PUBLIC OPINION SURVEY ON HOME EDUCATION: APPLICATION OF LOCATION PROBLEMS WITH RECTILINEAR NORM

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Abstract. In this paper, a public opinion survey on home education is considered an application of location problems with rectilinear norm. First, we search the structure of consciousness for "the educational item which is necessary for infants" and "the learning item which is necessary for parents having an infant" by applying quantification method III to the results of the questionnaire on home education. Next, location problems with rectilinear norm for the results of quantification method III are considered. As the learning contents which are near the required learning contents as much as possible, we propose concrete learning contents which should be provided by institutions providing learning.

1. Introduction As a survey research on lifelong learning and social education, Aomori Prefectural Community Education Center conducted a public opinion survey on home education by using questionnaires to adults in Aomori prefecture, Japan. For the fullness of home education power, this survey was conducted in order to make clear what requests the people of the prefecture have for the learning contents and activity on home education and to present each institution providing learning with the report as basic data [1].

Survey name: Survey Research on Lifelong Learning and Social Education —Survey Research on Home Education Power—

Survey period: August 14 - August 31, 2001

Survey method: (1)Mailing questionnaires (2)Sampling method (two-stage random sampling)

The number of valid results was 577(19.2%). We analyze on "the educational item which is necessary for infants" and "the learning item which is necessary for parents having an infant" by using questionnaire's answers of the following questions only for infants.

Questions

Q1 What are *the most important education items* in home for *infants, schoolchildren, junior high school students and high school students*, respectively? Choose *at most three numbers* of the following.

1 Basic living habitude (washing a face, uprising, greeting, etc)
2 Living experience (wringing a towel, caring for small children, using a knife, etc)
3 Natural experience (playing at the sea or the river, observing nature, mountaineering, etc)
4 Independence (an attitude to act by one's own judgement)
5 Self-control (suppressing feelings and desire)
6 Self-dependence (a mental attitude to do by oneself without help)
7 Abundant sentiment (feeling that the beautiful one is beautiful)
8 Consideration for others
9 A moral sense
10 Social manners
11 A sense of justice
12 Making human relations
13 A view of occupation
14 Sex education
15 Other (please describe concretely)
16 Don't know

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Q2 What are the most important *learning items for parents* to bring up their child? When the child is *an infant, a schoolchild, a junior high school student and a high school student*, respectively, choose *at most three numbers of necessary learning items for parents* of the following.

1 Fixation of basic living habitude 2 A way of developing self-dependence of a child
3 A way of communicating with a child 4 A way of growing as parents
5 Matters between husband and wife 6 Matters about a family 7 Social morality
8 Developmental stage of a child 9 Life style 10 Activity in community
11 A way of enriching natural experience 12 A way of nourishing tolerance and applicability
13 Sex education 14 About cooperative participation of man and woman
15 School maladjustment (non-going school, staying indoors, bullying, psychosomatic disease, etc)
16 Cruelty to children 17 Delinquency 18 Developmental disorder
19 No learning is necessary 20 Other (please describe concretely)

In this paper, the public opinion survey on home education is considered. First, we transform the results of questionnaires into categorized data and apply quantification method III to the data. Then we search the structure of consciousness for "the educational item which is necessary for infants" and "the learning item which is necessary for parents having an infant". Next, for further analysis, a multicriteria and a minisum location problem with rectilinear norm for the results of quantification method III are considered. As the learning contents which are near the learning contents required by the people of the prefecture as much as possible, we try to propose concrete learning contents which should be provided by institutions providing learning.

In section 2, the results of quantification method III for the results of the questionnaire are given. In section 3, location problems with rectilinear norm, which are useful to analyze the results of quantification method III, are introduced. In section 4, the results of a multicriteria and a minisum location problem for the results of quantification method III are given. Finally, some conclusions are given in section 5.

2. The results of quantification method III In this section, the results of quantification method III for the results of the questionnaire are given.

First, categories (answer items) that not many people answered are eliminated for each question. As the objects of analysis, we choose categories

1 Basic living habitude 2 Living experience 3 Natural experience 7 Abundant sentiment

for Q1 and categories

1 Fixation of basic living habitude 2 A way of developing self-dependence of a child
3 A way of communicating with a child 4 A way of growing as parents
8 Developmental stage of a child 11 A way of enriching natural experience

for Q2. Furthermore, eliminating results which have missing answers, we had categorized data as in Table 1 and 2. In the following, the results of quantification method III for each data are given.

Table 1. The results of Q1.

Individual No.	1 Living habitude	2 Living experience	3 Nature	7 Sentiment
1	1	0	0	1
2	1	1	1	0
⋮	⋮	⋮	⋮	⋮
492	1	1	1	0

Table 2. The results of Q2.

Individual No.	1 Living habitude	2 Self -dependence	3 Communication	4 Growing	8 Development	11 Nature
1	1	0	1	0	1	0
2	0	0	1	0	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
485	0	0	1	0	1	1

Quantification method III for Q1

Table 3 shows the results of quantification method III for the data of Table 1. Table 4 and 5 show categorical and individual scores, respectively.

Interpretation 1 (Categorical scores): From Table 4, categorical scores can be interpreted as follows:

- The smaller the first categorical score $y_1^{(1)}$ is, the more mental (sentimental) it means, and the larger $y_1^{(1)}$ is, the more physical (active) it means;
- The smaller the second categorical score $y_2^{(1)}$ is, the more non-usual place it means, and the larger $y_2^{(1)}$ is, the more usual place it means.

Interpretation 2 (Categorical space): Under Interpretation 1, each point in a categorical space (see Figure 1) represents the learning contents. Comparing a given point in the categorical space with points for categories, the learning contents for the given point can be known.

Interpretation 3 (Individual scores): Interpretations of the first individual score $x_1^{(1)}$ and the second individual score $x_2^{(1)}$ are the same as those of the first categorical score $y_1^{(1)}$ and the second categorical score $y_2^{(1)}$, respectively.

Interpretation 4 (Individual space): Under Interpretation 3, a point for each individual represents the learning contents which the individual requires and thinks important most. Each point in an individual space (see Figure 2) represents the learning contents. A given point $(x_1^{(1)}, x_2^{(1)})$ in the individual space can not be compared directly with points for categories. However, if we transform the given point into $(y_1^{(1)}, y_2^{(1)})$ in the categorical space by using Table 6 and compare the transformed point with points for categories, then the learning contents for the given point can be known. Transforming a point in the individual space into a point in the categorical space by using Table 6 means estimating the point in the categorical space from the point in the individual space by using regression analysis.

Consideration: From Table 5, points 2, 4, 7 and 10 have large individual frequencies. So we shall consider the learning contents for these points. The learning contents for these points mean the learning contents which many people of the prefecture require for "the educational item which is necessary for infants". Using Table 6, points 2,4,7 and 10 are transformed, respectively, into $(-0.859405, 0.364403)$, $(-0.458612, -0.330616)$, $(0.070827, 0.314693)$ and $(0.594021, -0.092548)$. Comparing these transformed points with points for categories, we can propose the learning contents for points 2, 4, 7 and 10 as follows:

- **"Sentiment cultivated in everyday life"** for the point 2;

- **"Sentiment cultivated by coming in contact with neighborhood nature"** for the point 4;
- **"Basic living habitude"** for the point 7;
- **"Play in nature"** for the point 10.

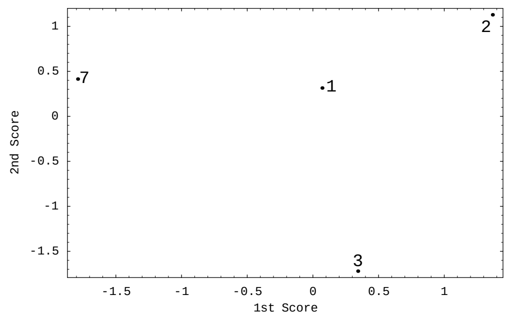

Figure 1. Categorical distribution for Q1.

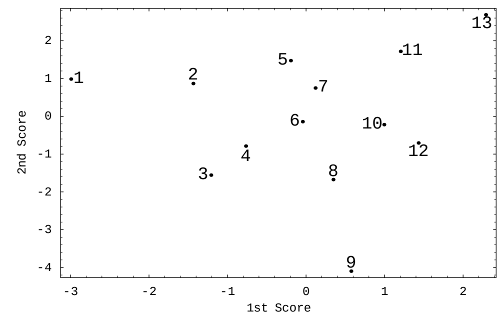

Figure 2. Individual ditribution for Q1.

Table 3. Quantification method III for Q1.

Order	Eigenvalue	Rate of contribution	Cumulative rate of contribution	Correlation coefficient
1	0.357379	53.28	53.28	0.597812
2	0.176344	26.29	79.58	0.419933
3	0.136980	20.42	100.00	0.370109

Table 4. Categorical scores for Q1.

Categorical No.	1st score $y_1^{(1)}$	2nd score $y_2^{(1)}$
1(Living habitude)	0.070827	0.314694
2(Living experience)	1.368262	1.128318
3(Nature)	0.342975	−1.720656
7(Sentiment)	−1.789637	0.414114

Table 5. Individual scores for Q1.

No.	1st score $x_1^{(1)}$	2nd score $x_2^{(1)}$	Individual frequency (persons)	Individual relative frequency (%)
1	−2.993646	0.986142	21	4.3
2	−1.437585	0.867765	62	12.6
3	−1.209965	−1.555654	12	2.4
4	−0.767151	−0.787306	91	18.5
5	−0.195462	1.474143	38	7.7
6	−0.043715	−0.141470	4	0.8
7	0.118477	0.749389	60	12.2
8	0.346096	−1.674030	34	6.9
9	0.573717	−4.097450	5	1.0
10	0.993659	−0.220388	125	25.4
11	1.203630	1.718143	35	7.1
12	1.431250	−0.705276	1	0.2
13	2.288784	2.686898	4	0.8

Table 6. Regression lines of $y_j^{(1)}$ on $x_j^{(1)}$ for Q1.

j	Regression line
1	$y_1^{(1)} = 0.597812x_1^{(1)}$
2	$y_2^{(1)} = 0.419933x_2^{(1)}$

Quantification method III for Q2

Table 7 shows the results of quantification method III for the data of Table 2. Table 8 and 9 show categorical and individual scores, respectively.

Interpretation 1 (Categorical scores): From Table 8, categorical scores can be interpreted as follows:

- The smaller the first categorical score $y_1^{(2)}$ is, the more mental (thinking) it means, and the larger $y_1^{(2)}$ is, the more physical (active) it means;
- The smaller the second categorical score $y_2^{(2)}$ is, the nearer to child's own matters it means, and the larger $y_2^{(2)}$ is, the more environmental around a child it means;
- The smaller the third categorical score $y_3^{(2)}$ is, the more physical growing of a child it means, and the larger $y_3^{(2)}$ is, the more mental growing of a child (and parents) it means.

Interpretation 2 (Categorical space): Under Interpretation 1, each point in a categorical space (see Figure 3) represents the learning contents. Comparing a given point in the categorical space with points for categories, the learning contents for the given point can be known.

Interpretation 3 (Individual scores): Interpretations of the first individual score $x_1^{(2)}$, the second individual score $x_2^{(2)}$ and the third individual score $x_3^{(2)}$ are the same, respectively, as those of the first categorical score $y_1^{(2)}$, the second categorical score $y_2^{(2)}$ and the third categorical score $y_3^{(2)}$.

Interpretation 4 (Individual space): Under Interpretation 3, a point for each individual represents the learning contents which the individual requires and thinks important most. Each point in an individual space (see Figure 4) represents the learning contents. A given point $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)})$ in the individual space can not be compared directly with points for categories. However, if we transform the given point into $(y_1^{(2)}, y_2^{(2)}, y_3^{(2)})$ in the categorical space by using Table 10 and compare the transformed point with points for categories, then the learning contents for the given point can be known. Transforming a point in the individual space into a point in the categorical space by using Table 10 means estimating the point in the categorical space from the point in the individual space by using regression analysis.

Consideration: From Table 9, points 17, 22 and 23 have large individual frequencies. The learning contents for these points mean the learning contents which many people of the prefecture require for "the learning item which is necessary for parents having an infant". Since the learning contents for points 17 and 22 are considered in section 4, we shall consider the learning contents for the point 23. Using Table 10, the point 23 is transformed into (0.141334, −0.227711, 0.081771). Comparing this transformed point with points for categories, we can propose the learning contents for the point 23 as follows:

• **"All-round matters about everyday life of a child"**.

From Figure 4, the required learning contents seem to form some groups each of which contains the similar required learning contents. If we can partition the required learning contents into such groups, then it is efficient to consider the learning contents which sould be provided for each group. Therefore, cluster analysis is applied to the data in Table 9. Table 11 shows the result of group average method with rectilinear norm. Since individual scores are normalized and a difference of each score between two points can be interpreted (explained) easily, rectilinear norm was used to measure the distance between two points. Each cluster represents the group of the learning contents in which the learning contents required for "the learning item which is necessary for parents having an infant" are similar.

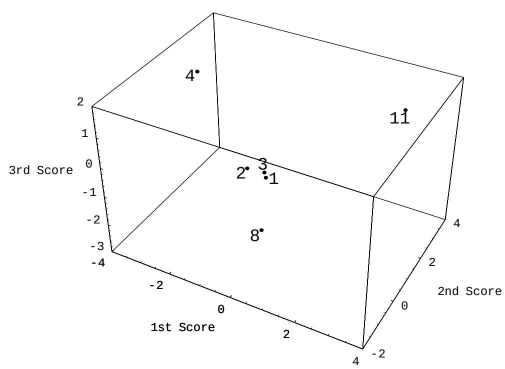

Figure 3. Categorical distribution for Q2.

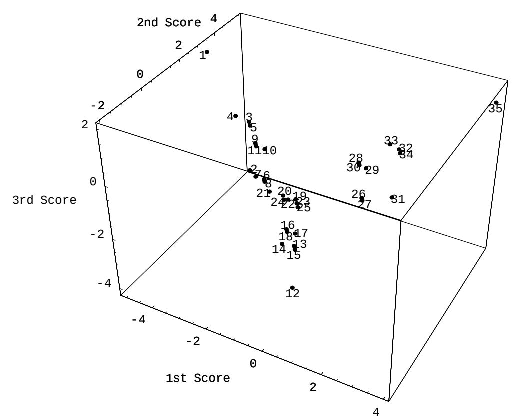

Figure 4. Individual ditribution for Q2.

Table 7. Quantification method III for Q2.

Order	Eigenvalue	Rate of contribution	Cumulative rate of contribution	Correlation coefficient
1	0.448307	29.20	29.20	0.669557
2	0.386511	25.18	54.38	0.621700
3	0.358713	23.37	77.75	0.598927
4	0.193679	12.62	90.36	0.440090
5	0.147927	9.64	100.00	0.384613

Table 8. Categorical scores for Q2.

Categorical No.	1st score $y_1^{(2)}$	2nd score $y_2^{(2)}$	3rd score $y_3^{(2)}$
1(Living habitude)	0.176262	−0.250243	0.021793
2(Self-dependence)	0.135729	−1.141599	0.894498
3(Communication)	0.106407	−0.205180	0.141748
4(Growing)	−3.284645	1.881493	1.234624
8(Development)	−0.344675	0.372356	−2.475496
11(Nature)	2.642057	3.369432	0.883005

Table 9. Individual scores for Q2.

No.	1st score $x_1^{(2)}$	2nd score $x_2^{(2)}$	3rd score $x_3^{(2)}$	Individual frequency (persons)	Individual relative frequency (%)
1	−4.905695	3.026367	2.061394	8	1.6
2	−2.710238	1.812650	−1.035913	4	0.8
3	−2.373387	1.348168	1.149032	8	1.6
4	−2.351490	0.595057	1.777448	3	0.6
5	−2.321222	1.311927	1.048891	4	0.8
6	−1.753851	1.098423	−0.611719	5	1.0
7	−1.739254	0.596349	−0.192775	1	0.2
8	−1.719075	1.074262	−0.678480	6	1.2
9	−1.514686	0.286694	1.263855	3	0.6
10	−1.494507	0.764608	0.778151	20	4.1
11	−1.479910	0.262533	1.197094	6	1.2
12	−0.514781	0.598933	−4.133220	7	1.4
13	−0.177930	0.134451	−1.948275	14	2.9
14	−0.156033	−0.618661	−1.319859	1	0.2
15	−0.125765	0.098209	−2.048416	24	4.9
16	−0.051048	−0.522451	−0.801016	6	1.2
17	−0.030870	−0.044537	−1.286721	51	10.5
18	−0.016272	−0.546612	−0.867777	13	2.7
19	0.158921	−0.330031	0.236671	19	3.9
20	0.180818	−1.083142	0.865086	9	1.9
21	0.202714	−1.836254	1.493501	2	0.4
22	0.208296	−0.856266	0.588853	107	22.1
23	0.211086	−0.366272	0.136529	60	12.4
24	0.232983	−1.119384	0.764944	20	4.1
25	0.263251	−0.402514	0.036387	29	6.0
26	1.196705	1.896203	−0.807413	3	0.6
27	1.231482	1.872042	−0.874174	3	0.6
28	1.435870	1.084474	1.068161	1	0.2
29	1.456049	1.562388	0.582457	23	4.7
30	1.470647	1.060313	1.001400	3	0.6
31	1.715597	3.009320	−1.329454	2	0.4
32	2.052448	2.544839	0.855491	6	1.2
33	2.074345	1.791727	1.483906	2	0.4
34	2.104613	2.508597	0.755349	9	1.9
35	3.945975	5.419708	1.474311	3	0.6

Table 10. Regression lines of $y_j^{(2)}$ on $x_j^{(2)}$ for Q2.

j	Regression line
1	$y_1^{(2)} = 0.669558x_1^{(2)}$
2	$y_2^{(2)} = 0.621700x_2^{(2)}$
3	$y_3^{(2)} = 0.598927x_3^{(2)}$

Table 11. The result of cluster anaysis for Q2.

Label	Cluster	Individual frequency (persons)	Individual relative frequency (%)
A	$\{1\}$	8	1.6
B	$\{2\}$	4	0.8
C	$\{3, 4, 5\}$	15	3.1
D	$\{6, 7, 8\}$	12	2.5
E	$\{9, 10, 11\}$	29	6.0
F	$\{12\}$	7	1.4
G	$\{13, 14, 15, 16, 17, 18\}$	109	22.5
H	$\{19, 20, 22, 23, 24, 25\}$	244	50.3
I	$\{21\}$	2	0.4
J	$\{26, 27\}$	6	1.2
K	$\{28, 29, 30\}$	27	5.6
L	$\{31\}$	2	0.4
M	$\{32, 33, 34\}$	17	3.5
N	$\{35\}$	3	0.6

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3. Location problems with rectilinear norm In this section, location problems with rectilinear norm, which are useful to analyze the results of quantification method III, are introduced.

In the previous section, for "the educational item which is necessary for infants" and "the learning item which is necessary for parents having an infant", the learning contents required by individuals are represented as points in the corresponding individual space. It is desirable for institutions providing learning to provide the learning contents which are near the required learning contents as much as possible. Since it is difficult to provide various learning contents required by all individuals, it is practical and important to propose such one or a few learning contents. If we want to find the learning contents which is near the required learning contents, then it is convenient to regard each learning contents as a point in the individual space. Because each point for the learning contents can be compared directly with the required learning contents. In the individual space, if the point which is near points for the required learning contents as much as possible is determined, it is possible to propose more concrete learning contents for the point. Such problem reduces to a problem to find the point which is near given points in $\mathbb{R}^n$ as much as possible, and can be formulated as a location problem.

In a general location model, a finite set of points called demand points in $\mathbb{R}^n$, modeling a set of existing facilities or customers, is given. Let $D \equiv \{\boldsymbol{d}_1, \boldsymbol{d}_2, \cdots, \boldsymbol{d}_m\} \subset \mathbb{R}^n$ be the set of demand points. Then a problem to locate a new facility in $\mathbb{R}^n$ is called a single facility location problem. Let $\boldsymbol{x} \in \mathbb{R}^n$ be the variable location of the facility. The problem is usually formulated as a minimization problem with an objective function involving distances between the facility and demand points. Let $\gamma : \mathbb{R}^n \to \mathbb{R}$ be a distance measure, that is, $\gamma(\boldsymbol{x} - \boldsymbol{d}_i)$ represents the distance from $\boldsymbol{d}_i$ to $\boldsymbol{x}$ for each $i \in M \equiv \{1, 2, \cdots, m\}$. For example, γ is a norm on $\mathbb{R}^n$ or a function satisfying that $\gamma(\boldsymbol{x} - \boldsymbol{d}_i) = d(\boldsymbol{x}, \boldsymbol{d}_i)$, $i \in M$ for a metric d on $\mathbb{R}^n$. As typical location problems, the following three types of location models are known.

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} (\gamma(\boldsymbol{x} - \boldsymbol{d}_1), \gamma(\boldsymbol{x} - \boldsymbol{d}_2), \cdots, \gamma(\boldsymbol{x} - \boldsymbol{d}_m)), \tag{1}$$

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} \sum_{i=1}^{m} \lambda_i \gamma(\boldsymbol{x} - \boldsymbol{d}_i), \tag{2}$$

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} \max\{\lambda_i \gamma(\boldsymbol{x} - \boldsymbol{d}_i) : i \in M\} \tag{3}$$

where, for each $i \in M$, λ_i is a positive weight which specifies the demand of $\boldsymbol{d}_i$. We put $\boldsymbol{\lambda} \equiv (\lambda_1, \lambda_2, \cdots, \lambda_m)$. (1), (2) and (3) are called, respectively, a multicriteria location problem (MCP), a minisum location problem (MSP) and a minimax location problem (MMP). MCP is a problem to find an efficient solution. A point $\boldsymbol{x}_0 \in \mathbb{R}^n$ is called *an efficient solution of* (1) if there is no $\boldsymbol{x} \in \mathbb{R}^n$ such that $\gamma(\boldsymbol{x}-\boldsymbol{d}_i) \leq \gamma(\boldsymbol{x}_0-\boldsymbol{d}_i)$ for all $i \in M$ and that $\gamma(\boldsymbol{x}-\boldsymbol{d}_j) < \gamma(\boldsymbol{x}_0-\boldsymbol{d}_j)$ for some $j \in M$. MCP, MSP and MMP with various distances or norms as γ are considered [2-7]. For example, MCP with rectilinear norm in [2, 4], MCP with asymmetric rectilinear distance in [6], MSP and MMP with asymmetric rectilinear distance in [3], MMP with A-distance in [7], MCP with the block norm in [5]. Rectilinear norm is a special case of the others. If the facility to be located is a public and non-emergency one like for example a warehouse, then MCP and MSP can be used. If it is a public and emergency one like for example a hospital or a fire station, then MMP can be used. So we consider MCP and MSP for further analysis. By the same reason mentioned in the last of the previous section, rectilinear norm $\|\cdot\|_1$ as γ is used to measure the distance between two points. Namely, we consider a multicriteria and a minisum location problem with rectilinear norm as follows:

$$\text{(P)} \qquad \min_{\boldsymbol{x}\in\mathbb{R}^n} (\|\boldsymbol{x}-\boldsymbol{d}_1\|_1, \|\boldsymbol{x}-\boldsymbol{d}_2\|_1, \cdots, \|\boldsymbol{x}-\boldsymbol{d}_m\|_1);$$

$$(\text{P}_{\boldsymbol{\lambda}}) \qquad \min_{\boldsymbol{x}\in\mathbb{R}^n} \sum_{i=1}^{m} \lambda_i \|\boldsymbol{x}-\boldsymbol{d}_i\|_1.$$

We denote sets of all efficient solutions of (P) and all optimal solutions of $(\text{P}_{\boldsymbol{\lambda}})$ as $E(D)$ and $S_{\boldsymbol{\lambda}}(D)$, respectively. In $\mathbb{R}^2$ and $\mathbb{R}^3$, the set of all efficient solutions of (P) can be determined by using algorithms in [2] and [4], respectively. The set of all optimal solutions of $(\text{P}_{\boldsymbol{\lambda}})$ can be determined by using an algorithm in [3]. It is known that

$$E(D) = \{\boldsymbol{x}^* \in \mathbb{R}^n : \boldsymbol{x}^* \in S_{\boldsymbol{\lambda}}(D) \text{ for some } \boldsymbol{\lambda} > \boldsymbol{0}\}$$

(see [5]).

In the individual space, if points for the required learning contents are regarded as demand points and individual frequencies for the points as weights for the demand points, then a multicriteria and a minisum location problem can be applied. The set $E(D)$ can be used as a standard when one considers the learning contents to be provided. In other words, one should choose the learning contents to be provided among the learning contents for points in $E(D)$. On the other hand, the learning contents for the optimal solution of $(\text{P}_{\boldsymbol{\lambda}})$ can be proposed more concretely as the representative learning contents of the learning contents required by the people of the prefecture. To our knowledge, it is a new approach to use quantification method III in order to determine demand points for location problems. Depending on the questionnaire, this suggests that we can also use other statistical methods for example quantification method IV, multidimensional scaling, principal component anasysis, factor analysis, etc. instead of quantification method III in order to determine demand points for location problems.

4. The results of location problems In this section, the results of a multicriteria and a minisum location problem for the results of quantification method III are given.

Location problems for Q1

In Table 5, we denote $(x_1^{(1)}, x_2^{(1)})$ and an individual frequency for each No. i as $\boldsymbol{d}_i^{(1)}$ and $\lambda_i^{(1)}$, respectively. Then we consider a multicriteria location problem

$$(\mathrm{P}^{(1)}) \qquad \min_{\boldsymbol{x} \in \mathbb{R}^2} (\|\boldsymbol{x} - \boldsymbol{d}_1^{(1)}\|_1, \|\boldsymbol{x} - \boldsymbol{d}_2^{(1)}\|_1, \cdots, \|\boldsymbol{x} - \boldsymbol{d}_{13}^{(1)}\|_1)$$

and a minisum location problem

$$(\mathrm{P}_{\boldsymbol{\lambda}}^{(1)}) \qquad \min_{\boldsymbol{x} \in \mathbb{R}^2} \sum_{i=1}^{13} \lambda_i^{(1)} \|\boldsymbol{x} - \boldsymbol{d}_i^{(1)}\|_1.$$

Let $E^{(1)}(D)$ be the set of all efficient solutions of $(\mathrm{P}^{(1)})$ and $S_{\boldsymbol{\lambda}}^{(1)}(D)$ be the set of all optimal solutions of $(\mathrm{P}_{\boldsymbol{\lambda}}^{(1)})$. Figure 5 and 6 show $E^{(1)}(D)$ and $S_{\boldsymbol{\lambda}}^{(1)}(D)$, respectively.

Consideration: We have $(0.118477, -0.220388)$ in the individual space as the optimal solution of the minisum location problem. The learning contents for the optimal solution is the representative learning contents for "the educational item which is necessary for infants" of the learning contents required by the people of the prefecture. Transforming the optimal solution by using Table 6, we have $(0.070827, -0.092548)$ in the categorical space. Comparing this transformed point with points for categories, we can propose the learning contents for the optimal solution as follows:

- **"Fixation of basic living habitude and play in neighborhood nature".**

On the other hand, the set $E^{(1)}(D)$ can be used as a standard when one considers the learning contents to be provided. In other words, one should choose the learning contents to be provided among the learning contents for points in $E^{(1)}(D)$.

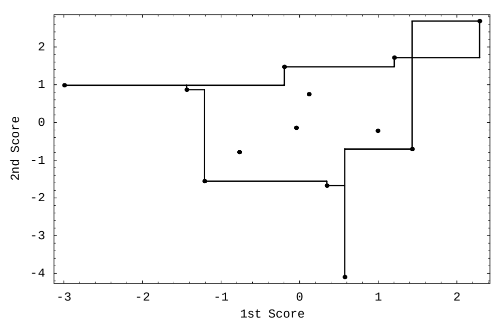

Figure 5. $E^{(1)}(D)$ for Q1.

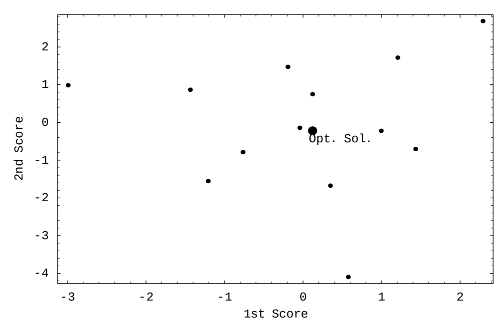

Figure 6. $S_{\boldsymbol{\lambda}}^{(1)}(D)$ for Q1.

Location problems for Q2

Applying cluster analysis, the required learning contents were partitioned into some groups each of which contains the similar required learning contents. Since clusters G and H have large individual frequencies, we shall consider location problems for each of these clusters. In Table 9, we denote $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)})$ and an individual frequency for each No. i as $\boldsymbol{d}_i^{(2)}$ and $\lambda_i^{(2)}$, respectively. For the cluster G, we consider a multicriteria location problem

$$(\mathrm{P}^{(G)}) \quad \min_{\boldsymbol{x} \in \mathbb{R}^3} (\|\boldsymbol{x} - \boldsymbol{d}_{13}^{(2)}\|_1, \|\boldsymbol{x} - \boldsymbol{d}_{14}^{(2)}\|_1, \|\boldsymbol{x} - \boldsymbol{d}_{15}^{(2)}\|_1, \|\boldsymbol{x} - \boldsymbol{d}_{16}^{(2)}\|_1, \|\boldsymbol{x} - \boldsymbol{d}_{17}^{(2)}\|_1, \|\boldsymbol{x} - \boldsymbol{d}_{18}^{(2)}\|_1)$$

and a minisum location problem

$$(\mathrm{P}_{\boldsymbol{\lambda}}^{(G)}) \qquad \min_{\boldsymbol{x}\in\mathbb{R}^3} \lambda_{13}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{13}^{(2)}\|_1 + \lambda_{14}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{14}^{(2)}\|_1 + \lambda_{15}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{15}^{(2)}\|_1 + \lambda_{16}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{16}^{(2)}\|_1 + \lambda_{17}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{17}^{(2)}\|_1 + \lambda_{18}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{18}^{(2)}\|_1.$$

For the cluster H, we consider a multicriteria location problem

$$(\mathrm{P}^{(H)}) \quad \min_{\boldsymbol{x}\in\mathbb{R}^3} (\|\boldsymbol{x}-\boldsymbol{d}_{19}^{(2)}\|_1, \|\boldsymbol{x}-\boldsymbol{d}_{20}^{(2)}\|_1, \|\boldsymbol{x}-\boldsymbol{d}_{22}^{(2)}\|_1, \|\boldsymbol{x}-\boldsymbol{d}_{23}^{(2)}\|_1, \|\boldsymbol{x}-\boldsymbol{d}_{24}^{(2)}\|_1, \|\boldsymbol{x}-\boldsymbol{d}_{25}^{(2)}\|_1)$$

and a minisum location problem

$$(\mathrm{P}_{\boldsymbol{\lambda}}^{(H)}) \qquad \min_{\boldsymbol{x}\in\mathbb{R}^3} \lambda_{19}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{19}^{(2)}\|_1 + \lambda_{20}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{20}^{(2)}\|_1 + \lambda_{22}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{22}^{(2)}\|_1 + \lambda_{23}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{23}^{(2)}\|_1 + \lambda_{24}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{24}^{(2)}\|_1 + \lambda_{25}^{(2)}\|\boldsymbol{x}-\boldsymbol{d}_{25}^{(2)}\|_1.$$

For each $* \in \{G, H\}$, let $E^{(*)}(D)$ be the set of all efficient solutions of $(\mathrm{P}^{(*)})$ and $S_{\boldsymbol{\lambda}}^{(*)}(D)$ be the set of all optimal solutions of $(\mathrm{P}_{\boldsymbol{\lambda}}^{(*)})$. Figure 7-10 show $E^{(*)}(D)$ and $S_{\boldsymbol{\lambda}}^{(*)}(D)$ for $* \in \{G, H\}$.

Consideration: We have (−0.030870, −0.044537, −1.286721) and (0.208296, −0.856266, 0.588853) in the individual space as optimal solutions of minisum location problems for clusters G and H, respectively. These optimal solutions coincide with points 17 and 22 in the individual space, respectively. The leraning contents for each optimal solution is the representative learning contents for "the learning item which is necessary for parents having an infant" of the required learning contents in each corresponding cluster. Transforming these optimal solutions by using Table 10, we have (−0.020669, −0.027689, −0.770652) and (0.139466, −0.532341, 0.352680) in the categorical space, respectively. Comparing these transformed points with points for categories, we can propose the learning contents for these optimal solutions as follows:

- **"A way of communicating with a child according to developmental stage of the child"** for the cluster G;
- **"A way of coming in contact with a child in order to develop self-dependence of the child in everyday life"** for the cluster H.

On the other hand, sets $E^{(G)}(D)$ and $E^{(H)}(D)$ can be used as standards when one considers the learning contents to be provided. In other words, one should choose the learning contents to be provided among the learning contents for points in $E^{(G)}(D)$ and $E^{(H)}(D)$ for clusters G and H, respectively.

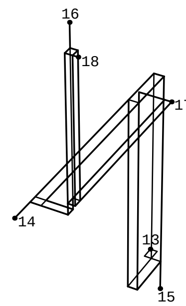

Figure 7. $E^{(G)}(D)$ for the cluster G of Q2.

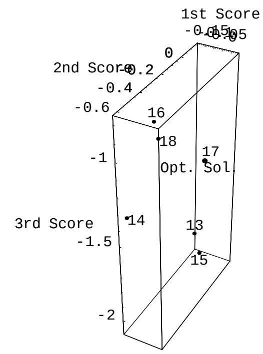

Figure 8. $S_{\boldsymbol{\lambda}}^{(G)}(D)$ for the cluster G of Q2.

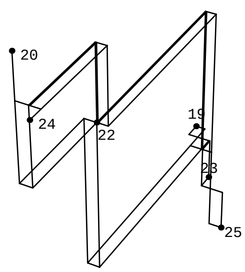

Figure 9. $E^{(H)}(D)$ for the cluster H of Q2.

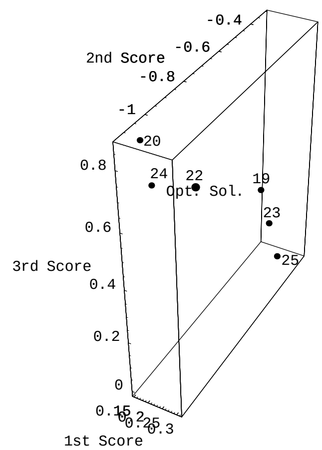

Figure 10. $S_{\boldsymbol{\lambda}}^{(H)}(D)$ for the cluster H of Q2.

5. Conclusions Based on the results of questionnaires on home education to adults in Aomori prefecture for the public opinion survey, which was conducted by Aomori Prefectural Community Education Center as a survey research on lifelong learning and social education, we analyzed on “the educational item which is necessary for infants” and “the learning item which is necessary for parents having an infant”. First, we applied quantification method III to the results of the questionnaire in order to research what requests the people of the prefecture have for the learning contents. Then questionnaire's categorical items and the learning contents required by individuals were represented as points in the categorical and the individual space, respectively. Interpreting each score for the results of quantification method III, it was shown that each learning contents could be represented as a point in the categorical or the individual space. Next, since it is desirable for institutions providing learning to provide the learning contents which are near the required learning contents as much as possible, a multicriteria and a minisum location problem with rectilinear norm were considered in the individual space, where demand points were points for the required learning contents and weights were individual frequencies for the points. As the learning contents for solutions of location problems, the learning contents which were near the learning contents required by the people of the prefecture as much as possible were determined. Consequently, for "the educational item which is necessary for infants" and "the learning item which is necessary for parents having an infant", some learning contents which should be provided by institutions providing learning were proposed concretely.

These results suggest the wide applicability of the procedure used in this paper to decision making like for example the development of new products or deciding policies. When one wants to find the decision object which is near the requests of individuals as much as possible, the procedure for the decision making is described briefly in the following. First, get a multivariate data as the requests of individuals by questionnaires for the object to individuals. In this paper, its object was "the educational item which is necessary for infants" or "the learning item which is necessary for parents having an infant". Now, it is assumed that quantification method III is suitable for the multivariate data. Otherwise, one should choose another suitable statistical method for example quantification method IV, multidimensional scaling, principal component analysis, factor analysis, etc. instead of quantification method III. Next, apply quantification method III to the multivariate data, and represent questionnaire's categorical items and the requests of individuals as points in the categorical and the individual space, respectively. Next, choose one of location models and of distance measures as γ. In the individual space, regarding points for individuals as demand points and individual frequencies for the points as weights for the demand points if one needs weights, and solve the location problem. To our knowledge, it is a new approach to use quantification method III in order to determine demand points for location problems. Next, estimate the point in the categorical space from the solution of the location problem in the individual space by using regression analysis. Finally, in the categorical space, comparing the estimated point with points for categories, the object which is near the requests of indivisuals as much as possible can be known.

References

[1] Aomori Prefectural Community Education Center, *Survey Report on Home Education Power* (in Japanese) (2002).

[2] L. G. Chalmet, R. L. Francis and A. Kolen, *Finding efficient solutions for rectilinear distance location problems efficiently*, European Journal of Operational Research, **6** (1981), 117-124.

[3] Z. Drezner and G. O. Wesolowsky, *The asymmetric distance location problem*, Transportation Science, **23** (1989), 201-207.

[4] M. Kon, *Efficient solutions of multicriteria location problems with rectilinear norm in* $\boldsymbol{R}^3$, Scientiae Mathematicae Japonicae, **54** (2001), 289-299.

[5] M. Kon and S. Kushimoto, *On efficient solutions of multicriteria location problems with the block norm*, Scientiae Mathematicae, **2** (1999), 245-254.

[6] T. Matsutomi and H. Ishii, *Fuzzy facility location problem with asymmetric rectilinear distance* (in Japanese), Journal of Japan Society for Fuzzy Theory and Systems, **8** (1996), 57-64.

[7] T. Matsutomi and H. Ishii, *Minimax location problem with A-distance*, Journal of the Operations Research Soc. of Japan, **41** (1998), 181-195.

[8] T. Sugihara and W. Fujita, *Multivariate Analysis* (in Japanese), Makino-Syoten, Japan (1998).

[9] Y. Tanaka and K. Wakimoto, *Methods of Multivariate Statistical Analysis* (in Japanese), Gendai-Sugakusya, Japan (1983).

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