BENCHMARKING METHOD CONSIDERING COMPLEMENTARY NETWORK EXTERNALITY

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Abstract. It is important to evaluate the performance of objects mathematically from various aspects, and to reflect the result on decision making of marketing. These aspects should influence consumers' lifestyle as economies that indicate characters of social structure. Economies of network is defined that an economic activities of some agents influences those of others, so called network externality. To put it another way, economies of network are those of scale of consumers. In this article, we propose a benchmarking method based on two ratios LP and PCA focusing on measuring complementary network externality.

1 Introduction In marketing, it is important to investigate the trend of a market. At information-communication society, it is important to take not only the strategy, which raises the evaluation to the own performance of a product, but also a strategy with emphasis on externality, such as other products, which adopt as its peripheral equipment, and the number of users. Benchmarking has been a popular method to compare objects relatively, to reflect marketing or decision making. Many mathematical methods such as Principal Component Analysis (PCA), Data Envelopment Analysis (DEA), Conjoint Analysis, and Analytic Hierarchy Process (AHP) are used usually in the situation of marketing or decision making.

DEA is a linear fractional programming originally developed for the estimation of the relative efficiency of a set of units (called decision making units, DMUs) producing a set of outputs from common inputs. DEA seeks set of weights for each unit that maximizes a weighted sum of variables, with the constraint that no units have a weighted sum larger than one. As a result, each unit receives a score between 0 and 1. PCA is widely used in signal processing, statistics, and neural computing. The basic goal in PCA is to reduce the dimension of the data. Adler and Golany [2001] proposed DEA-PCA model to select the most efficient networks configurations form the many that are possible in the deregulated European Union airline market. To overcome the difficulties that DEA encounters when there is an excessive number of inputs or outputs, they employ PCA to aggregate certain, clustered data.

These methods should be based on criteria reflecting consumers' lifestyle and structures of societies. These criteria have been presented as economies of scale, economies of scope, and economies of network. Economies of scale focus on production size to improve production efficiency. Economies of scale are also known as returns to scale. Increasing returns to scale in production means that an increase in resource usage, by say $x\%$, results of an increase in output by more than $x\%$. Constant returns to scale means that an increase in resource usage, by say $x\%$, results of an increase in output by $x\%$. And decreasing returns

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to scale means that an increase in resource usage, by say $x\%$, results in an increase of output by less than $x\%$. Sueyoshi [1997] measured the scale efficiency of Nippon Telephone & Telegraph by applying DEA.

Economies of scope focus how kinds of services we provide to maximize benefit. Economies of scope arise when an increase in the range of goods produced brings a decrease in average total cost. In other words, economies of scope indicate cost of joint production is less than producing each separately, or are decreases in average cost made possible by increasing the number of different goods produced.

Economies of network are well known as network externality. Liebowits [1998] defined Network externality as a change in the benefit, or surplus, that an agent derives from a good when the number of other agents consuming the same kind of good changes and direct network effects have been defined as those generated through a direct physical effect of the number of purchasers on the value of a product. Indirect network effects are "market mediated effects" such as cases where complementary goods are more readily available or lower in price as the number of users of good increases.

To put it another way, economies of network are those of scale of consumers. We should take into not only economies of suppliers but also the consumers if there exists network externality. In marketing, it is important to develop indices that indicate existence of network externality, because network externality influences marketing strategy which considers how allocate resources into quality raising and spread strategy. In this article, we propose new procedure to deal with complement network externality applying LP and PCA.

2 Network Externality In this section we introduce network externality.

Kats and Shapiro [1994] showed indirect influence through the diversity and the price of a component, although selection of the system among consumers does not necessarily have direct influence on a utility, using relations such as a nut and a bolt, an ATM machine and a card, and the main part of a camera and a lens.

Gandal [1994] measured worth of file compatibility standards in PC software market using hedonic approach and verified existence of complementary network externality in this market. Cottell and Koput [1998] estimated the effects of software provision on the valuation of hardware using hedonic approach.

In Tsuji and Nishiwaki [1996], Network is defined as the aggregate of the arc (edge) and the node (point) as the whole economy and society, and a lead pipe of exchange of man, money or information. Not only a physical combination of a communication circuit, an electric wire, a gas pipe, etc. but the relation with people and a man, the business tie-up between companies, etc. can be considered to be a network. In a consumption externality the utility of one consumer is directly affected by the actions of another consumer. And in production externality the production set of one firm is directly affected by the actions of another agent (Varian [1992]).

About a network, it is possible that the number of members and the number of participants itself affect a utility. It is expected that the way with many members and participants can receive so many information, the reliance to a network becomes large, and sense of security is given.

There are two companies, company 1 and company 2, and suppose that different products Y_1 and Y_2 are produced in the following production functions.

$$Y_1 = F(X_1)$$
$$Y_2 = F(X_2)$$

Here, X_1, X_2 indicate input such as labor or capital, and the product price be P, the input price W. If it does not exhibit network externality, each company makes decisions independently, and profit of company 1 and company 2, π_1, π_2 are expressed as follows:

$$\pi_1 = PY_1 - WX_1$$
$$\pi_2 = PY_2 - WX_2$$

On the other hand, supposing the information shared, by acquiring the information that a partner has, it is thought that the sales of each company are as follows:

$$Y_1^{'} = F(X_1) + \alpha Y_2$$
$$Y_2^{'} = F(X_2) + \alpha Y_1$$
$$where \quad \alpha > 0.$$

By network of company 1 and company 2, each company can access now the external resources which a partner company holds, Added value of each product of company become high by αY_i $(i = 1, 2)$. It becomes possible by conforming to a market to produce the larger quantity of production $Y_i^{'}$ than Y_i. Thus, the profits $\pi_i^{'}$ of each company when companies 1 and 2 connect by network are as follows:

$$\pi_1^{'} = P(Y_1 + \alpha Y_2) - WX_1$$
$$\pi_2^{'} = P(Y_2 + \alpha Y_1) - WX_2$$

Thus, profits are increasing by network.

3 Framework of Benchmarking In this section, we propose one procedure to measure complement network externality. In considering network externality, we should take into two points, existence of network externality in markets, and evaluate spread strategy of objects.

Gandal [1994] measured worth of file compatibility standards in PC software market using hedonic approach and verified existence of complementally network externality in this market. And Cottell and Koput [1998] estimated the effects of software provision on the valuation of hardware using hedonic approach. Many researches have been focused on existing of network externality, but research, which verifies the action to each object, is seldom done.

We assume that each element of the internal factor for analysis and an external factor does not have correlation and the data of two categories is independent. So we are able to treat as a LP problem, which changed the two ratios FP.

The evaluation value by the internal factor is carried out based on the target characteristic and the evaluation value by the external factor is based on evaluation from the outside at an object. Although the evaluation value by the internal factor for evaluation and the evaluation value by the external factor constitute one candidate for analysis, it is thought in calculation of each factor that it is independent.

3.1 Characters of objects We consider the following situation. Object$_j$ has two categorized data called internal category and external category. Internal category indicates the evaluation value of units own, for example, size, weight, and quantity of units. External category indicates the evaluation value of consumers', for example number of users or number of peripherals. There is no correlation between internal factors and external factors. Internal category consisted of m input items $x_j^i = (x_{1j}^i, \ldots, x_{mj}^i)$ and s output items $y_j^i = (y_{1j}^i, y_{2j}^i, \ldots, y_{sj}^i)$. External category is consisted of m' input items

$x_j^e = (x_{1j}^e, \ldots, x_{m'j}^e)$ and s' output items $y_j^e = (y_{1j}^e, y_{2j}^e, \ldots, y_{s'j}^e)$. The input data matrix X^i, X^e and output data matrix Y^i, Y^e can be arranged as follows;

$$X^i = \begin{pmatrix} x_{11}^i & \cdots & x_{1n}^i \\ \vdots & & \vdots \\ x_{m1}^i & \cdots & x_{mn}^i \end{pmatrix}, X^e = \begin{pmatrix} x_{11}^e & \cdots & x_{1n}^e \\ \vdots & & \vdots \\ x_{m'1}^e & \cdots & x_{m'n}^e \end{pmatrix}, Y^i = \begin{pmatrix} y_{11}^i & \cdots & y_{1n}^i \\ \vdots & & \vdots \\ y_{s1}^i & \cdots & y_{sn}^i \end{pmatrix},$$

$$Y^e = \begin{pmatrix} y_{11}^e & \cdots & y_{1n}^e \\ \vdots & & \vdots \\ y_{s'1}^e & \cdots & y_{s'n}^e \end{pmatrix}.$$

We solve the following fractional programming $< FP1 >$ to obtain values for input weights $u^i = (u_1^i, u_2^i, \ldots, u_m^i)$ and $u^e = (u_1^i, u_2^i, \ldots, u_{m'}^i)$, and output weights $v^i = (v_1^i, v_2^i, \ldots, v_s^i)$ and $v^e = (v_1^e, v_2^e, \ldots, v_{s'}^e)$ as variables.

$$< FP1 > \max \ \frac{u^i y_o^i}{v^i x_o^i} + \frac{u^e y_o^e}{v^e x_o^e} \tag{1}$$

$$\begin{aligned} \textit{subject to} \quad & \frac{u^i y_j^i}{v^i x_j^i} \le 1 \ (j = 1, \ldots, n) \\ & \frac{u^e y_j^e}{v^e x_j^e} \le 1 \ (j = 1, \ldots, n) \\ & u^i, v^i, u^e, v^e \ge 0 \end{aligned}$$

We transforms the above fractional program $< FP1 >$ into linear program $< LP1 >$ since these elements of two fractions are independent each other.

$$< LP1 > \max \ u^i y_o^i + u^e y_o^e \tag{2}$$

$$\begin{aligned} \textit{subject to} \quad & -v^i X^i + u^i Y^i \le 0 \\ & -v^e X^e + u^e Y^e \le 0 \\ & v^i x_o^i = 1 \\ & v^e x_o^e = 1 \\ & u^i, v^i, u^e, v^e \ge 0 \end{aligned}$$

The dual problem $< DLP1 >$ of $< LP1 >$ is expressed with a real variable θ^i and θ^e, and a nonnegative vector λ^i and λ^e of variables as follows;

$$< DFP1 > \min \ \theta^i + \theta^e \tag{3}$$

$$\begin{aligned} \textit{subject to} \quad & \theta^i x_o^i - X^i \lambda^i \ge 0 \\ & \theta^e x_o^e - X^e \lambda^e \ge 0 \\ & Y^i \lambda^i \ge y_o^i \\ & Y^e \lambda^e \ge y_o^e \\ & \lambda^i, \lambda^e \ge 0 \end{aligned}$$

3.2 Tendency of objects PCA is a method to integrate from many number of data standing for character of unit, to a few number of data without losing feature of unit. On the other words, the basic goal in PCA is to reduce the dimension of the data (Adler and Golany [2001]). In this article, we focus on two variables into one component. We formulate principal component $f = w_1\theta^i + w_2\theta^e$ from weighted sum of 2 valuables θ^i and θ^e, where θ^i and θ^e are solutions of $< DLP1 >$ and evaluation values of internal and external category.

Mean of composite value f and variance are expressed as follows:

$$m(\bar{f}) = w_1\bar{\theta^i} + w_2\bar{\theta^e}$$

$$v(f) = \frac{1}{n-1}\sum_{i=1}^{n}(f_i - \bar{f})^2 = w_1^2 s_{11} + 2w_1 w_2 s_{12} + w_2^2 s_{22}$$

To seek set of weights (w_1, w_2) that maximize the variance, we consider the following Lagrange multiplier method with Lagrange multiplier λ:

$$\max_{w_1,w_2,\lambda} L(w_1, w_2, \lambda) = v(f) - (w_1^2 + w_2^2 - 1) \tag{4}$$

¿From (4), we are able to gain two variables λ_1 and λ_2 and eigenvectors corresponding to two roots, w_1 and w_2, we call $f_1 = w_1^T\theta$ first principal component and $f_2 = w_2^T\theta$ second principal component.

3.3 Evaluation Next, we consider two points; existence of network externality in markets and objects that focused on network externality.

Based on the result searched for the foregoing paragraph, we consider that whether network externality is working in the whole group for analysis and whether each object looks at a network external effect.

First, by solving the LP problem of two categories, the evaluation value by the internal factor for evaluation and each external factor is calculated. Next, by principal component analysis, when two integrated indices show the evaluation value by the internal factor for evaluation, and the evaluation value by the external factor, network externality is considered from the evaluation value acquired, respectively.

The aim of PCA in our method is to plot objects into two-dimensional plane.

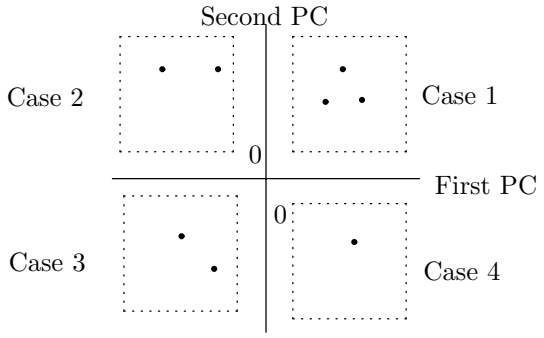

Figure 1 Objects ploted into two-dimensional plane

We consider that there exhibits network externality when the signs of eigenvector (X2) which indicates external category (factor), is positive in both first PC and second PC.

We discuss in following 4 cases whether network externality works for objects or not. In these cases, we assume that first PC is internal category oriented and second PC is external category oriented.

Case 1: Both first PC and second PC are positive; Objects plotted in area are evaluated high whether network externality exhibits or not. These adopt not only spread strategy but also high spec products.
Case 2: Second PC is positive, first PC is negative; Objects plotted in area are evaluated high with network effect, but evaluation by object own is not high, these objects should improve objects own.
Case 3: Both first PC and second PC are negative; Objects plotted in area are evaluated low whether network externality exhibits or not. These objects should not only improve objects own but adopt spread strategy.
Case 4: First PC is positive, second PC is negative; Objects plotted in area are evaluated high without network effect, but with network effect evaluation are not high. These objects should adopt spread strategy making use of network effect.

Otherwise, when the sign of eigenvector (X2), which indicates external category (factor), is negative in either first PC or second PC there does not exhibit network externality.

4 Example In this section we demonstrate our benchmarking method applying to flash memory market of digital camera.

In consideration of the externality as recording medium of the digital camera, which is one of the main use places, benchmarking is performed to evaluation of a flash memory.

Objects are Compact Flash (CF), Smart Media(SM), Memory Stick (MS)), SD Memory Card (SD), Multi Media Card (MMC), and Micro Drive (MD). Elements for benchmarking are as follows; The number of users, the number of the digital cameras which are using the media as the recording media, the number of users using digital camera which are using the media as the recording media, the number of selling agency, the price per 1MB, the sum of 3 methods, weight, capacity, write-in speed, and read-out speed.

Table 1 Specification and property of flash memories

	i-input	i-input	i-input	i-output	i-output	i-output
	price	sum	weight	capacity	write	read
	[yen /MB]	[mm]	[g]	[MB]	[MB/s]	[MB/s]
CF	33.4	82.50	11.4	1000	1.8	5.62
SM	41.4	82.76	2.0	128	2.2	9.80
MS	54.5	74.30	4.0	128	1.8	2.45
SD	50.4	58.10	2.0	512	10.0	10.00
xD	53.8	46.70	2.0	128	3.0	5.00
MMC	71.9	57.40	1.6	64	2.0	1.70
MD	26.9	84.20	16.0	1000	4.1	4.10

	e-output user [number]	e-output user [number]	e-output camera [number]	e-output maker [number]
CF	276	669	57	14
SM	265	506	46	10
MS	284	325	21	5
SD	250	564	40	13
xD	26	119	10	2
MMC	77	564	35	7
MD	465	410	38	6

¿From these data, we estimate characters of objects into evaluation value of the internal factor and the external factor by the method shown in section 3.

Table 2 Evaluation value of each factor

	Internal	External
CF	1.00000	1.00000
SM	0.98158	0.85712
MS	0.21767	0.67761
SD	1.00000	0.89035
xD	0.53193	0.16601
MMC	0.00255	0.75628
MD	1.00000	1.00000

¿From result of Table 2, we evaluated tendency of objects, and figured scatter diagram about principal component in Figure 2.

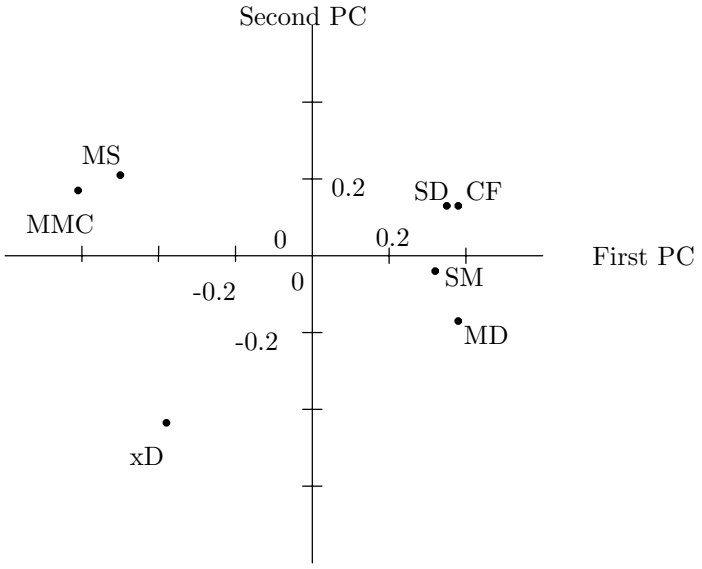

Figure 2 Scatter diagram about principal component

Table 3 The result of PCA and component scores

EIGENVECTOR (X1)	0.92	-0.42
EIGENVECTOR (X2)	0.42	0.92
EIGENVALUE	0.21	0.06
PROPORTION	0.79	0.21
CUM. PROP	0.79	1.00

	First PC	Second PC
CF	0.39	0.13
SM	0.32	-0.04
MS	-0.43	0.21
SD	0.35	0.13
xD	-0.38	-0.44
MMC	-0.61	0.17
MD	0.39	-0.17

We consider that whether their exhibits network externality or not. In this case the signs of eigenvector (X2) indicating external category (factor), are positive in both first PC (0.42) and second PC (0.92), and so in this market we consider there exhibits network externality. This result when consumers are going to buy flash memories for digital camera as the recording media, are they will select the media which is compatible.
Case 1: Both first PC and second PC are positive; Objects plotted in area are evaluated high whether network externality exhibits or not.
Case 2: Second PC is positive, first PC is negative; Objects plotted in area are evaluated high with network effect, but evaluation by object own is not high, these objects should improve objects own.
Case 3: Both first PC and second PC are negative; Objects plotted in area are evaluated low whether network externality exhibits or not. These objects should not only improve objects own but also adopt spread strategy.
Case 4: First PC is positive, second PC is negative; Objects plotted in area are evaluated high without network effect, but with network effect evaluation are not high. These objects should adopt spread strategy making use of network effect.

Next, we consider each object. Compact Flash and SD memory card are evaluate highly in both first PC, which is weighted internal factor, and second PC, which is weighted external factor. That is, these are high goods of evaluation both a performance side and a spread strategy. Memory Stick and Multi Media Card are evaluated low in first PC but evaluated high in second PC. Although the evaluation of the goods itself of those are high, evaluation of an external factor are low. It is thought that these two have adopted the strategy; whose concept is that network externality is important. Although the evaluations of the goods itself of Smart Media and Micro Drive is high, evaluation of an external factor is low. It is thought that the strategy putting emphasis on goods spread should be taken.

5 Conclusions We proposed a new benchmarking method considering complement network externality. The framework of this method is to analyze the selling strategy of each product by applying the two ratio linear programming problem for the evaluation value from the own evaluation and that of outside on a product is calculated, and using principal component analysis, through considering the tendency of the whole objects, we discussed whether network externality is working to the whole objects.

We examined flash memory market whether network externality exhibits or not by using our procedure, considering evaluation of internal category whose factor composed is specification, and evaluation of externality whose factor composed the number of users and a number of peripheral equipment of a product. Further we showed that there does not exhibit network externality and some memories have adopted the spread strategy which takes network externality into account.

Major contribution of this article is this method is able to discuss not only existence of network externality in markets which works all objects in market, but also spread strategy of each object by this method.

In this article, we deal with relative comparison, but in accounting, absolute evaluation such as measurement of corporate value or invisible assets is called for. It is a future subject to adapt the framework of the analysis to a scene like value evaluation of invisible assets. We want to perform application to the electronic commerce from the argument on standardization, such as an electronic payment system and electronic money.

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