ON (α, β)-FUZZY IDEALS OF BCK/BCI-ALGEBRAS

YOUNG BAE JUN

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ABSTRACT. Using the *belongs to* relation ($\in$) and *quasi-coincidence with* relation (q) between fuzzy points and fuzzy sets, the concept of (α, β)-fuzzy ideals where α, β are any two of $\{\in, \mathrm{q}, \in\vee\mathrm{q}, \in\wedge\mathrm{q}\}$ with $\alpha \neq\, \in\wedge\mathrm{q}$ is introduced, and related propertie s are discussed. Relations between $(\in\vee\mathrm{q}, \in\vee\mathrm{q})$-fuzzy ideals and $(\in, \in\vee\mathrm{q})$-fuzzy ideals are investigated, and conditions for an $(\in, \in\vee\mathrm{q})$-fuzzy ideal to be an $(\in, \in)$-fuzzy ideal are provided. Characterizations of $(\in, \in\vee\mathrm{q})$-fuzzy ideals a re given, and conditions for a fuzzy set to be a $(\mathrm{q}, \in\vee\mathrm{q})$-fu zzy ideal are provided.

1. Introduction

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [5], played a vital role to generate some different types of fuzzy subgroups, called (α, β)-fuzzy subgroups, introduced by Bhakat and Das [2]. In particular, $(\in, \in\vee\mathrm{q})$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. It is now natural to investigate similar type of generalizations of the existing fuzzy subsystems of other algebraic structures. With this objective in view, we introduce the concept of (α, β)-fuzzy ideal of a BCK/BCI-algebra and discuss related results. We investigate relations between $(\in\vee\mathrm{q}, \in\vee\mathrm{q})$-fuzzy ideals and $(\in, \in\vee\mathrm{q})$-fuzzy ideals, and give conditions for an $(\in, \in\vee\mathrm{q})$-fuzzy ideal to be an $(\in, \in)$-fuzzy ideal. We establish characterizations of $(\in, \in\vee\mathrm{q})$-fuzzy ideals, and provide conditions for a fuzzy set to be a $(\mathrm{q}, \in\vee\mathrm{q})$-fuzzy ideal.

2. Preliminaries

By a *BCI-algebra* we mean an algebra $(X, *, 0)$ of type $(2, 0)$ satisfying the axioms:

(i) $(\forall x, y, z \in X)\,(((x * y) * (x * z)) * (z * y) = 0)$,
(ii) $(\forall x, y \in X)\,((x * (x * y)) * y = 0)$,
(iii) $(\forall x \in X)\,(x * x = 0)$,
(iv) $(\forall x, y \in X)\,(x * y = y * x = 0 \Rightarrow x = y)$.

We can define a partial ordering $\leq$ by $x \leq y$ if and only if $x * y = 0$. If a BCI-algebra X satisfies $0 * x = 0$ for all $x \in X$, then we say that X is a *BCK-algebra*. In what follows let X denote a BCK/BCI-algebra unless otherwise specified. A nonempty subset S of X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A nonempty subset A of X is called an *ideal* of X if it satisfies

- $0 \in A$,
- $(\forall x, y \in X)$ $(x * y \in A,\, y \in A \Rightarrow x \in A)$.

We refer the reader to the book [3] for further information regarding BCK/BCI-algebras.

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A fuzzy set μ in a set X of the form

$$\mu(y) := \begin{cases} t \in (0,1] & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is said to be a *fuzzy point* with support x and value t and is denoted by x_t.

For a fuzzy point x_t and a fuzzy set μ in a set X, Pu and Liu [5] gave meaning to the symbol $x_t\alpha\mu$, where $\alpha \in \{\in, \mathrm{q}, \in\vee\,\mathrm{q}, \in\wedge\,\mathrm{q}\}$.

To say that $x_t \in \mu$ (resp. $x_t\mathrm{q}\mu$) means that $\mu(x) \geq t$ (resp. $\mu(x) + t > 1$), and in this case, x_t is said to *belong to* (resp. *be quasi-coincident with*) a fuzzy set μ.

To say that $x_t \in\vee\,\mathrm{q}\,\mu$ (resp. $x_t \in\wedge\,\mathrm{q}\,\mu$) means that $x_t \in \mu$ or $x_t\mathrm{q}\mu$ (resp. $x_t \in \mu$ and $x_t\mathrm{q}\mu$). For all $t_1, t_2 \in [0,1]$, $\min\{t_1, t_2\}$ and $\max\{t_1, t_2\}$ will be
denoted by $m(t_1, t_2)$ and $M(t_1, t_2)$, respectively.

A fuzzy set μ in X is called a *fuzzy subalgebra* of X if it satisfies

$$(\forall x, y \in X)\,(\mu(x * y) \geq m(\mu(x), \mu(y))). \tag{1}$$

A fuzzy set μ in X is called a *fuzzy ideal* of X if it satisfie s

(a1) $(\forall x \in X)\ (\mu(0) \geq \mu(x))$,
(a2) $(\forall x, y \in X)\ (\mu(x) \geq m(\mu(x * y), \mu(y)))$.

Proposition 2.1. *Let μ be a fuzzy set in X. Then μ is a fuzzy subalgebra/ideal of X if and only if $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is a subalgebra/ideal of X for all $t \in (0,1]$, for our convenience, the empty set $\emptyset$ is regarded as a subalgebra/ideal of X.*

3. (α, β)-FUZZY IDEALS

In what follows let α and β denote any one of $\in$, q, $\in\vee\,\mathrm{q}$, or $\in\wedge\,\mathrm{q}$ unless otherwise specified. To say that $x_t\,\overline{\alpha}\,\mu$ means that $x_t\,\alpha\,\mu$ does not hold.

Proposition 3.1. [4] *For any fuzzy set μ in X, the condition* (1) *is equivalent to the following condition*

$$(\forall x, y \in X)\,(\forall t_1, t_2 \in (0,1])\,(x_{t_1}, y_{t_2} \in \mu \;\Rightarrow\; (x * y)_{m(t_1,t_2)} \in \mu). \tag{2}$$

Note that if μ is a fuzzy set in X defined by $\mu(x) \leq 0.5$ for all $x \in X$, then the set $\{x_t \mid x_t \in\wedge\,\mathrm{q}\,\mu\}$ is empty.

A fuzzy set μ in X is said to be an (α, β)*-fuzzy subalgebra* of X, where $\alpha \neq\ \in\wedge\,\mathrm{q}$, if it satisfies the following conditions (see [4]):

$$(\forall x, y \in X)\,(\forall t_1, t_2 \in (0,1])\,(x_{t_1}\,\alpha\,\mu,\ y_{t_2}\,\alpha\,\mu \;\Rightarrow\; (x * y)_{m(t_1,t_2)}\,\beta\,\mu). \tag{3}$$

Theorem 3.2. *Let μ be a fuzzy set in X. Then $U(\mu; t)$ is an ideal of X for all $t \in (0.5, 1]$ if and only if μ satisfies the following conditions:*

(i) $(\forall x \in X)\ (M(\mu(0),\, 0.5) \geq \mu(x))$,
(ii) $(\forall x, y \in X)\ (M(\mu(x),\, 0.5) \geq m(\mu(x * y),\, \mu(y)))$.

Proof. Assume that $U(\mu; t)$ is an ideal of X for all $t \in (0.5, 1]$. If there is $a \in X$ such that the condition (i) is not valid, that is,

$$(\exists a \in X)(M(\mu(0),\, 0.5) < \mu(a)),$$

then $\mu(a) \in (0.5, 1]$ and $a \in U(\mu; \mu(a))$. But $\mu(0) < \mu(a)$ implies $0 \notin U(\mu; \mu(a))$, a contradiction. Hence (i) is valid. Suppose that

$$M(\mu(a), 0.5) < m(\mu(a * b),\, \mu(b)) = s$$

for some $a, b \in X$. Then $s \in (0.5, 1]$ and $a * b$, $b \in U(\mu; s)$. But $a \notin U(\mu; s)$ since $\mu(a) < s$. This is a contradiction, and therefore (ii) is valid. Conversely, assume that μ satisfies conditions (i) and (ii). Let $t \in (0.5, 1]$. For any $x \in U(\mu; t)$, we have $M(\mu(0),\, 0.5) \geq \mu(x) \geq t > 0.5$ and so $\mu(0) \geq t$. Thus $0 \in U(\mu; t)$. Let $x, y \in X$ be such that $x * y \in U(\mu; t)$ and $y \in U(\mu; t)$. Then

$$M(\mu(x),\, 0.5) \geq m(\mu(x * y),\, \mu(y)) \geq t > 0.5,$$

and thus $\mu(x) \geq t$, that is, $x \in U(\mu; t)$. Hence $U(\mu; t)$ is an ideal of X for all $t \in (0.5, 1]$. □

Definition 3.3. A fuzzy set μ in X is called an (α, β)-*fuzzy ideal* of X, where $\alpha \neq\, \in \wedge\, \mathrm{q}$, if it satisfies

(b1) $(\forall x \in X)\ (\forall t \in (0, 1])\ (x_t\, \alpha\, \mu \Rightarrow 0_t\, \beta\, \mu)$,
(b2) $(\forall x, y \in X)\ (\forall t_1, t_2 \in (0, 1])\ ((x * y)_{t_1}\, \alpha\, \mu,\ y_{t_2}\, \alpha\, \mu \Rightarrow x_{m(t_1,\, t_2)}\, \beta\, \mu)$.

Example 3.4. Let $X = \{0, a, b, c, d\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	a
b	b	b	0	b	0
c	c	a	c	0	c
d	d	d	b	d	0

(1) A fuzzy set μ in X given by $\mu(0) = 0.7$, $\mu(a) = \mu(c) = 0.3$ and $\mu(b) = \mu(d) = 0.2$ is an $(\in, \in\vee\, \mathrm{q})$-fuzzy ideal as well as a fuzzy ideal of X.

(2) A fuzzy set ν in X given by $\nu(0) = 0.6$, $\nu(a) = \nu(c) = 0.7$ and $\nu(b) = \nu(d) = 0.2$ is an $(\in, \in\vee\, \mathrm{q})$-fuzzy ideal which is not a fuzzy ideal of X.

Theorem 3.5. *For any fuzzy set μ in X, the conditions* (a1) *and* (a2) *are equivalent to the conditions*

(i) $(\forall x \in X)\ (\forall t \in (0, 1])\ (x_t \in \mu \Rightarrow 0_t \in \mu)$,
(ii) $(\forall x, y \in X)\ (\forall t_1, t_2 \in (0, 1])\ ((x * y)_{t_1} \in \mu,\ y_{t_2} \in \mu \Rightarrow x_{m(t_1,\, t_2)} \in \mu)$

respectively.

Proof. Assume that (a1) is valid and let $x \in X$ and $t \in (0, 1]$ be such that $x_t \in \mu$. Then $\mu(0) \geq \mu(x) \geq t$, and so $0_t \in \mu$. Suppose that (i) is true. Since $x_{\mu(x)} \in \mu$ for all $x \in X$, it follows from (i) that $0_{\mu(x)} \in \mu$ so that $\mu(0) \geq \mu(x)$ for all $x \in X$. Assume that the condition (a2) holds. Let $x, y \in X$ and $t_1, t_2 \in (0, 1]$ be such that $(x * y)_{t_1} \in \mu$ and $y_{t_2} \in \mu$. Then $\mu(x * y) \geq t_1$ and $\mu(y) \geq t_2$. It follows from (a2) that

$$\mu(x) \geq m(\mu(x * y),\, \mu(y)) \geq m(t_1,\, t_2)$$

so that $x_{m(t_1,\, t_2)} \in \mu$. Finally suppose that (ii) is valid. Note that for every $x, y \in X$, $(x * y)_{\mu(x*y)} \in \mu$ and $y_{\mu(y)} \in \mu$. Hence $x_{m(\mu(x*y),\, \mu(y))} \in \mu$ by (ii), and thus

$$\mu(x) \geq m(\mu(x * y),\, \mu(y)).$$

This completes the proof. □

Remark. Theorem 3.5 shows that every $(\in, \in)$-fuzzy ideal is precisely a fuzzy ideal and vice versa. Obviously, every $(\in, \in)$-fuzzy ideal is an $(\in, \in\vee\, \mathrm{q})$-fuzzy ideal.

Theorem 3.6. *Every $(\in\vee\, \mathrm{q}, \in\vee\, \mathrm{q})$-fuzzy ideal is an $(\in, \in\vee\, \mathrm{q})$-fuzzy ideal.*

Proof. Let μ be an $(\in\vee\, \mathrm{q}, \in\vee\, \mathrm{q})$-fuzzy ideal of X. Let $x \in X$ and $t \in (0, 1]$ be such that $x_t \in \mu$. Then $x_t \in\vee\, \mathrm{q}\ \mu$, and so $0_t \in\vee\, \mathrm{q}\ \mu$. Let $x, y \in X$ and $t_1, t_2 \in (0, 1]$ be such that $(x * y)_{t_1} \in \mu$ and $y_{t_2} \in \mu$. Then $(x * y)_{t_1} \in\vee\, \mathrm{q}\ \mu$ and $y_{t_2} \in\vee\, \mathrm{q}\ \mu$ which imply that $x_{m(t_1,\, t_2)} \in\vee\, \mathrm{q}\ \mu$. Therefore μ is an $(\in, \in\vee\, \mathrm{q})$-fuzzy ideal of X. □

Remark. The converse of Theorem 3.6 is not true in general. In fact, the $(\in, \in\vee\mathrm{q})$-fuzzy ideal ν in Example 3.4(2) is not an $(\in\vee\mathrm{q}, \in\vee\mathrm{q})$-fuzzy ideal of X since $(b*a)_{0.82} \in\vee\mathrm{q}\ \nu$ and $a_{0.7} \in\vee\mathrm{q}\ \nu$, but $b_{m(0.82,\,0.7)}\ \overline{\in\vee\mathrm{q}}\ \nu$.

Remark. Let X be a BCK-algebra. We know that every $(\in,\in)$-fuzzy ideal of X is an $(\in,\in)$-fuzzy subalgebra of X. But an $(\in, \in\vee\mathrm{q})$-fuzzy ideal of X may not be an $(\in,\in)$-fuzzy subalgebra of X. For example, the $(\in, \in\vee\mathrm{q})$-fuzzy ideal ν in Example 3.4(2) is not an $(\in,\in)$-fuzzy subalgebra because $a_{0.65} \in \nu$ and $c_{0.67} \in \nu$ but $(a*c)_{m(0.65,\,0.67)} = 0_{0.65}\ \overline{\in}\ \nu$.

Theorem 3.7. *A fuzzy set μ in X is an $(\in, \in\vee\mathrm{q})$-fuzzy ideal of X if and only if it satisfies*

(i) $(\forall x \in X)\ (\mu(0) \geq m(\mu(x),\, 0.5))$,
(ii) $(\forall x, y \in X)\ (\mu(x) \geq m(\mu(x*y),\, \mu(y),\, 0.5))$.

Proof. Suppose that μ is an $(\in, \in\vee\mathrm{q})$-fuzzy ideal of X. Let $x \in X$ and assume that $\mu(x) < 0.5$. If $\mu(0) < \mu(x)$, then $\mu(0) < t < \mu(x)$ for some $t \in (0, 0.5)$ and so $x_t \in \mu$ and $0_t\ \overline{\in}\ \mu$. Since $\mu(0)+t < 1$, we have $0_t\ \overline{\mathrm{q}}\ \mu$. It follows that $0_t\ \overline{\in\vee\mathrm{q}}\ \mu$, a contradiction. Hence $\mu(0) \geq \mu(x)$. Now if $\mu(x) \geq 0.5$, then $x_{0.5} \in \mu$ and thus $0_{0.5} \in\vee\mathrm{q}\,\mu$. Thus $\mu(0) \geq 0.5$. Otherwise, $\mu(0)+0.5 < 0.5+0.5 = 1$, a contradiction. Consequently, $\mu(0) \geq m(\mu(x),\, 0.5)$ for all $x \in X$. Let $x, y \in X$ and suppose that $m(\mu(x*y),\, \mu(y)) < 0.5$. Then $\mu(x) \geq m(\mu(x*y),\, \mu(y))$. If not, then $\mu(x) < t < m(\mu(x*y),\, \mu(y))$ for some $t \in (0, 0.5)$. It follows that $(x*y)_t \in \mu$ and $y_t \in \mu$ but $x_{m(t,\,t)} = x_t\ \overline{\in\vee\mathrm{q}}\ \mu$ which is a contradiction. Hence $\mu(x) \geq m(\mu(x*y),\, \mu(y))$ whenever $m(\mu(x*y),\, \mu(y)) < 0.5$. If $m(\mu(x*y),\, \mu(y)) \geq 0.5$, then $(x*y)_{0.5} \in \mu$ and $y_{0.5} \in \mu$, which implies that $x_{0.5} = x_{m(0.5,\,0.5)} \in\vee\mathrm{q}\ \mu$. Therefore $\mu(x) \geq 0.5$ because if $\mu(x) < 0.5$ then $\mu(x)+0.5 < 0.5+0.5 = 1$, a contradiction. Hence $\mu(x) \geq m(\mu(x*y),\, \mu(y),\, 0.5)$ for all $x, y \in X$. Conversely assume that μ satisfies conditions (i) and (ii). Let $x \in X$ and $t \in (0, 1]$ be such that $x_t \in \mu$. Then $\mu(x) \geq t$. Suppose that $\mu(0) < t$. If $\mu(x) < 0.5$, then $\mu(0) \geq m(\mu(x),\, 0.5) = \mu(x) \geq t$, a contradiction. Hence we know that $\mu(x) \geq 0.5$ and so

$$\mu(0) + t > 2\mu(0) \geq 2m(\mu(x),\, 0.5) = 1.$$

Thus $0_t \in\vee\mathrm{q}\ \mu$. Let $x, y \in X$ and $t_1, t_2 \in (0, 1]$ be such that $(x*y)_{t_1} \in \mu$ and $y_{t_2} \in \mu$. Then $\mu(x*y) \geq t_1$ and $\mu(y) \geq t_2$. Suppose that $\mu(x) < m(t_1,\, t_2)$. If $m(\mu(x*y),\, \mu(y)) < 0.5$ then

$$\mu(x) \geq m(\mu(x*y),\, \mu(y),\, 0.5) = m(\mu(x*y),\, \mu(y)) \geq m(t_1,\, t_2).$$

This is a contradiction, and so $m(\mu(x*y),\, \mu(y)) \geq 0.5$. It follows that

$$\mu(x) + m(t_1,\, t_2) > 2\mu(x) \geq 2m(\mu(x*y),\, \mu(y),\, 0.5) = 1$$

so that $x_{m(t_1,\,t_2)} \in\vee\mathrm{q}\,\mu$. Consequently μ is an $(\in, \in\vee\mathrm{q})$-fuzzy ideal of X. □

Theorem 3.8. *Let μ be an $(\in, \in\vee\mathrm{q})$-fuzzy ideal of X such that $\mu(x) < 0.5$ for all $x \in X$. Then μ is an $(\in,\in)$-fuzzy ideal of X.*

Proof. Let $x \in X$ and $t \in (0, 1]$ be such that $x_t \in \mu$. Then $\mu(x) \geq t$, and so

$$\mu(0) \geq m(\mu(x),\, 0.5) = \mu(x) \geq t.$$

Hence $0_t \in \mu$. Now let $x, y \in X$ and $t_1, t_2 \in (0, 1]$ be such that $(x*y)_{t_1} \in \mu$ and $y_{t_2} \in \mu$. Then $\mu(x*y) \geq t_1$ and $\mu(y) \geq t_2$. It follows from Theorem 3.7(ii) that

$$\mu(x) \geq m(\mu(x*y),\, \mu(y),\, 0.5) = m(\mu(x*y),\, \mu(y)) \geq m(t_1,\, t_2)$$

so that $x_{m(t_1,\,t_2)} \in \mu$. Therefore μ is an $(\in,\in)$-fuzzy ideal of X. □

Theorem 3.9. *A fuzzy set μ in X is an $(\in, \in\vee\mathrm{q})$-fuzzy ideal of X if and only if the set*

$$U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$$

is an ideal of X for all $t \in (0, 0.5]$.

Proof. Assume that μ is an $(\in, \in\vee\,\mathrm{q})$-fuzzy ideal of X and let $t \in (0, 0.5]$. Using Theorem 3.7(i), we have $\mu(0) \geq m(\mu(x), 0.5)$ for any $x \in U(\mu; t)$. It follows that $\mu(0) \geq m(t, 0.5) = t$ so that $0 \in U(\mu; t)$. Let $x, y \in X$ be such that $x * y \in U(\mu; t)$ and $y \in U(\mu; t)$ for $t \in (0, 0.5]$. Then $\mu(x * y) \geq t$ and $\mu(y) \geq t$. Using Theorem 3.7(ii), we get

$$\mu(x) \geq m(\mu(x * y), \mu(y), 0.5) \geq m(t, 0.5) = t,$$

and so $x \in U(\mu; t)$. Hence $U(\mu; t)$, $t \in (0, 0.5]$, is an ideal of X. Conversely let μ be a fuzzy set in X such that $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is an ideal of X for all $t \in (0, 0.5]$. If there is $a \in X$ such that $\mu(0) < m(\mu(a), 0.5)$, then $\mu(0) < t < m(\mu(a), 0.5)$ for some $t \in (0, 0.5)$, and so $0 \notin U(\mu; t)$. This is a contradiction. Hence $\mu(0) \geq m(\mu(x), 0.5)$ for all $x \in X$. Assume that there exist $a, b \in X$ such that $\mu(a) < m(\mu(a * b), \mu(b), 0.5)$. Taking

$$t := \frac{1}{2}(\mu(a) + m(\mu(a * b), \mu(b), 0.5)),$$

we get $t \in (0, 0.5)$ and $\mu(a) < t < m(\mu(a*b), \mu(b), 0.5)$. Thus $a*b \in U(\mu; t)$ and $b \in U(\mu; t)$, but $a \notin U(\mu, ; t)$ This is a contradiction. Hence

$$\mu(x) \geq m(\mu(x * y), \mu(y), 0.5).$$

It follows from Theorem 3.7 that μ is an $(\in, \in\vee\,\mathrm{q})$-fuzzy ideal of X. □

Theorem 3.10. *Let A be an ideal of X and let μ be a fuzzy set in X such that*

(i) $(\forall x \in X \setminus A)\ (\mu(x) = 0)$,
(ii) $(\forall x \in A)\ (\mu(x) \geq 0.5)$.

Then μ is a $(\mathrm{q}, \in\vee\,\mathrm{q})$-fuzzy ideal of X.

Proof. Let $x \in X$ and $t \in (0, 1]$ be such that $x_t\, \mathrm{q}\, \mu$. Then $\mu(x) + t > 1$ and so $x \in A$. Thus $\mu(x) \geq 0.5$ and $t > 0.5$. Since $0 \in A$, it follows that $\mu(0) + t > 0.5 + 0.5 = 1$ so that $0_t \in\vee\,\mathrm{q}\,\mu$. Let $x, y \in X$ and $t_1, t_2 \in (0, 1]$ be such that $(x*y)_{t_1}\mathrm{q}\,\mu$ and $y_{t_2}\mathrm{q}\,\mu$, i.e., $\mu(x*y) + t_1 > 1$ and $\mu(y) + t_2 > 1$. Then $x * y \in A$ and $y \in A$. For, if $x * y \notin A$ (resp. $y \notin A$), then $\mu(x * y) = 0$ (resp. $\mu(y) = 0$) and so $t_1 > 1$ (resp. $t_2 > 1$), a contradiction. Since A is an ideal, it follows that $x \in A$ so that $\mu(x) \geq 0.5$. If $t_1 \leq 0.5$ or $t_2 \leq 0.5$, then $\mu(x) \geq 0.5 \geq m(t_1, t_2)$. Hence $x_{m(t_1, t_2)} \in \mu$. If $t_1 > 0.5$ and $t_2 > 0.5$, then $\mu(x) + m(t_1, t_2) > 0.5 + 0.5 = 1$ and so $x_{m(t_1, t_2)}\mathrm{q}\,\mu$. Consequently $x_{m(t_1, t_2)} \in\vee\,\mathrm{q}\,\mu$. Therefore μ is a $(\mathrm{q}, \in\vee\,\mathrm{q})$-fuzzy ideal of X. □

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Department of Mathematics Education, Gyeongsang National University, Chinju 660-701, Korea

E-mail address: ybjun@gsnu.ac.kr icedoor@hotmail.com