A GEOMETRIC MEAN IN THE FURUTA INEQUALITY

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Abstract. Uchiyama discussed the Furuta inequality from the viewpoint of the Jensen inequality. Recently Furuta and Kamei improved it as follows: Suppose that $A, B, C > 0$ and $r, s \geq 0$. If $A^t \ll B^t \;\nabla_\mu\; C^t$ for all $t \geq 0$, then

$$f(t) = A^{-r} \;\sharp_{\frac{s+r}{t+r}}\; (B^t \;\nabla_\mu\; C^t)$$

is an increasing function of $t \geq s$. On the other hand, if $A^t \ll B^t \;!_\mu\; C^t$ for all $t \geq 0$, then

$$h(t) = A^{-r} \;\sharp_{\frac{s+r}{t+r}}\; (B^t \;!_\mu\; C^t)$$

is a decreasing function of $t \geq s$.

In this note, we pay our attention to the assumptions in above and point out that the operator function $F(s) = ((1-\mu)A^s + \mu B^s)^{\frac{1}{s}}$ ($s \in \mathbb{R}$) for given $A, B > 0$ and $\mu \in [0, 1]$ is monotone increasing under the chaotic order $X \gg Y$ defined by $\log X \geq \log Y$ and consequently s-$\lim_{h \to 0} F(h) = e^{(1-\mu)\log B + \mu \log C}$.This means that we can see another geometric mean $B \diamondsuit_\mu C = e^{(1-\mu)\log B + \mu \log C}$ in the Furuta inequality. Moreover we consider Uchiyama's result in a general setting.

1. Introduction

First of all, we cite the Löwner-Heinz inequality (LH) which is one of the most fundamental operator inequalities: If A and B are positive operators acting on a Hilbert space H and satisfy $A \geq B$, then $A^p \geq B^p$ for all $p \in [0, 1]$. In 1987, Furuta [8] established the following historical extension of (LH), see [13], [9], [2] and [15].

The Furuta inequality

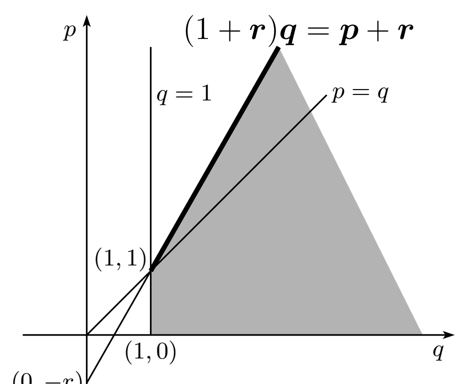
If $A \geq B \geq 0$, then for each $r \geq 0$,

(i) $$(B^{\frac{r}{2}} A^p B^{\frac{r}{2}})^{\frac{1}{q}} \geq (B^{\frac{r}{2}} B^p B^{\frac{r}{2}})^{\frac{1}{q}}$$

and

(ii) $$(A^{\frac{r}{2}} A^p A^{\frac{r}{2}})^{\frac{1}{q}} \geq (A^{\frac{r}{2}} B^p A^{\frac{r}{2}})^{\frac{1}{q}}$$

hold for $p \geq 0$ and $q \geq 1$ with $(1+r)q \geq p + r$.



Figure

Motivated by Ando's inequality [1], we introduced the chaotic order among positive invertible operators [7]: For $A, B > 0$, we denote by $A \gg B$ if $\log A \geq \log B$. Finally we

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obtained the following chaotic version (FC) of the Furuta inequality, [3] and [10], see also [4], [5] and [6]:

For $A, B > 0$, $A \gg B$, i.e., $\log A \geq \log B$, if and only if

$$\left(A^{\frac{r}{2}} A^p A^{\frac{r}{2}}\right)^{\frac{r}{p+r}} \geq \left(A^{\frac{r}{2}} B^p A^{\frac{r}{2}}\right)^{\frac{r}{p+r}}$$

for all $p,\ r \geq 0$. This is expressed in terms of the monotonicity of an operator function.

Theorem A . *For $A, B > 0$, $A \ll B$ if and only if for each $s \geq 0$, $G(t,r) = A^{-r} \sharp_{\frac{s+r}{t+r}} B^t$ is an increasing function of both $t \geq s$ and $r \geq 0$, where $\sharp_\alpha$ is the α-geometric mean.*

Recently, Uchiyama [16] gave a new viewpoint to the Furuta inequality. He explained that it is from the Jensen inequality for operator concave functions.

Theorem B . *If $A \leq B\ !_\mu\ C$ for $A, B, C > 0$, then*

$$B^s\ \nabla_\mu\ C^s \leq A^{-r}\ \sharp_{\frac{s+r}{t+r}}\ (B^t\ \nabla_\mu\ C^t)$$

for $r \geq 0$ and $t \geq s \geq 0$, where $!_\mu$ and ∇_μ are μ-harmonic and arithmetic means respectively.

Afterwards, we were given an opportunity to see a paper [11] by Furuta and Kamei, in which Theorem B is improved from the viewpoint of Theorem B.

Theorem C . *Suppose that $A, B, C > 0$ and $r, s \geq 0$. If $A^t \ll B^t\ \nabla_\mu\ C^t$ for all $t \geq 0$, then*

$$f(t) = A^{-r}\ \sharp_{\frac{s+r}{t+r}}\ (B^t\ \nabla_\mu\ C^t)$$

is an increasing function of $t \geq s$. On the other hand, if $A^t \ll B^t\ !_\mu\ C^t$ for all $t \geq 0$, then

$$h(t) = A^{-r}\ \sharp_{\frac{s+r}{t+r}}\ (B^t\ !_\mu\ C^t)$$

is a decreasing function of $t \geq s$.

In this note, we pay our attention to the assumptions of Theorems B and C. Namely we discuss the monotonicity of the operator function

$$F(s) = ((1-\mu)A^s + \mu B^s)^{\frac{1}{s}} \quad (s \in \mathbb{R})$$

for given $A, B > 0$ and $\mu \in [0,1]$. It is not monotone increasing under the usual operator order, but we can prove that it is monotone increasing under the chaotic order and moreover s-$\lim_{h\to 0} F(h) = e^{(1-\mu)\log A + \mu \log B}$. We call it the chaotically μ-geometric mean $A \diamondsuit_\mu B$ of A and B. So we can reformulate Theorem C and generalize Theorem B. This means that we find, in the Furuta inequality, another geometric mean different from the geometric mean $\sharp_\mu$ in the sense of Kubo-Ando. Of course, they coincide if A and B commute.

2. The chaotically geometric mean

In this section, we discuss the monotonicity of the operator function $F(s)$. First of all, we do it under the usual operator order.

Lemma 1. *Let $B,\ C > 0$ and $\mu \in [0,1]$ be given. Then the operator function $F(s) = ((1-\mu)B^s + \mu C^s)^{\frac{1}{s}}$ $(s \in \mathbb{R})$ is monotone increasing on $[1, \infty)$, i.e., $F(s) \leq F(t)$ if $1 \leq s \leq t$. In addition, $F(s) \leq F(t)$ if $1 \leq t \leq 2s$, and $F(s)$ is not monotone increasing on $(0,1]$ in general.*

Proof. The first assertion follows from the operator concavity of the function x^r ($r \in [0,1]$): If $1 \le s \le t$, then

$$((1-\mu)B^t + \mu C^t)^{\frac{s}{t}} \ge (1-\mu)B^s + \mu C^s$$

and so $F(t) \ge F(s)$ by (LH). On the other hand, the second one follows from the operator convexity of x^r for $1 \le r \le 2$: If $1 \le t \le 2s$, then

$$((1-\mu)B^s + \mu C^s)^{\frac{t}{s}} \le (1-\mu)B^t + \mu C^t$$

and so $F(s) \le F(t)$ by (LH).

Finally we give a simple counterexample to the third one as follows:

$$B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^3, \quad C = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}^3.$$

Then

$$F(1) = \frac{1}{2}(B+C) = \begin{pmatrix} 14 & 14 \\ 14 & 20 \end{pmatrix}$$

and

$$F(\frac{1}{3}) = [\frac{1}{2}(B^{\frac{1}{3}} + C^{\frac{1}{3}})]^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^3 = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix},$$

so that

$$F(1) - F(\frac{1}{3}) = \begin{pmatrix} 0 & 1 \\ 1 & 6 \end{pmatrix} \not\ge 0.$$

Next we discuss it under the chaotic order.

Lemma 2. *The operator function $F(s)$ is monotone increasing under the chaotic order, i.e., $F(s) \ll F(t)$ if $s < t$. In particular,*

$$\text{s} - \lim_{h \to 0} F(h) = e^{(1-\mu)\log B + \mu \log C}.$$

Proof. It suffices to show that for $s < t$ with $s, t \ne 0$

$$\frac{1}{s}\log((1-\mu)B^s + \mu C^s) \le \frac{1}{t}\log((1-\mu)B^t + \mu C^t).$$

To prove this, the operator concavity of x^r for $r \in [0,1]$ is available. We first assume $0 < s < t$. Then

$$\log((1-\mu)B^t + \mu C^t)^{\frac{s}{t}} \ge \log((1-\mu)B^s + \mu C^s),$$

and so $\log F(t) \ge \log F(s)$. Next, if $s < t < 0$, then $\frac{t}{s} \in (0,1)$ and hence

$$\log((1-\mu)B^s + \mu C^s)^{\frac{t}{s}} \ge \log((1-\mu)B^t + \mu C^t).$$

Noting $t < 0$, we have $\log F(s) \le \log F(t)$.

Now we prove the second assertion. By the operator concavity of $\log x$ and the Krein inequality $x - 1 \geq \log x$, it implies that for any $t > 0$

$$\begin{aligned}
&(1-\mu)\log B + \mu \log C \\
&= \frac{1}{t}((1-\mu)\log B^t + \mu \log C^t) \\
&\leq \frac{1}{t}\log((1-\mu)B^t + \mu C^t) \\
&\leq \frac{1}{t}((1-\mu)B^t + \mu C^t - 1) \\
&= (1-\mu)\frac{B^t - 1}{t} + \mu \frac{C^t - 1}{t} \\
&\to (1-\mu)\log B + \mu \log C \quad (t \to +0).
\end{aligned}$$

Therefore it follows that

$$s - \lim_{t \to +0} \log((1-\mu)B^t + \mu C^t)^{\frac{1}{t}} = (1-\mu)\log B + \mu \log C,$$

so that

$$s - \lim_{t \to +0} ((1-\mu)B^t + \mu C^t)^{\frac{1}{t}} = e^{(1-\mu)\log B + \mu \log C}.$$

On the other hand, it follows from the identity obtained above that for $s > 0$

$$\begin{aligned}
F_{B,C}(-s) &= F_{B^{-1},C^{-1}}(s)^{-1} \\
&\to [e^{(1-\mu)\log B^{-1} + \mu \log C^{-1}}]^{-1} \\
&= e^{(1-\mu)\log B + \mu \log C}.
\end{aligned}$$

Hence we have the second assertion, which says that s-$\lim_{h \to 0} F(h)$ can be regarded as $F(0)$. Therefore, if $s < 0 < t$, then

$$F(s) \ll F(0) \ll F(t).$$

Consequently we have the monotonicity of $F(s)$.

For the sake of convenience, we define another geometric mean:

Definition 3. *For $B, C > 0$ and $\mu \in [0,1]$,*

$$B \diamondsuit_\mu C = e^{(1-\mu)\log B + \mu \log C}$$

is said to be the chaotically μ-geometric mean of B and C.

Theorem 4. *For $B, C > 0$ and $\mu \in [0,1]$, both $(B^t \nabla_\mu C^t)^{\frac{1}{t}}$ and $(B^t \,!_\mu\, C^t)^{\frac{1}{t}}$ converge to the chaotically μ-geometric mean $B \diamondsuit_\mu C$ as $t \searrow 0$. Consequently*

$$s - \lim_{t \searrow 0} (B^t \sharp_\mu C^t)^{\frac{1}{t}} = B \diamondsuit_\mu C.$$

Proof. The first assertion follows from Lemma 2. To prove the second one, it suffices to show that $\log(B^t \sharp_\mu C^t)^{\frac{1}{t}}$ converges to $(1-\mu)\log B + \mu \log C$. By the well-known arithmetic-geometric mean inequality, we have

$$B^t \,!_\mu\, C^t \leq B^t \sharp_\mu C^t \leq B^t \nabla_\mu C^t,$$

so that

$$\log(B^t \,!_\mu\, C^t) \leq \log(B^t \sharp_\mu C^t) \leq \log(B^t \nabla_\mu C^t).$$

By multiplying $\frac{1}{t}$ on each term, it follows from Lemma 2 that the middle term $\frac{1}{t}\log(B^t \sharp_\mu C^t)$ must converge to $(1-\mu)\log B + \mu \log C$.

Remark. The second assertion of Theorem 4 appeared in [12, Lemma 3.3].

3. Uchiyama's generalization on the Furuta inequality

As stated in Theorem B, Uchiyama gave an interesting viewpoint to the Furuta inequality. Recently it is considered under the chaotic order by Furuta-Kamei, which we cite as Theorem C. We now reformulate it by using the chaotically μ-geometric mean.

Theorem 5. *For $A, B, C > 0$ and $\mu \in [0,1]$, the following statements are mutually equivalent:*

(1) $A \ll B \diamondsuit_\mu C$.

(2) $B^s \nabla_\mu C^s \leq A^{-r} \sharp_{\frac{s+r}{t+r}} (B^t \nabla_\mu C^t)$ for $r \geq 0$ and $t \geq s \geq 0$.

(3) For each $r, s \geq 0$, $f(t) = A^{-r} \sharp_{\frac{s+r}{t+r}} (B^t \nabla_\mu C^t)$ is an increasing function of $t \geq s$.

Proof. First of all, we note that (1) is equivalent to the condition $A^t \ll B^t \nabla_\mu C^t$ for all $t \geq 0$ by Lemma 2 and Theorem 4. That is, (1) implies (3) has been proved in Theorem C. If (3) holds, then (2) is obtained by putting $t = s$. Finally, if (2) holds for $s = 0$, then for each $t > 0$, $1 \leq A^{-r} \sharp_{\frac{r}{t+r}} (B^t \nabla_\mu C^t)$ for all $r \geq 0$. It is equivalent to (1) by (FC) stated in §1.

The following theorem is a complement to Theorem 5, which is corresponding to the second assertion of Theorem C.

Theorem 6. *For $A, B, C > 0$ and $\mu \in [0,1]$, the following statements are mutually equivalent:*

(1) $A \gg B \diamondsuit_\mu C$.

(2) $B^s \,!_\mu\, C^s \leq A^{-r} \sharp_{\frac{s+r}{t+r}} (B^t \,!_\mu\, C^t)$ for $r \geq 0$ and $t \geq s \geq 0$.

(3) For each $r, s \geq 0$, $h(t) = A^{-r} \sharp_{\frac{s+r}{t+r}} (B^t \,!_\mu\, C^t)$ is a decreasing function of $t \geq s$.

Proof. Clearly (1) is equivalent to the condition $A^{-1} \ll B^{-1} \diamondsuit_\mu C^{-1}$. So it follows from Theorem 5 that (1) means $f_{A^{-1},B^{-1},C^{-1}}(t)$ is monotone increasing. Moreover, since $h_{A,B,C}(t)^{-1} = f_{A^{-1},B^{-1},C^{-1}}(t)$, (1) holds if and only if $h(t)$ is monotone decreasing, i.e., (3) holds. The proof of the others is similar to that of Theorem 5.

We note that Theorems 4 - 6 require an improvement of Theorem B. As a matter of fact, we can reply as follows:

Theorem 7. *Suppose that $A, B, C > 0$ satisfy $A \ll (B^{t_0} \nabla_\mu C^{t_0})^{1/t_0}$ for some t_0. If $t_0 \geq 0$, then*

$$B^s \nabla_\mu C^s \leq A^{-r} \sharp_{\frac{s+r}{t+r}} (B^t \nabla_\mu C^t)$$

for all $r \geq 0$ and $t \geq s \geq 0$ with $t \geq t_0$. On the other hand, if $t_0 < 0$, then

$$(B^t \,!_\mu\, C^t)^{\frac{s}{t}} \leq A^{-r} \sharp_{\frac{s+r}{t+r}} (B^t \,!_\mu\, C^t)$$

for all $r \geq 0$ and $-t_0 \geq t \geq s \geq 0$.

Proof. We need the following fact [14, Theorem 2 (3)] obtained by (FC): If $A \ll B$, then $B^s \leq A^{-r} \sharp_{\frac{s+r}{t+r}} B^t$ for all $r \geq 0$ and $t \geq s \geq 0$. We first suppose that $A \ll F(t_0)$ for some $t_0 > 0$. Since $A \ll F(t)$ for $t \geq t_0$ by Lemma 2, we have

$$F(t)^s \leq A^{-r} \sharp_{\frac{s+r}{t+r}} F(t)^t$$

for all $r \geq 0$ and $t \geq s \geq 0$ with $t \geq t_0$. On the other hand, since $F(t)^s = (B^t \nabla_\mu C^t)^{\frac{s}{t}} \geq B^s \nabla_\mu C^s$ by $t \geq s \geq 0$, it follows that

$$B^s \nabla_\mu C^s \leq F(t)^s \leq A^{-r} \natural_{\frac{s+r}{t+r}} F(t)^t = A^{-r} \natural_{\frac{s+r}{t+r}} B^t \nabla_\mu C^t.$$

Next we suppose that $A \ll F(t_0)$ for some $t_0 < 0$. Since

$$A \ll F(t_0) \ll F(-t) = (B^t\ !_\mu\ C^t)^{\frac{1}{t}}$$

for $-t_0 \geq t \geq s \geq 0$, we have the desired inequality

$$(B^t\ !_\mu\ C^t)^{\frac{s}{t}} \leq A^{-r} \natural_{\frac{s+r}{t+r}} (B^t\ !_\mu\ C^t)$$

by applying the inequality cited in above again.

For the sake of convenience, we cite a mean theoretic proof of the inequality [14, Theorem 2 (3)] used above: For this, (FC) under $A \ll B$ is expressed as $1 \leq A^{-r} \natural_{\frac{r}{t+r}} B^t = B^t \natural_{\frac{t}{t+r}} A^{-r}$ for $r,\ t \geq 0$. Thus, if $A \ll B$ and $r \geq 0$, then for $t \geq s \geq 0$

$$A^{-r} \natural_{\frac{s+r}{t+r}} B^t = B^t \natural_{\frac{t-s}{t+r}} A^{-r} = B^t \natural_{\frac{t-s}{t}} (B^t \natural_{\frac{t}{t+r}} A^{-r}) \geq B^t \natural_{\frac{t-s}{t}} 1 = 1 \natural_{\frac{s}{t}} B^t = B^s.$$

We now remark that Theorem 7 can be rephrased as a similar form to Theorem C.

Corollary 8. *Suppose that $A, B, C > 0$, $\mu \in [0,1]$ and $t_0 > 0$. Then the following statements are mutually equivalent:*

(1) $A \ll (B^{t_0} \nabla_\mu C^{t_0})^{1/t_0}$.

(2) $B^s \nabla_\mu C^s \leq A^{-r} \natural_{\frac{s+r}{t+r}} (B^t \nabla_\mu C^t)$ for all $r \geq 0$ and $t \geq s \geq 0$ with $t \geq t_0$.

(3) For each $r, s \geq 0$, $f(t) = A^{-r} \natural_{\frac{s+r}{t+r}} (B^t \nabla_\mu C^t)$ is an increasing function of t, precisely, $f(t) \geq f(t_1)$ for $t \geq t_1 \geq s$ with $t \geq t_0$.

Proof. (1) $\to$ (3): It is similar to that of Theorem C. Since $A \ll F(t_0) \ll F(t)$ for $t \geq t_0$ by Lemma 2, Theorem A implies that

$$A^{-r} \natural_{\frac{s+r}{t_1+r}} F(t)^{t_1} \leq A^{-r} \natural_{\frac{s+r}{t+r}} F(t)^t = f(t)$$

for $t \geq t_1 \geq s \geq 0$. Moreover, since the operator concavity of x^α ($\alpha \in [0,1]$) ensures that

$$F(t)^{t_1} = (B^t \nabla_\mu C^t)^{\frac{t_1}{t}} \geq B^{t_1} \nabla_\mu C^{t_1} = F(t_1)^{t_1},$$

we have

$$f(t_1) = A^{-r} \natural_{\frac{s+r}{t_1+r}} F(t_1)^{t_1} \leq A^{-r} \natural_{\frac{s+r}{t_1+r}} F(t)^{t_1} \leq f(t).$$

(3) $\to$ (2): If we take $t_1 = s$ in (3), then $f(s) \leq f(t)$ for $t \geq t_0$. Since $f(s) = B^s \nabla_\mu C^s$, we have (2). (2) $\to$ (1): We take $s = 0$ and $t = t_0$ in (2).

Corollary 9. *Suppose that $A, B, C > 0$, $\mu \in [0,1]$ and $t_0 < 0$. Then the following statements are mutually equivalent:*

(1) $A \ll (B^{t_0} \nabla_\mu C^{t_0})^{1/t_0}$.

(2) $(B^s\ !_\mu\ C^s)^{\frac{s}{t}} \leq A^{-r} \natural_{\frac{s+r}{t+r}} (B^t\ !_\mu\ C^t)$ for all $r \geq 0$ and $-t_0 \geq t \geq s \geq 0$.

(3) For each $t \in [s, -t_0]$ and $s \geq 0$, $k(r) = A^{-r} \natural_{\frac{s+r}{t+r}} (B^t\ !_\mu\ C^t)$ is an increasing function of $r \geq 0$.

Proof. (1) $\to$ (3): Lemma 2 implies that $A \ll F(-t)$ for $t \leq -t_0$. Since $F(-t)^t = B^t\ !_\mu\ C^t$, it follows from Theorem A that $k(r)$ is an increasing function of $r \geq 0$. Moreover (3) implies that $k(0) \leq k(r)$ for $r \geq 0$, that is, (2) holds, and (2) $\to$ (1) follows from putting $s = 0$ in (2).

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