FUZZY FANTASTIC FILTERS OF LATTICE IMPLICATION ALGEBRAS

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Abstract. We fuzzify the concept of fantastic filters of lattice implication algebras and give the relations among fuzzy filter, fuzzy positive implicative filter and fuzzy fantastic filter.

1 Introduction and Preliminaries In order to research the logical system whose propositional value is given in a lattice, Xu [4] proposed the concept of lattice implication algebras, and discussed some of their properties. Xu and Qin [5] introduced the notion of a filter in a lattice implication algebra, and investigated their properties. Y.B.Jun [3] introduced the concept of a positive implicative filter and associative filter in a lattice implication algebra, and obtained some related properties. Also, Y.B.Jun [2] fuzzify the concept of positive implicative filters and associative filters in lattice implication algebras, and investigate some results. In [1], Y.B.Jun introduced the notion of a fantastic filter in a lattice implication algebra and gave some results. In this paper, we fuzzify the concept of fantastic filters of lattice implication algebras and give the relations among fuzzy filter , fuzzy positive implicative filter and fuzzy fantastic filter.

By a lattice implication algebra we mean a bounded lattice $(L, \vee, \wedge, 0, 1)$ with order-reversing involution "$'$" and a binary operation " $\rightarrow$ " satisfying the following conditions:

(I1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$

(I2) $x \rightarrow x = 1$

(I3) $x \rightarrow y = y' \rightarrow x'$

(I4) $x \rightarrow y = y \rightarrow x = 1 \Rightarrow x = y$

(I5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$

(L1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$

(L2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$

for all $x, y, z \in L$.

Note that the condition (L1) and (L2) are equivalent to the condition (L1) and (L2) are equivalent to the conditions:

(L3) $x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)$, and

(L4) $x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$, respectively.

In the sequel the binary operation " $\rightarrow$ " will be denoted by juxtaposition. We can define a partial ordering " $\leq$ " on a lattice implication algebra L by $x \leq y$ if and only if $xy = 1$.

In a lattice implication algebra L the following hold:

(1) $0x = 1, 1x = x$ and $x1 = 1$.

(2) $x' = x0$

(3) $xy \leq (yz)(xz)$

(4) $x \vee y = (xy)y$

(5) $((yx)y')' = x \wedge y = ((xy)x')'$

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(6) $x \leq y$ implies $yz \leq xz$ and $zx \leq zy$

(7) $x \leq (xy)y$

In what follows L denotes a lattice implication algebra unless otherwise specified.

Definition 1.1([5]) A subset F of L is called a filter of L if it satisfies:

(F1) $1 \in F$

(F2) $x \in F$ and $xy \in F$ imply $y \in F$ for all $x, y \in L$.

Definition 1.2 ([3]) A subset F of L is called a positive implicative filter of L if it satisfies (F1) and

(F3) $x((yz)y) \in F$ and $x \in F$ imply $y \in F$ for all $x, y, z \in L$.

Definition 1.3([1]) A subset F of L is called a fantastic filter of L if it satisfies (F1) and

(F4) $z(yx) \in Fandz \in F$ imply $((xy)y)x \in F$ for all $x, y, z \in L$.

Definition 1.4 [6]) Let μ be a fuzzy set in L. Then μ is called a fuzzy filter of L if

(FF1) $\mu(1) \geq \mu(x)$

(FF2) $\mu(y) \geq min\{\mu(x), \mu(xy)\}$ for all $x, y \in L$.

Definition 1.5 ([6]) A fuzzy set μ in L is called a fuzzy positive implicative filter of L if it satisfies:

(FF1) and

(PF1) $\mu(y) \geq min\{\mu(x((yz)y)), \mu(x)\}$ for all $x, y, z \in L$.

2 Main Results

Definition 2.1 A fuzzy set μ in L is called a fuzzy fantastic filter of L if it satisfies (FF1) and

(FF3) $\mu(((xy)y)x) \geq min\{\mu(z(yx)), \mu(z)\}$ for all $x, y, z \in L$.

Example 2.2 let $L = \{0, a, b, c, d, 1\}$ be a partial ordering as follows:

$0 \leq d \leq a \leq 1$, $0 \leq c \leq b \leq 1$ and $0 \leq d \leq b \leq 1$.

Define a unary operation "$'$" and a binary operation denoted by juxtaposition on L as follows:

x	x'
0	1
a	c
b	d
c	a
d	b
1	0

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

Define $\vee$- and $\wedge$-operation on L as follows:

$$x \vee y = (xy)y, \quad x \wedge y = ((x'y')y')'$$

Then L is a lattice implication algebra . Define a fuzzy set in L by

$$\mu(x) = \begin{cases} 0.8 & \text{if} \quad x \in \{b, c, 1\} \\ 0.2 & \text{otherwise} \end{cases}$$

Then it is easy to verify that μ is a fuzzy fantastic filter of L.

Theorem 2.3 Every fuzzy fantastic filter of L is a fuzzy filter.

Proof. Let μ be a fuzzy fantastic filter of a lattice implication algebra L. Taking $y = 1$ in (FF3), we have $\mu(x) = \mu(((x1)1)x) \geq min\{\mu(z(1x)), \mu(z)\} = min\{\mu(zx), \mu(z)\}$. Hence μ is a fuzzy filter of L.

We now give an equivalent condition for a fuzzy filter to be a fuzzy fantastic filter.

Theorem 2.4 A fuzzy filter μ of L is fuzzy fantastic filter if and only if it satisfies

(FF4) $\mu(((xy)y)x) \geq \mu(yx)$ for all $x, y \in L$.

Proof. Assume that μ is a fuzzy fantastic filter of L and $x, y \in L$. Then $\mu(((xy)y)x) \geq min\{\mu(1(yx)), \mu(1)\} = min\{\mu(yx), \mu(1)\} = \mu(yx)$. Conversely, supposed that μ satisfies inequality (FF4). For any $x, y, z \in L$, we have $\mu(((xy)y)x) \geq \mu(yx) \geq min\{\mu(z), \mu(z(yx))\}$. Hence μ is a fuzzy fantastic filter of L.

Lemma 2.5([2]) Every fuzzy positive implicative filter of a lattice implication algebra is a fuzzy filter.

Lemma 2.6([2]) A fuzzy filter of L is a fuzzy positive implicative filter of a lattice implication algebra L if and only if it satisfies:

(PF2) $\mu(x) \geq \mu((xy)x)$ for all $x, y \in L$.

Lemma 2.7([6]) Let μ be a fuzzy filter of a lattice implication algebra L, then μ is order preserving.

Theorem 2.8 Every fuzzy positive implication filter of L is fuzzy fantastic.

Proof. Let μ be a fuzzy positive implicative filter of L. Then μ is a fuzzy filter of L by Lemma 2.5. Since $x \leq ((xy)y)x$, we get $(((xy)y)x)y \leq xy$. Hence

$((((xy)y)x)y)(((xy)y)x) \geq (xy)(((xy)y)x) = ((xy)y)((xy)x) \geq yx$, and thus

$\mu(yx) \leq \mu(((((xy)y)x)y)(((xy)y)x))$ [Lemma 2.7]

$\leq \mu(((xy)y)x)$ [Lemma 2.6]

that is, $\mu(yx) \leq \mu(((xy)y)x)$. Therefore , by Theorem 2.4, μ is a fuzzy fantastic filter of L.

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