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ON THE BOUNDARY LAYER FLOW OF AN INELASTIC FLUID NEAR A MOVING WALL

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Abstract. The paper presents a mathematical model for an inelastic fluid whose apparent viscosity is a polynomial function of the invariants of the rate of strain tensor. Starting with the constitutive equation, the two-dimensional boundary layer equations for the flow near a moving flat surface have been derived. The system of nonlinear partial differential equations for this flow has been subjected to a similarity analysis. This leads to a nonlinear ordinary differential equation (ODE) involving the similarity functions as well as a non-Newtonian parameter K and another parameter $r(= W/U$, W and U being the wall velocity and the free-stream velocity, respectively). For $r \ll 1$, two nonlinear ODEs governing the similarity functions have been derived. These equations have been subjected to a perturbation analysis in terms of K. The resulting systems of two-point boundary value problems have been solved using standard numerical techniques. The boundary layer velocity profiles have been presented graphically.

1 Introduction The non-Newtonian fluids for which the relationships between the stress tensor and the deformation rate tensor are nonlinear, arise in several industrial applications such as chemical, biochemical and mineral processing [1 – 3]. The extent to which the rheological properties of the fluids influence the flow features varies according to the specific fluid flow dynamics. This in turn is related to the manner in which the apparent viscosity of the fluid under consideration is defined with respect to the shear rate. It is also known that there are classes of incompressible inelastic fluids for which the apparent viscosity may decrease with shear rate (shear thinning or pseudoplastic behaviour), increase with shear rate (shear thickening or dilatant behaviour); or the fluid may possess yield stress, e.g., Bingham plastic material. In this respect, it may be remarked that major efforts in literature have been devoted to the study of pseudoplastic and viscoplastic behaviours. This has apparently been due to the frequent occurrences of such fluid behaviours in applications. However, with the increasing interest in the processing of highly concentrated suspensions and pastes, the analysis of dilatant fluids has assumed greater importance [4 – 6].

Our aim in this work is to consider a special type of dilatant fluid [7] which has received less attention in the literature. To this end, we have investigated fluid dynamic characteristics of a steady, laminar boundary layer flow over a moving wall. The governing boundary layer equations of the fluid model have first been derived using standard boundary layer approximations. Subsequently, these equations have been subjected to a similarity analysis under the assumption that the ratio of the wall velocity to the free-stream velocity, r, is small. The resulting ordinary differential equations have been solved using a regular perturbation technique followed by numerical integration.

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2 The Fluid Model A constitutive equation of an inelastic non-Newtonian fluid can be represented in the form [1]

$$\tau_{ij} = \mu(I_1, I_2, I_3)e_{ij} \tag{1}$$

where I_1, I_2 and I_3 are scalar invariants of the rate of strain tensor. For the two-dimensional flow being considered here, I_1 and I_3 vanish identically. Furthermore, we assume that $\mu(I_2)$ can be approximated as a power series in I_2:

$$\mu(I_2) = \mu_0 + \mu_1 I_2 + \mu_2 I_2^2 + \cdots \tag{2}$$

where μ_0 is the conventional zero-shear viscosity, while other coefficients μ_i (> 0) are the rheological parameters of the fluid. The series expansion (2) allows us to account for the shear thickening behaviour. In the present case, we retain only up to the first degree terms in I_2, and thus

$$\mu = \mu_0 + \mu_1 I_2 \tag{3}$$

The model given by equation (1), in conjunction with equation (3), is employed to theoretically analyze the boundary layer flow mentioned above.

3 Governing Equations Consider the steady, laminar flow of the rheological fluid, satisfying the constitutive equations (1) and (3), near a two-dimensional stagnation point on a wall moving in its own plane with velocity W_1. In the cartesian coordinate system, with a suitably chosen origin, the x-axis is taken along the plate, while the y-axis is perpendicular to it, into the fluid. The density ρ of the fluid is assumed to be constant. The equations governing the x- and y- components of velocity, $u = u(x, y)$, $v = v(x, y)$, and the pressure $p = p(x, y)$, are the usual momentum and continuity equations. Taking into account the influence of μ_0 and μ_1 on the stress components τ_{ij}, the x- and y- components of the momentum equations can be written as

$$\begin{aligned} \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) &= -\frac{\partial p}{\partial x} + \mu_0\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ &\quad + \mu_1\left[\left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 4\left(\frac{\partial u}{\partial x}\right)^2\right\}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right. \\ &\quad + 16\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2} + 2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial x^2}\right) \\ &\quad \left. + 4\frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\left(\frac{\partial^2 v}{\partial x^2} - 3\frac{\partial^2 v}{\partial y^2}\right)\right] \end{aligned} \tag{4}$$

$$\begin{aligned} \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) &= -\frac{\partial p}{\partial y} + \mu_0\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \\ &\quad + \mu_1\left[\left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 4\left(\frac{\partial u}{\partial x}\right)^2\right\}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right. \\ &\quad + 16\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 v}{\partial y^2} + 2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial y^2}\right) \\ &\quad \left. + 4\frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\left(3\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right)\right] \end{aligned} \tag{5}$$

The continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

In order to analyze the fluid motion near the solid boundary, it is necessary to derive the boundary layer equations of the flow. This is best done by considering the non-dimensional forms of the governing equations. To this end, we write

$$\begin{aligned} \Re &= U_1L/\nu_0, \quad \tilde{y}=\sqrt{\Re}y/L, \quad \tilde{x}=x/L \\ \tilde{u} &= u/U_1, \quad \tilde{v}=\sqrt{\Re}v/U_1, \quad \tilde{p}=p/(\rho U_1^2) \end{aligned} \tag{7}$$

where $\nu_0=\mu_0/\rho$ is the Newtonian kinematic viscosity, $\Re$ is the Reynolds number, and L and U_1 are the characteristic scales of length and velocity, respectively. Using equation (7) in equations (4) and (5) and neglecting tildes on the variables for convenience, we obtain

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+\frac{1}{\Re}\frac{\partial^2 u}{\partial x^2}+\bar{K}\left[3\Re\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}\right. \\ &+6\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}-\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial x^2}+4\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 u}{\partial y^2} \\ &-12\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+\frac{1}{\Re}\left\{20\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2}+3\left(\frac{\partial v}{\partial x}\right)^2\frac{\partial^2 u}{\partial y^2}\right. \\ &\left.-2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2}+4\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2}-12\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial y^2}\right\} \\ &\left.+\frac{1}{\Re^2}\left\{4\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2}-\left(\frac{\partial v}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2}\right\}\right] \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{1}{\Re}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) &= -\frac{\partial p}{\partial y}+\frac{1}{\Re}\frac{\partial^2 v}{\partial y^2}+\frac{1}{\Re^2}\frac{\partial^2 v}{\partial x^2}-\bar{K}\left[\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 v}{\partial y^2}\right. \\ &+4\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}-\frac{1}{\Re}\left\{3\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 v}{\partial x^2}-2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial y^2}\right. \\ &\left.+20\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 v}{\partial y^2}+12\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}-4\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}\right\} \\ &-\frac{1}{\Re^2}\left\{6\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2}-\left(\frac{\partial v}{\partial x}\right)^2\frac{\partial^2 v}{\partial y^2}\right. \\ &\left.+4\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 v}{\partial x^2}+12\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2}\right\} \\ &\left.-\frac{3}{\Re^3}\left(\frac{\partial v}{\partial x}\right)^2\frac{\partial^2 v}{\partial x^2}\right] \end{aligned} \tag{9}$$

where $\bar{K}=(\mu_1U_1^2)/(\mu_0L^2)$ is a non-dimensional parameter characterising the ratio of non-Newtonian to ordinary viscous forces. One may also note that the continuity equation (6) remains unchanged in the non-dimensional form. Thus, equations (8) and (9) together with equation (6) are the governing equations for the two-dimensional flow in non-dimensional forms. Equations (8) and (9) can be further simplified under boundary layer approximation by assuming that the Reynolds number $\Re$ is several orders of magnitude larger than unity. Following the usual boundary layer analysis [8], we retain the lowest order effect of $\Re$. To this order, and noting that the non-Newtonian parameter $\bar{K}$ is assumed to be of order $1/\Re$ [7], equations (8) and (9) reduce, respectively, to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + 3\bar{K}\left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} \tag{10}$$

$$\frac{\partial p}{\partial y} = 0 \tag{11}$$

The boundary conditions to be satisfied by the velocity variables are

$$u = W, \quad v = 0 \ \text{ at } \ y = 0; \quad u \to U \ \text{ as } \ y \to \infty \tag{12}$$

where U and W are the non-dimensional free-stream velocity and wall velocity, respectively.

4 Method of Solution Following Gorla [9], we seek similarity solutions using the transformations

$$u = Uf'(y) + Wg'(y), \quad v = -f(y) \tag{13}$$

where prime denotes derivative with respect to y. It can be easily verified that the continuity equation (6) is automatically satisfied by equation (13). Using equation (13) in equation (10), we obtain

$$\begin{aligned} &f'^2 - ff'' - f''' - 1 + r\,(f'g' - fg'' - g''') - K\Big(f''^2 f''' + rf''^2 g''' \\ &\quad + 2rf''g''f''' + 2r^2 f''g''g''' + r^2 g''^2 f''' + r^3 g''^2 g'''\Big) = 0 \end{aligned} \tag{14}$$

where $K = 3\bar{K}U^2$, and $r = W/U$. In equation (14), K is a modified non-Newtonian parameter and r is a measure of the speed of the wall motion compared to the free-stream velocity. In order to solve the highly nonlinear ordinary differential equation (14), it is necessary to make further assumptions on the flow. In this paper, we shall obtain the solution corresponding to $r \ll 1$. Equating to zero the coefficients of r^0 and r^1, we obtain a set of two coupled nonlinear ordinary differential equations

$$f''' + ff'' - f'^2 + 1 + Kf''^2 f''' = 0 \tag{15}$$

$$g''' + fg'' - f'g' + K\left(f''^2 g''' + 2f''g''f'''\right) = 0 \tag{16}$$

The transformed boundary conditions are

$$f(0) = f'(0) = g(0) = g'(\infty) = 0, \quad f'(\infty) = g'(0) = 1 \tag{17}$$

Our prime objective in this work is to analyze the effect of the non-Newtonian parameter K on the stagnation point flow. The velocity functions are now governed by equations (15)–(17). In order to solve them, we further assume that the non-Newtonian effects are small so that a perturbation solution can be obtained. We write

$$f(y) = f_0(y) + Kf_1(y) + K^2 f_2(y) + \cdots \tag{18}$$

$$g(y) = g_0(y) + Kg_1(y) + K^2 g_2(y) + \cdots \tag{19}$$

On using equations (18) and (19) in equations (15) and (16) and equating to zero the coefficients of each power of K, we obtain a set of ordinary differential equations governing f_i and g_i, $(i = 0, 1, 2, \dots)$. It is evident from the available literature on similar studies of other fluid models (see, e.g., [9 – 12]) that higher order terms f_i and g_i, $(i = 3, 4, \dots)$ in the respective series for f and g, do not contribute significantly to the function values. Moreover, the algebra involved in the derivation of the corresponding set of coupled ordinary differential equations and then finding their solutions is a cumbersome process with little consequence to the flow analysis. In Section 5, we have also given justification (see Tables 1 and 2) for neglecting terms of degree 3 and above in the series (18) and (19). We shall thus consider here terms up to f_2 and g_2 only. The relevant equations are

$$f_0''' + f_0 f_0'' - {f_0'}^2 + 1 = 0 \tag{20}$$

$$f_1''' + f_0 f_1'' - 2f_0' f_1' + f_0'' f_1 = -{f_0''}^2 f_0''' \tag{21}$$

$$f_2''' + f_0 f_2'' - 2f_0' f_2' + f_0'' f_2 = {f_1'}^2 - f_1 f_1'' - {f_0''}^2 f_1''' - 2f_0'' f_0''' f_1'' \tag{22}$$

$$g_0''' + f_0 g_0'' - f_0' g_0' = 0 \tag{23}$$

$$g_1''' + f_0 g_1'' - f_0' g_1' = f_1' g_0' - f_1 g_0'' - {f_0''}^2 g_0''' - 2f_0'' g_0'' f_0''' \tag{24}$$

$$\begin{aligned} g_2''' + f_0 g_2'' - f_0' g_2' = {} & f_1' g_1' + f_2' g_0' - f_1 g_1'' - f_2 g_0'' - {f_0''}^2 g_1''' \\ & - 2\left(f_0'' f_1'' g_0''' + f_0'' f_0''' g_1'' + f_1'' f_0''' g_0'' + f_0'' f_1''' g_0''\right) \end{aligned} \tag{25}$$

The boundary conditions become

$$f_0(0) = f_0'(0) = g_0(0) = g_0'(\infty) = 0, \quad f_0'(\infty) = g_0'(0) = 1 \tag{26}$$

$$f_1(0) = f_1'(0) = f_1'(\infty) = g_1(0) = g_1'(0) = g_1'(\infty) = 0 \tag{27}$$

$$f_2(0) = f_2'(0) = f_2'(\infty) = g_2(0) = g_2'(0) = g_2'(\infty) = 0 \tag{28}$$

It is worth noting that the above analysis applies to the case when the wall velocity $W \neq 0$. For the stagnation point flow near a stationary wall ($W = 0$), the problem can be analyzed in terms of a single similarity function f [7].

5 Numerical Results The set of boundary value problems, equations (20)–(28), governing the functions f_i and g_i ($i = 0,\ 1,\ 2$) have been numerically integrated using the NAG Subroutine D02AGFE. As stated in the previous section, in order to justify the perturbation approximation of the solutions for small deviations from the Newtonian profiles, we shall first present the relative errors in the partial sums of the series (18) and (19) up to and including the terms of degree 2 in K. Let S_n, $(n = 0, 1, 2)$, denote the partial sums of either series up to this order. Obviously, S_0 denotes the Newtonian value while S_1 and S_2 refer, respectively, to the first order and second order contributions due to the non-Newtonian effects. As our study is devoted to the analysis of the non-Newtonian effects over the corresponding Newtonian flows, it is instructive to compute the relative errors between S_1 $(= \ f_0 + Kf_1$ or $g_0 + Kg_1)$ and S_2 $(= \ f_0 + Kf_1 + K^2 f_2$ or $g_0 + Kg_1 + K^2 g_2)$.

The numerical values of the partial sums S_1 and S_2 as also their relative percentage errors have thus been given in Tables 1 and 2. It may be noted that the inclusion of the second order approximation in either series gives acceptable accuracy in comparison to the first order approximation. In particular, when K is very small $(0.1 \sim 0.3)$, the maximum error between the first and second order approximations for both f and g is generally less than 2%. However, as should be expected, the relative percentage errors increase marginally as K assumes larger values. The results for the velocity in the boundary layer and the missing wall derivatives presented below should thus be seen in this perspective.

Table 1: Partial sums of f-series and percentage errors

K	y	$f_0 + Kf_1$ (S_1)	$f_0 + Kf_1 + K^2 f_2$ (S_2)	$\frac{\lvert S_1 - S_2 \rvert}{S_1} \times 100$
0.1	0.54	0.1512	0.1515	0.20
	1.08	0.5174	0.5180	0.12
	1.62	0.9930	0.9936	0.06
	2.17	1.5139	1.5146	0.05
	2.71	2.0501	2.0508	0.03
	3.25	2.5905	2.5912	0.03
	3.79	3.1316	3.1323	0.02
	4.33	3.6730	3.6737	0.02
	4.87	4.2141	4.2148	0.02
	5.42	4.6886	4.6893	0.01
0.3	0.54	0.1439	0.1468	2.02
	1.08	0.5019	0.5069	1.00
	1.62	0.9733	0.9791	0.60
	2.17	1.4927	1.4987	0.40
	2.71	2.0283	2.0345	0.31
	3.25	2.5684	2.5746	0.24
	3.79	3.1091	3.1154	0.20
	4.33	3.6498	3.6563	0.18
	4.87	4.1903	4.1969	0.16
	5.42	4.6661	4.6722	0.13
0.5	0.54	0.1366	0.1448	6.00
	1.08	0.4863	0.5003	2.88
	1.62	0.9537	0.9697	1.68
	2.17	1.4715	1.4883	1.14
	2.71	2.0066	2.0236	0.85
	3.25	2.5463	2.5635	0.68
	3.79	3.0865	3.1040	0.57
	4.33	3.6267	3.6446	0.49
	4.87	4.1665	4.1848	0.44
	5.42	4.6435	4.6604	0.36

In order to analyze the effects of the governing parameters K and r, we have plotted the boundary layer velocity profiles (u/W *vs* y) for both Newtonian ($K = 0$) and non-

Table 2: Partial sums of g-series and percentage errors

K	y	$g_0 + Kg_1$ (S_1)	$g_0 + Kg_1 + K^2 g_2$ (S_2)	$\frac{\lvert S_1 - S_2 \rvert}{S_1} \times 100$
0.1	0.54	0.4349	0.4347	0.05
	1.08	0.6765	0.6761	0.06
	1.62	0.7861	0.7856	0.06
	2.17	0.8266	0.8259	0.08
	2.71	0.8386	0.8379	0.08
	3.25	0.8416	0.8408	0.10
	3.79	0.8424	0.8415	0.11
	4.33	0.8428	0.8418	0.12
	4.87	0.8431	0.8420	0.13
	5.42	0.8424	0.8413	0.13
0.3	0.54	0.4521	0.4506	0.33
	1.08	0.7168	0.7138	0.42
	1.62	0.8398	0.8353	0.54
	2.17	0.8857	0.8800	0.64
	2.71	0.8997	0.8930	0.74
	3.25	0.9035	0.8958	0.85
	3.79	0.9048	0.8964	0.93
	4.33	0.9057	0.8967	0.99
	4.87	0.9066	0.8973	1.03
	5.42	0.9045	0.8953	1.02
0.5	0.54	0.4692	0.4651	0.87
	1.08	0.7571	0.7487	1.11
	1.62	0.8934	0.8810	1.39
	2.17	0.9449	0.9291	1.67
	2.71	0.9609	0.9420	1.97
	3.25	0.9654	0.9440	2.22
	3.79	0.9672	0.9438	2.42
	4.33	0.9686	0.9437	2.57
	4.87	0.9701	0.9442	2.67
	5.42	0.9666	0.9412	2.63

Newtonian ($K = 0.3$) fluids. These are shown in Figs. 1 and 2, respectively, for a range of values of α ($= 1/r$), and include the physical situations corresponding to the free-stream velocity U being in the same or opposite directions of the wall velocity W. It may be observed that the velocity profiles of our dilatant fluid model are of uniform pattern, and converges to their free-stream values in the region near $y \approx 2$. This indicates that the boundary layer thickness is essentially independent of the magnitude of α, which is in agreement with a similar study for a viscoelastic fluid [9]. However, it is of interest to note that the overshooting effects for velocities reported in the literature for viscoelastic fluid models [9 – 12] are absent for the present non-Newtonian model. This implies that the velocity in the boundary layer for the present dilatant model cannot exceed the free-stream velocity. Although the non-Newtonian velocity profiles do not show much deviations from the Newtonian profiles within the perturbation approximations and the consequent small parameter values chosen here, we have nevertheless shown comparison of a couple of profiles in Fig. 3.

In many practical applications, prediction of the local wall shear stress is of great importance. The wall shear stress depends on the missing wall derivatives $f''(0)$ and $g''(0)$. The values of these derivatives have thus been computed and are given in Table 3 for a range of values of the rheological parameter K.

Table 3: Wall derivatives $f''(0)$ and $g''(0)$

K	$f''(0)$	$g''(0)$	K	$f''(0)$	$g''(0)$
0.00	1.2326	-0.8113	0.50	1.1542	-0.4258
0.05	1.2110	-0.7657	0.55	1.1632	-0.3960
0.10	1.1924	-0.7216	0.60	1.1752	-0.3677
0.15	1.1770	-0.6791	0.65	1.1903	-0.3409
0.20	1.1645	-0.6382	0.70	1.2085	-0.3158
0.25	1.1552	-0.5989	0.75	1.2297	-0.2922
0.30	1.1489	-0.5611	0.80	1.2540	-0.2702
0.35	1.1456	-0.5249	0.85	1.2813	-0.2498
0.40	1.1454	-0.4903	0.90	1.3117	-0.2310
0.45	1.1483	-0.4573	0.95	1.3452	-0.2137

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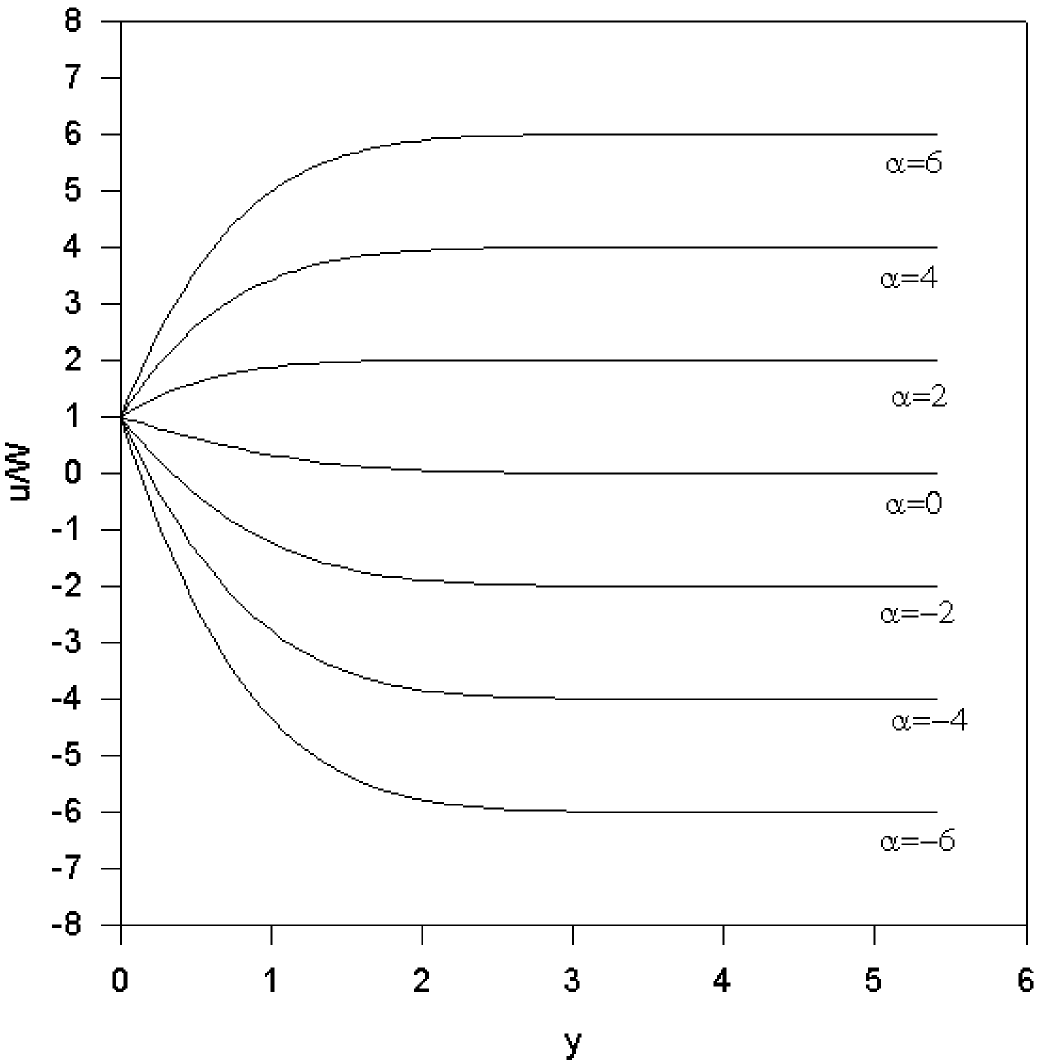

Figure 1: Boundary layer velocity profiles ($K = 0$)

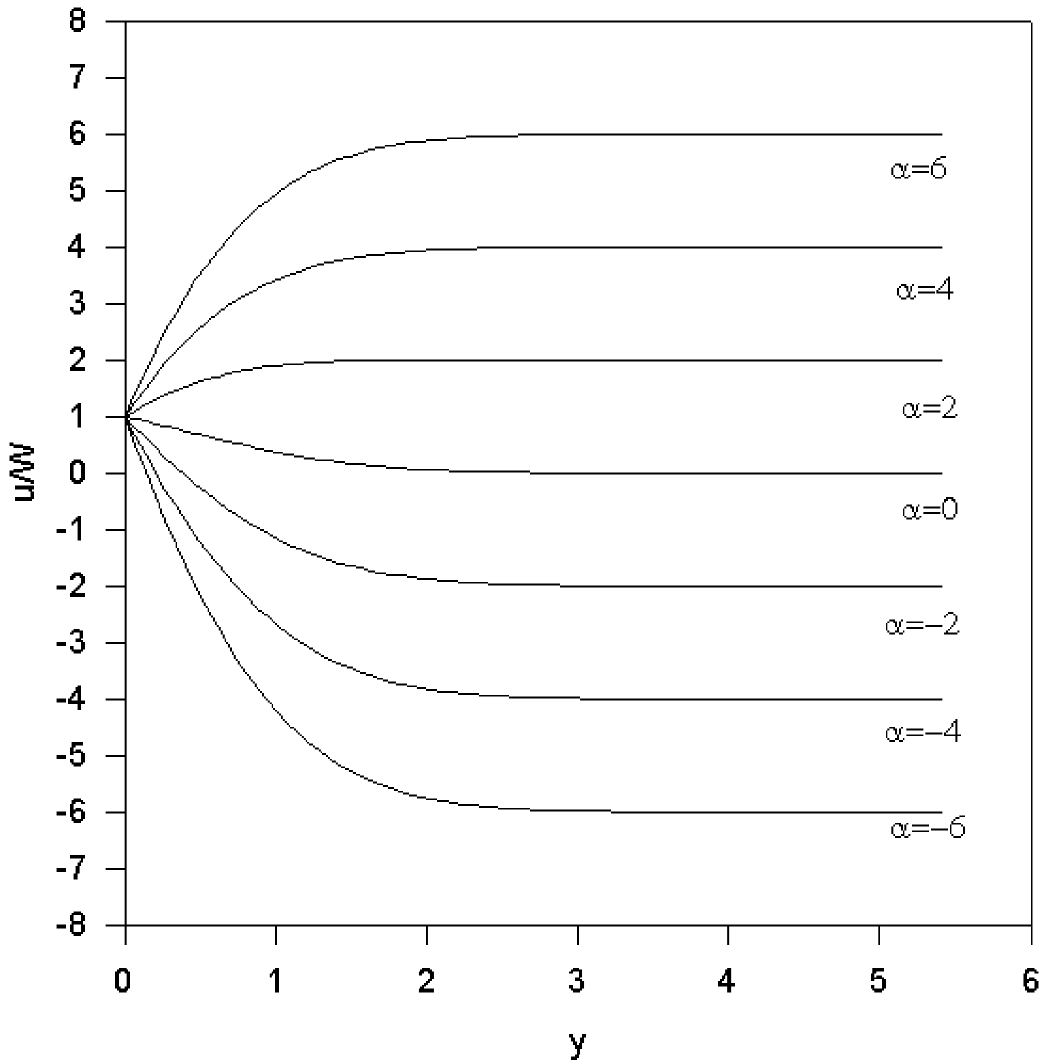

Figure 2: Boundary layer velocity profiles ($K = 0.3$)

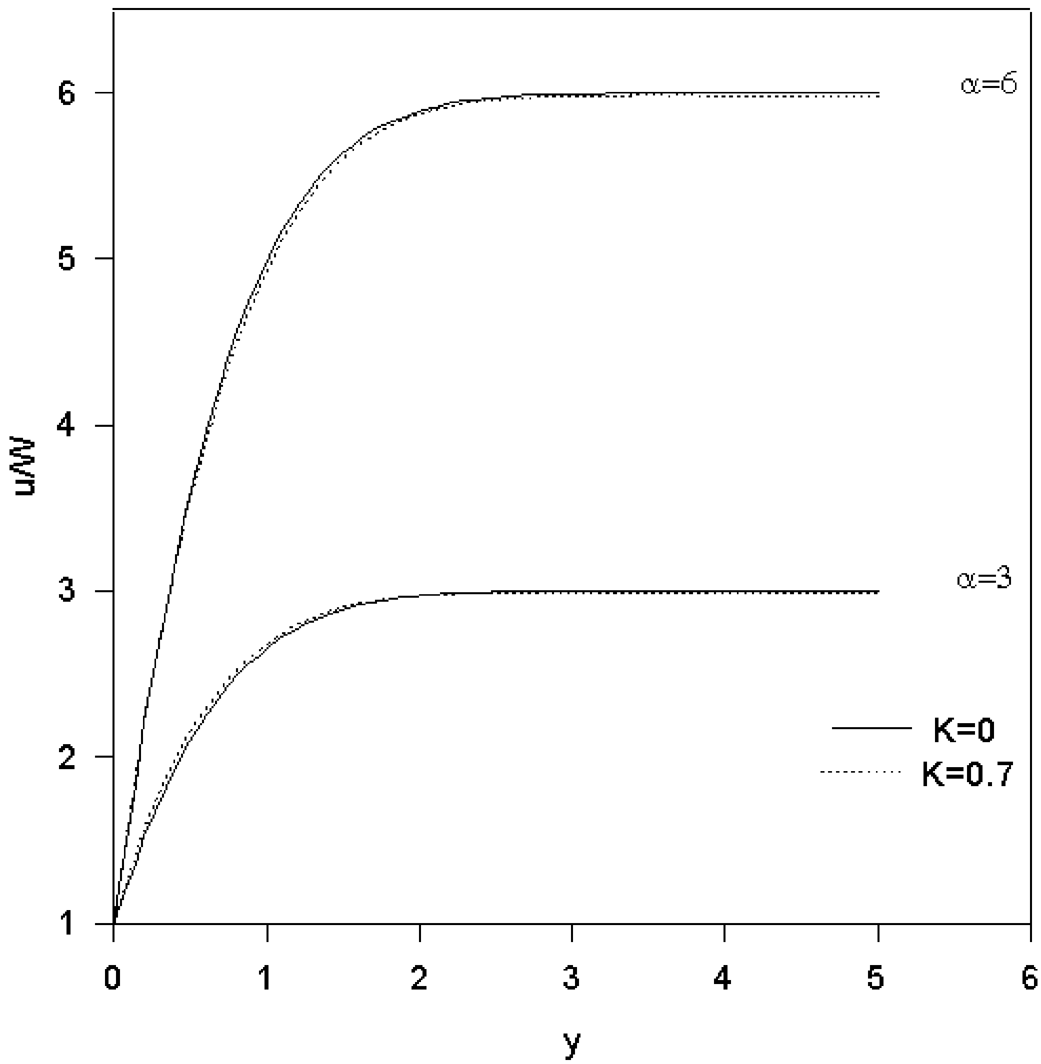

Figure 3: Comparison of Newtonian and non-Newtonian velocity profiles

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