algebra without the assumption of the existence of a unit element, and M an A/K-bimodule.

3 Properties of Brešar generalized derivations

A K-homomorphism $f : A \longrightarrow M$ is said to be a *left multiplier* if $f(ab) = f(a)b$ for all $a,\ b \in A$. We denote by $\mathrm{Mul}(A,\ M)$ the set of left multipliers from A to M. If f and g are left multipliers and $\alpha \in K$, then $f + g$ and αf are also left multipliers and hence, $\mathrm{Mul}(A,\ M)$ is a K-module. Note that, for an arbitrary Brešar generalized derivation $(f,\ d)$ from A to M, $f - d \in \mathrm{Mul}(A,\ M)$.

Next theorem gives us a necessary and sufficient condition for $\mathrm{BDer}(A,\ M)$ to be isomorphic to $\mathrm{gDer}(A,\ M)$ as a K-module:

Theorem 3.1. *Let $\Phi : \mathrm{gDer}(A,\ M) \longrightarrow \mathrm{BDer}(A,\ M)$ and $\Psi : M \longrightarrow \mathrm{Mul}(A, M)$ be K-homomorphisms such that $\Phi((f,\ m)) = (f,\ f + m_\ell)$ and $\Psi(m) = m_\ell$. Then Φ is a K-isomorphism if and only if Ψ is a K-isomorphism.*

Proof. Let $\psi_1 : \mathrm{Mul}(A,\ M) \longrightarrow \mathrm{BDer}(A,\ M)$ and $\psi_2 : \mathrm{BDer}(A,\ M) \longrightarrow \mathrm{Der}(A,\ M)$ be K-homomorphisms such that $\psi_1(g) = (g,\ 0)$ and $\psi_2((f,\ d)) = d$. Let $\iota_2 : \mathrm{Der}(A,\ M) \to \mathrm{BDer}(A,\ M)$ be a K-homomorphism such that $\iota_2(d) = (d,\ d)$. Then $\psi_2\iota_2 = id_{\mathrm{Der}(A,\ M)}$, and hence, we have the following split exact sequence of K-modules:

$$0 \longrightarrow \mathrm{Mul}(A,\ M) \xrightarrow{\psi_1} \mathrm{BDer}(A,\ M) \xrightarrow{\psi_2} \mathrm{Der}(A,\ M) \longrightarrow 0.$$

Then we have the following commutative diagram:

$$\begin{array}{ccccccccc}
0 \longrightarrow & M & \xrightarrow{\varphi_1} & \mathrm{gDer}(A,M) & \xrightarrow{\varphi_2} & \mathrm{Der}(A,M) & \longrightarrow 0 \\
& {\scriptstyle\Psi}\big\downarrow & & {\scriptstyle\Phi}\big\downarrow & & {\scriptstyle id}\big\downarrow & \\
0 \longrightarrow & \mathrm{Mul}(A,M) & \xrightarrow{\psi_1} & \mathrm{BDer}(A,M) & \xrightarrow{\psi_2} & \mathrm{Der}(A,M) & \longrightarrow 0.
\end{array}$$

Diagram 1

By using Five Lemma, we complete the proof of Theorem 3.1. $\square$

In Diagram 1, using Snake Lemma, we have the following exact sequence:

$$0 \longrightarrow \mathrm{Ker}\Psi \longrightarrow \mathrm{Ker}\Phi \longrightarrow 0 \longrightarrow \mathrm{Coker}\Psi \longrightarrow \mathrm{Coker}\Phi \longrightarrow 0,$$

and hence, $\mathrm{Ker}\Psi \cong \mathrm{Ker}\Phi$ and $\mathrm{Coker}\Psi \cong \mathrm{Coker}\Phi$. In fact, we can also easily check that

$$\begin{aligned} \mathrm{Ker}\Psi &= \{ \ m \in M \mid mA = 0 \ \}, \\ \mathrm{Ker}\Phi &= \{ \ (0, \ m) \in \mathrm{gDer}(A, \ M) \mid mA = 0 \ \}, \end{aligned}$$

and

$$\begin{aligned} \mathrm{Im}\Psi &= \{ \ m_\ell \in \mathrm{Mul}(A, \ M) \mid m \in M \ \}, \\ \mathrm{Im}\Phi &= \{ \ (f, \ d) \in \mathrm{BDer}(A, \ M) \mid f - d = m_\ell \text{ for some } m \in M \ \}. \end{aligned}$$

The following two examples show that there exists a K-algebra A such that $\Psi : A \longrightarrow \mathrm{Mul}(A, \ A)$ is not a K-isomorphism:

Example 1. Let $A = \begin{pmatrix} K[x] & 0 \\ K[x] & 0 \end{pmatrix}$, where $K[x]$ is the polynomial ring in one variable x. Then A is a non-commutative K-algebra, which is a subring of $\mathrm{M}_2(K[x])$, and A does not have a unit element. Let $f_i : K[x] \longrightarrow K[x]$ $(i = 1, \ 2)$ be K-homomorphisms satisfying $f_i(ab) = f_i(a)b$ for all $a, \ b \in K[x]$ (i.e. f_1 and f_2 are left multipliers of $K[x]$). Then $f_i(a) = f_i(1)a$ since $1 \in K[x]$. We take f_i such that $f_1(1) \neq f_2(1) = 1$. Let $F : A \longrightarrow A$ be a K-homomorphism defined by

$$F \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} = \begin{pmatrix} f_1(a) & 0 \\ f_2(b) & 0 \end{pmatrix}.$$

Put $P = \begin{pmatrix} p_1 & 0 \\ p_2 & 0 \end{pmatrix}$, $Q = \begin{pmatrix} q_1 & 0 \\ q_2 & 0 \end{pmatrix} \in A$. Then

$$\begin{aligned} F(PQ) &= F\left(\begin{pmatrix} p_1 & 0 \\ p_2 & 0 \end{pmatrix} \begin{pmatrix} q_1 & 0 \\ q_2 & 0 \end{pmatrix} \right) = F\left(\begin{pmatrix} p_1 q_1 & 0 \\ p_2 q_1 & 0 \end{pmatrix} \right) \\ &= \begin{pmatrix} f_1(p_1 q_1) & 0 \\ f_2(p_2 q_1) & 0 \end{pmatrix} = \begin{pmatrix} f_1(p_1) q_1 & 0 \\ f_2(p_2) q_1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} f_1(p_1) & 0 \\ f_2(p_2) & 0 \end{pmatrix} \begin{pmatrix} q_1 & 0 \\ q_2 & 0 \end{pmatrix} = F(P)Q. \end{aligned}$$

This means that F is a left multiplier of A. Now we consider $\Psi : A \longrightarrow \mathrm{Mul}(A, \ A)$. We can easily check that $\mathrm{Ker}\Psi = 0$, and hence, Ψ is a monomorphism. Assume that there exists $L = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \in A$ such that $F(P) = LP$ for all $P \in A$, then

$$F(P) = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \begin{pmatrix} p_1 & 0 \\ p_2 & 0 \end{pmatrix} = \begin{pmatrix} ap_1 & 0 \\ bp_1 & 0 \end{pmatrix},$$

hence, $f_2(p_2) = 1 \cdot p_2 = p_2 = bp_1$ for all $p_1, \ p_2 \in K[x]$, a contradiction. This means that $F \notin \mathrm{Im}\Psi$. Hence, Ψ is not an epimorphism.

Example 2. Let $A = \begin{pmatrix} K[x] & K[x] \\ 0 & 0 \end{pmatrix}$. Then A is also a non-commutative K-algebra, and does not have a unit element. Let $F : A \longrightarrow A$ be a left multiplier. Put $P = \begin{pmatrix} p_1 & p_2 \\ 0 & 0 \end{pmatrix} \in A$. Then $P = \begin{pmatrix} p_1 & p_2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} p_1 & p_2 \\ 0 & 0 \end{pmatrix}$ and hence, $F(P) = F\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) P$. This means that $\Psi : A \longrightarrow \mathrm{Mul}(A,\ A)$ is an epimorphism. However, since $\Psi\left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right) = 0$, we have $\mathrm{Ker}\Psi \neq 0$. Hence, Ψ is not a monomorphism.

4 Extensions of Brešar generalized derivations

Let A be a K-algebra without a unit element. Let $\widehat{A} = \{\ (n,\ a) \mid n \in K,\ a \in A\ \}$ be a direct product $K \times A$ with multiplication $(n_1,\ a_1)(n_2,\ a_2) = (n_1 n_2,\ n_1 a_2 + n_2 a_1 + a_1 a_2)$ for $n_1,\ n_2 \in K,\ a_1,\ a_2 \in A$. Then $\widehat{A}$ is a K-algebra with a unit element $(1,\ 0)$. Let M be an A/K-bimodule. Then M is an $\widehat{A}/K$-bimodule with $(n_1,\ a_1) \cdot m_1 = n_1 m_1 + a_1 m_1$ and $m_2 \cdot (n_2,\ a_2) = n_2 m_2 + m_2 a_2$ for $n_i \in K$, $a_i \in A$ and $m_i \in M$.

Let $d : A \longrightarrow M$ be a K-derivation. Then there exists a unique K-derivation $\tilde{d} : \widehat{A} \longrightarrow M$ such that its restriction $\tilde{d}|_A$ is equal to d. This $\tilde{d}$ is defined by $\tilde{d}(n,\ a) = d(a)$.

Now we consider the extendability of Brešar generalized derivations. An extension $(F,\ D)$ of $(f,\ d) \in \mathrm{BDer}(A,\ M)$ means a Brešar generalized derivation $(F,\ D)$ from $\widehat{A}$ to M such that its restriction $(F,\ D)|_A$ is equal to $(f,\ d)$, where $D \in \mathrm{Der}(\widehat{A},\ M)$ and $D|_A = d \in \mathrm{Der}(A,\ M)$.

Theorem 4.1. *Let A be a K-algebra, M an A/K-bimodule and $\widehat{A}$ a K-algebra defined as above. Let $(f,\ d)$ be a Brešar generalized derivation from A to M. Then the following conditions are equivalent:*

(1) *A Brešar generalized derivation $(f,\ d)$ from A to M can be extended to a Brešar generalized derivation from $\widehat{A}$ to M.*

(2) *A left multiplier $f - d$ from A to M can be extended to a left multiplier from $\widehat{A}$ to M.*

(3) *There exists an element $m \in M$ such that $f - d = m_\ell$.*

Proof. (1) $\Rightarrow$ (2) : For $(f,\ d) \in \mathrm{BDer}(A,\ M)$, there exists its extension $(F,\ D) \in \mathrm{BDer}(\widehat{A},\ M)$. Then $F - D \in \mathrm{Mul}(\widehat{A},\ M)$ and we can easily check that $(F - D)|_A = f - d$.

(2) $\Rightarrow$ (3) : For $f - d \in \mathrm{Mul}(A,\ M)$, there exists its extension $G \in \mathrm{Mul}(\widehat{A},\ M)$. Then, for all $a \in A$,

$$(f - d)(a) = G((0,\ a)) = G((1,\ 0)) \cdot (0,\ a) = G((1,\ 0))a.$$

By putting $m = G((1,\ 0)) \in M$, the result follows.

(3) $\Rightarrow$ (1) : Let $D \in \mathrm{Der}(\widehat{A},\ M)$ be a unique extension of $d \in \mathrm{Der}(A,\ M)$. Let $F : \widehat{A} \longrightarrow M$ be a K-homomorphism defined by $F((n,\ a)) = nm + f(a)$ for all $(n,\ a) \in \widehat{A}$. Then

$$\begin{aligned}
&F((n_1,\ a_1)(n_2,\ a_2)) \\
&= n_1 n_2 m + n_1 f(a_2) + n_2 f(a_1) + f(a_1)a_2 + a_1 d(a_2) + n_1 d(a_2) - n_1 d(a_2) \\
&= n_2(n_1 m + f(a_1)) + n_1(f - d)(a_2) + f(a_1)a_2 + a_1 d(a_2) + n_1 d(a_2) \\
&= n_2(n_1 m + f(a_1)) + n_1 m a_2 + f(a_1)a_2 + n_1 D((0,\ a_2)) + a_1 D((0,\ a_2)) \\
&= (n_1 m + f(a_1)) \cdot (n_2,\ a_2) + (n_1,\ a_1) \cdot D((0,\ a_2)) \\
&= F((n_1,\ a_1)) \cdot (n_2,\ a_2) + (n_1,\ a_1) \cdot D((n_2,\ a_2)).
\end{aligned}$$

Hence, $(F,\ D) \in \mathrm{BDer}(\widehat{A},\ M)$ and we can easily see that $(F,\ D)|_A = (f,\ d)$. $\square$

Remark. When the equivalent condition of Theorem 4.1 holds, we can handle a Brešar generalized derivation $(f,\ d)$ as a generalized derivation $(f,\ -m)$ even if A does not contain a unit element.

5 Jordan derivations In this section, we treat generalized Jordan derivations (cf., [4]). A K-homomorphism $J : A \longrightarrow M$ is called a *Jordan derivation* if $J(a^2) = J(a)a + aJ(a)$ for all $a \in A$. We denote the set of Jordan derivations from A to M by $\mathrm{JDer}(A,\ M)$. Firstly, we generalize the notion of Brešar's generalized derivation to Jordan derivation.

Let $J : A \to M$ be a Jordan derivation, and $f : A \longrightarrow M$ a K-homomorphism. A pair $(f,\ J)$ is called a *generalized J-Jordan derivation* (or a *Brešar generalized Jordan derivation*) if, for any $a \in A$,

$$f(a^2) = f(a)a + aJ(a),$$

and a pair (f, m) $(m \in M)$ is called a *generalized Jordan derivation* if

$$f(a^2) = f(a)a + af(a) + ama.$$

We denote by $\mathrm{BJDer}(A,\ M)$ the set of generalized J-Jordan derivations for all Jordan derivations J and by $\mathrm{gJDer}(A,\ M)$ the set of generalized Jordan derivations. As usual, $\mathrm{BJDer}(A,\ M)$ and $\mathrm{gJDer}(A,\ M)$ are K-modules and $\mathrm{BDer}(A,\ M)$ (resp. $\mathrm{gDer}(A,\ M)$) is a K-submodule of $\mathrm{BJDer}(A,\ M)$ (resp. $\mathrm{gJDer}(A,\ M)$).

A K-homomorphism $f : A \longrightarrow M$ is called a *Jordan left multiplier* if $f(a^2) = f(a)a$ for any $a \in A$. Left multipliers and left multiplications are also Jordan left multipliers. We denote by $\mathrm{JMul}(A,\ M)$ the set of Jordan left multipliers and it is also a K-module. Moreover, $\mathrm{Mul}(A,\ M)$ is a K-submodule of $\mathrm{JMul}(A,\ M)$. Under these notations, we have the results similar to those in §3. Since the proofs are very similar, we omit them.

Theorem 5.1. *Let A be a K-algebra and M an A/K-bimodule. Then the following diagram is commutative and rows are split as K-modules:*

$$\begin{array}{ccccccccc}
0 \longrightarrow & M & \xrightarrow{\varphi_1} & \mathrm{gJDer}(A,\ M) & \xrightarrow{\varphi_2} & \mathrm{JDer}(A,\ M) & \longrightarrow 0 \\
& \Psi\Big\downarrow & & \Phi\Big\downarrow & & id\Big\downarrow & \\
0 \longrightarrow & \mathrm{JMul}(A,\ M) & \xrightarrow{\psi_1} & \mathrm{BJDer}(A,\ M) & \xrightarrow{\psi_2} & \mathrm{JDer}(A,\ M) & \longrightarrow 0,
\end{array}$$

where $\varphi_1(m) = (m_\ell,\ -m)$, $\varphi_2((f,\ m)) = f + m_\ell$, $\psi_1(g) = (g,\ 0)$, $\psi_2((f,\ J)) = J$, $\Psi(m) = m_\ell$, $\Phi((f,\ m)) = (f,\ f + m_\ell)$. *Moreover,* $\mathrm{Ker}\Phi \cong \mathrm{Ker}\Psi = \{\ m \in M \mid mA = 0\ \}$.

Corollary 5.2. $\Phi : \mathrm{gJDer}(A,\ M) \longrightarrow \mathrm{BJDer}(A,\ M)$ *is a K-isomorphism if and only if* $\Psi : M \longrightarrow \mathrm{JMul}(A,\ M)$ *is a K-isomorphism.*

Moreover, we have the following:

Theorem 5.3. *Let A be a K-algebra without a unit element, M an A/K-bimodule and $\widehat{A}$ a K-algebra defined as in §4. Let $(f,\ J)$ be a generalized J-Jordan derivation from A to M. Then the following conditions are equivalent:*

(1) *A generalized J-Jordan derivation $(f,\ J)$ from A to M can be extended to a generalized $\widehat{J}$-Jordan derivation from $\widehat{A}$ to M, where $\widehat{J} : \widehat{A} \longrightarrow M$ is a unique Jordan derivation such that $\widehat{J}|_A = J$.*

(2) *A Jordan left multiplier $f - J$ from A to M can be extended to a Jordan left multiplier from $\widehat{A}$ to M.*

(3) *There exists an element $m \in M$ such that $f - J = m_\ell$.*

Proof. (1) $\Rightarrow$ (2) and (3) $\Rightarrow$ (1) are similar to those in the proof of Theorem 4.1. So it remains to prove (2) $\Rightarrow$ (3). For $f - J \in \mathrm{JMul}(A,\ M)$, there exists its extension $G \in \mathrm{JMul}(\widehat{A},\ M)$. By the property of Jordan left multipliers, we have

$$G((n_1,\ a_1)(n_2,\ a_2) + (n_2,\ a_2)(n_1,\ a_1)) = G((n_1,\ a_1)) \cdot (n_2,\ a_2) + G((n_2,\ a_2)) \cdot (n_1,\ a_1)$$

for all $(n_1,\ a_1)$, $(n_2,\ a_2) \in \widehat{A}$. Then we get

$$\begin{aligned}
&G((0,\ a)) + G((0,\ a)) \\
&= G((0,\ a)(1,\ 0) + (1,\ 0)(0,\ a)) \\
&= G((0,\ a)) \cdot (1,\ 0) + G((1,\ 0)) \cdot (0,\ a) \\
&= G((0,\ a)) + G((1,\ 0))a.
\end{aligned}$$

Hence, we have, for all $a \in A$,

$$(f - J)(a) = G((0,\ a)) = G((1,\ 0))a.$$

By putting $m = G((1,\ 0)) \in M$, the result follows.

$\square$

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