ISOMETRIC ISOMORPHISMS BETWEEN LOCALLY COMPACT HYPERGROUPS AND THEIR RELATED ALGEBRAS

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Introduction

An interesting result of Ghahramani, Lau and Losert **[3]** asserts that if G_1 and G_2 are two locally compact groups such that $LUC(G_1)^*$ is isometric isomorphic with $LUC(G_2)^*$, then G_1 and G_2 are topologically isomorphic. In the present paper we shall extend this result to locally hypergroups by proving that if K_1 and K_2 are two locally compact hypergroups such that $LUC(K_1)^*$ is isometrically isomorphic to $LUC(K_2)^*$, then $G(K_1)$ is topologically isomorphic with $G(K_2)$, where $G(K_i)$ denotes the maximum subgroup of $K_i (i = 1, 2)$.

Preliminaries

Throughout this paper, K will denote a locally compact hypergroup (Same as convo in **[4]**) with a fixed left Haar measre λ. The following notations are different form those in **[4]**:

δ_x The point mass at $x \in K$

$C_b(K)$ The bounded continuous complex velued functions on K

$\|f\|_\infty$ $\sup\{|f(x)| : x \in K\}$.

The involution on K is denoted by $x \to \check{x}$. If $f \in B_\infty(K)$ (the space of bounded complex valued Borel measurable functions on K) and $x, y \in K$, the left translation x^f or $\ell_x f$ is defined by

$$x^f(y) = \ell_x f(y) = \int_K f d\delta_x * \delta_y = f(x * y),$$

if the integral exists. For $f \in B_\infty(K)$ the two functions $\check{f}, \tilde{f}$ which are given by $\check{f}(x) = f(\check{x}), \tilde{f}(x) = \overline{f(\check{x})}$ respectively, are in $B_\infty(K)$. For μ in $M(K)$ the measure $\check{\mu}$ is defined by

$$\check{\mu}(f) = \int_K \tilde{f}(x) d\mu(x) \quad (f \in B_\infty(K)).$$

We also recall that if $\mu, \nu \in M(K)$ and $f \in B_\infty(K)$ then

$$\int_K f d(\mu * \nu) = \int_K \int_K f(x * y) d\mu(x) d\nu(y)$$

and

$$\mu * f(x) = \int_K f(\check{y} * x) d\mu(y)$$
$$f * \mu(x) = \int_K f(x * \check{y}) d\mu(y)$$

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The functions $\mu * f$ and $f * \mu$ are in $B_\infty(K)$. If f is also in $C_b(K)$ then both $\mu * f$ and $f * \mu$ are in $C_b(K)$ with $\|\mu * f\|_\infty \leq \|\mu\|\|f\|_\infty$ and if $f \in C_0(K)$ then $\mu * f \in C_0(K)$. Since for $f \in B_\infty(K)$, $(\mu * f)\check{} = \check{f} * \check{\mu}$, it follows that $f * \mu \in C_0(K)$ whenever $f \in C_0(K)$ and $\mu \in M(K)$.

Not that $\check{\mu} * f(x) = \int f(y * x) d\mu(x) = \langle f, \delta_x * \mu\rangle$ and similarly $f * \check{\mu}(x) = \langle f, \mu * \delta_x\rangle$ $(x \in K$ and $\mu \in M(K), f \in B_\infty(K))$. For simplicity we denote $\check{\mu} * f$ and $f * \check{\mu}$ by μf and $f\mu$ respectively. So if $f \in C_0(K)$, then both μf and $f\mu$ are in $C_0(K)$.

We also recall that if K is a hypergroup with a left Haar measure λ, then $L^1(K) = M_a(K) = \{\nu \in M(K) : x \longmapsto \delta_x * \nu$ from K into $M(K)$ is norm continuous$\}$ and furthermore $M_a(K)$ is a closed two sided $L-$ideal of $M(K)$ (c.f. [**5**]).

Since $M_a(K)$ has a bounded approximate identity an application of Cohen Factorization theorem [**4**. Theorem 32.22] shows that

$$\begin{aligned} C_0(K) &= \{f\mu : f \in C_0(K), \mu \in M_a(K)\} \\ &= \{\mu f : f \in C_0(K), \mu \in M_a(K)\}. \end{aligned}$$

For a hypergroup K we denote by $LUC(K)$ the space of all funtions $f \in C_b(K)$ for which the mapping : $x \longmapsto x^f$ is continuous from K to $(C_b(K), \| \|_\infty)$. Note that $LUC(K) = L_1(K) * L_\infty(K) = L_1(K) * LUC(K)$ (see Lemma 2.2 of [**10**]).

Let $G(K) = \{x \in K : \delta_x * \delta_{\check{x}} = \delta_{\check{x}} * \delta_x = \delta_e\}$. Then $G(K)$ is a (closed) subhypergroup of K and a locally compact group [**5** , 10.4C]. It is called the *maximum subgroup* of K. For each $x \in K$ and $y \in G(K)$, there exists a unique $z \in K$ such that $\delta_x * \delta_y = \delta_z$ [**5**, 10.4B]. We write $z = xy$. For more information on hypergroups we refer the interested reader to [**2**] and [**9**].

A closed linear subspace X of $C_b(K)$ is called *left introverted* if $\ell_x(X) \subseteq X$ for all $x \in K$, and for each $m \in X^*$ and $f \in X$ the function $m_\ell(f)$ on K is defined by $m_\ell(f)(x) = m(\ell_x f)$ $(x \in K)$ is also in X. In this case the Arens multiplication on X^* is defined by $\langle nm, f\rangle = \langle n, m_\ell(f)\rangle$ $(f \in X,\ n,\ m \in X^*)$ makes sense. Furthermore, X^* with this multiplication is a Banach algebra (see [**1**]). Trivial examples of left introverted subspaces of $C_b(K)$ are $C_0(K)$ and $LUC(K)$. In the case where $X = C_0(K)$, then $C_0(K)^* = M(K)$ and the multiplication on $M(K)$ is precisely the convolution of the measures as defined above.

The results

We start with the following result which is a generalization of 5.6B of [**5**].

Theorem 1. *Let K be a hypergroup. Let (μ_α) be a net in $M(K)$ which converges to μ in $M(K)$ in the weak * - topology with $\| \mu_\alpha \| \longrightarrow \| \mu \|$. Then $\| \mu_\alpha * \nu - \mu * \nu \| \longrightarrow 0$, for every $\nu \in M_a(K)$.*

Proof. Given $\epsilon > 0$, by Theorem 3.3 of [**8**] there exist an α_0 and a compact subset F of K such that for all $\alpha \geq \alpha_0$

$$(| \mu_\alpha | + | \mu |)(K \backslash F) < \epsilon. \tag{1}$$

Let $\mathcal{A} = \{\nu f : f \in C_b(K)$ and $\| f \|_\infty \leq 1\}$. Since $\| \nu f \|_\infty \leq \| \nu \| \| f \|_\infty$ it follows that $\mathcal{A}$ is uniformly bounded in $C_b(K)$. We claim that $\mathcal{A}$ is equicontinuous. To see this, take x_0 fixed in K. So there is a neighbourhood U of x_0 such that $\| \delta_x * \nu - \delta_{x_0} * \nu \| < \epsilon$ for all

$x \in U$. If $f \in C_b(K)$ with $\| f \|_\infty \leq 1$, then for every $x \in U$

$$\begin{aligned} | \nu f(x) - \nu f(x_0) | &= | \int_K f(y) d(\delta_x * \nu - \delta_{x_0} * \nu)(y) | \\ &\leq \| f \|_\infty \| \delta_x * \nu - \delta_{x_0} * \nu \| < \epsilon. \end{aligned}$$

That is $\mathcal{A}$ is equicontinuous. Let $\mathcal{A}_F$ denote the set of all elements in $\mathcal{A}$ restricted to F. By the Ascoli Theorem [**6**, p.233 Theorem 17] the uniform closure of $\mathcal{A}_F$ is compact in $C(F)$ (the space of all continuous complex-valued functions on F), and so it is totally bounded. Let $\{\nu f_1, \ldots, \nu f_N\}$ be an ϵ-net for this compact metric space. Let $\nu f \in \mathcal{A}$; then for some j $(1 \leq j \leq N)$
$\| \nu f - \nu f_j \|_F < \epsilon$, where $\| \, . \, \|_F$ denotes the sup-norm on F. Since $\mu_\alpha \longrightarrow \mu$ in the weak *-topology, there exists an $\alpha_1 (\alpha_1 \geq \alpha_0)$ such that for all $i = 1, \ldots, N$

$$| \textstyle\int_F \nu f_i d\mu_\alpha - \int_F \nu f_i d\mu | < \epsilon \text{ for all } \alpha \geq \alpha_1.$$

Thus for all $\alpha \geq \alpha_1$ we have

$$\begin{aligned} | (\mu_\alpha * \nu - \mu * \nu)(f) | &\leq | \int_{K \backslash F} f(x) d(\mu_\alpha * \nu - \mu * \nu)(x) | \\ &\quad + | \int_F f(x) d(\mu_\alpha * \nu - \mu * \nu)(x) | \\ &\leq \| \nu \| (| \mu_\alpha | + | \mu |)(K \backslash F) \\ &\quad + | \int_F [\nu f(x) - \nu f_j(x)] d\mu_\alpha(x) | \\ &\quad + | \int_F \nu f_j(x) d\mu_\alpha(x) - \int_F \nu f_j d\mu(x) | \\ &\quad + | \int_F [\nu f_j(x) - \nu f(x)] d\mu(x) | \\ &< \epsilon \| \nu \| + \| \nu f - \nu f_j \|_F \| \mu_\alpha \| + \epsilon \\ &\quad + \| \mu \| \| \| \nu f - \nu f_j \|_F \\ &< \epsilon(3M + 1), \end{aligned}$$

where $M > 0$ is chosen so that $\| \mu \| < M$, $\| \nu \| < M$ and $\| \mu_\alpha \| < M$ for all α. This implies that $\| \mu_\alpha * \nu - \mu * \nu \| \leq \epsilon(3M + 1)$. ■

In view of the above theorem we introduce the following definition.

Definition 2. Let $\{m_\alpha\}$ be a net in $LUC(K)^*$. We say that (m_α) converges to $m \in LUC(K)^*$ strictly if $\| m_\alpha \mu - m\mu \| \longrightarrow 0$ for every $\mu \in M_a(K)$.

As a consequence of Theorem 1, we obtain the following result.

Corollary 3. *Let K be a hypergroup. If (μ_α) is a net in $M(K)$ which converges to $\mu \in M(K)$ in the weak *-topology with $\| \mu_\alpha \| \longrightarrow \| \mu \|$, then (μ_α) converges to μ strictly.*

Lemma 4. *For any locally compact hypergroup K,*

$$LUC(K)^* = M(K) \oplus C_0(K)^\perp,$$

where $C_0(K)^\perp = \{m \in LUC(K)^* : m(f) = 0$ for all $f \in C_0(K)\}$. If $m \in LUC(K)^*$ and $m = \mu + m_1$ for $\mu \in M(K)$ and $m_1 \in C_0(K)^\perp$, then $\|m\| = \|\mu\| + \|m_1\|$ and $C_0(K)^\perp$ is a *closed ideal in* $LUC(K)^*$.

Proof. We only need to show that $C_0(K)^\perp$ is an ideal in $LUC(K)^*$, since the proof of the other parts is the same as that of Lemma 1.1 of **[3]**.

Let $n \in C_0(K)^\perp$ and $h \in C_0(K)$. Since $\ell_x h \in C_0(K)$ for every $x \in K$, it follows that $n(\ell_x h) = 0$ for all $x \in K$. Thus for every $x \in K$

$$\langle nh, x\rangle = n(\ell_x h) = 0.$$

Hence $nh = 0$. So for every $m \in LUC(K)^*$ and $h \in C_0(K)$ we have $\langle mn, h\rangle = \langle m, nh\rangle = 0$. Thus $mn \in C_0(K)^\perp$. So $C_0(K)^\perp$ is a left ideal in $LUC(K)^*$.

In order to prove that $C_0(K)^\perp$ is a right ideal in $LUC(K)^*$, we choose $n \in C_0(K)^\perp$. For every $\mu \in M(K)$ and $h \in C_0(K)$, since $\mu h \in C_0(K)$ we have $\langle n\mu, h\rangle = \langle n, \mu h\rangle = 0$. Thus $n\mu \in C_0(K)^\perp$. Let $m \in LUC(K)$ and $m = \mu + m_1$, for $\mu \in M(K)$ and $m_1 \in C_0(K)^\perp$. Then $nm = n\mu + nm_1$. Since by the second paragraph nm_1 is also in $C_0(K)^\perp$, we conclude that $nm \in C_0(K)^\perp$. That is, $C_0(K)^\perp$ is also a right ideal of $LUC(K)^*$. The proof is complete. ■

Lemma 5. *Let K be a hypergroup. Then for m in $LUC(K)^*$ the following are equivalent:*

(i) m is invertible and $\|m\| = \|m^{-1}\| = 1$,

(ii) there exists $\alpha \in \mathbb{C}$ with $|\alpha| = 1$ and $x \in G(K)$ such that $m = \alpha\delta_x$.

Proof. It is clear that (ii) implies (i). It remains to prove that (i) implies (ii). To see this we invoke Lemma 4 in order to write $m = \mu + m_1$ and $m^{-1} = \nu + m_2$ with $\mu, \nu \in M(K), m_1, m_2 \in C_0(K)^\perp$. Then $\delta_e = \mu * \nu + (\mu m_2 + m_1\nu + m_1 m_2)$. Again by Lemma 4 the part in parentheses belongs to $C_0(K)^\perp$ and hence equals 0. Hence $\| \mu*\nu \| = 1 = \| \mu \| = \| \nu \|$, so $m_1 = 0 = m_2$, by Lemma 4. For every $h \in C_0(K)$ with $h(e) = 1$ and $0 \le h \le 1$ we have

$$1 = \langle \delta_e, h\rangle = \langle \mu * \nu, h\rangle.$$

Thus

$$1 = \int_K hd(\mu * \nu) \le \int_K hd \mid \mu \mid * \mid \nu \mid = \int_K \mid \mu \mid hd \mid \nu \mid \le \| \mu \| \| \nu \| = 1.$$

Hence

$$\int_K [1 - \mid \mu \mid h] d \mid \nu \mid = 0.$$

Since $0 \le |\mu|h \le 1$, it follows that $|\mu|h(t) = 1$ for all $t \in \mathrm{supp}(\nu)$. Since

$$|\mu|h(t) = 1 = \int_K h(\delta_t * \delta_s) d|\mu|(s),$$

we have

$$\int_K [1 - h(\delta_t * \delta_s)] d|\mu|(s) \quad = 0.$$

Thus $h(\delta_t * \delta_s) = 1$ for all $t \in \mathrm{supp}(\nu)$ and $s \in \mathrm{supp}(\mu)$. Form this it follows that $e \in \mathrm{supp}(\delta_x * \delta_y)$ for all $x \in \mathrm{supp}(\mu)$ and $y \in \mathrm{supp}(\nu)$. So $x = \check{y}$ for every $x \in \mathrm{supp}(\nu)$ and $y \in \mathrm{supp}(\mu)$. Hence there exists $x \in K$ such that $\mathrm{supp}(\mu) = \{x\}$ and $\mathrm{supp}(\nu) = \{\check{x}\}$. Since $\delta_e = \mu * \nu$, it follows that $x \in G(K)$. This establishes the proof. ■

The proof of the following Lemma is the same as that of Lemma 1 of **[7]**.

Lemma 6. *Let X be a locally compact Hausdorff space and $m \in C_0(X)^*$. Then m has a unique norm preserving extension to a continuous linear functional on $C_b(X)$.*

Using Lemma 6 in place of Lemma 1 of **[7]** in the Proof of Lemma 1.4 of **[3]**, we obtain the following result. The proof is omitted.

Lemma 7. *Let K_1, K_2 be two locally compact hypergroups and let T be an isometric isomorphism from $LUC(K_1)^*$ onto $LUC(K_2)^*$. Let (m_α) be a net in $M(K_1)$ converging strictly to m in $M(K_2)$ and $\|m_\alpha\| = \|m\| = 1$. Then $T(m_\alpha)$ converges to $T(m)$ in the weak $*-$topology of $LUC(K_2)^*$.*

Remark. It should be remarked that in the above lemma $M(K_i)$ $(i = 1,\ 2)$ is considered as a subspace of $LUC(K_i)^*$ in the obvious way.

The following is the main result of this paper and it gives a generalization of Theorem 1.6 of **[3]**.

Theorem 8. *Let K_1 and K_2 be two locally compact hypergroups and $LUC(K_1)^*$ is isometrically isomorphic with $LUC(K_2)^*$. Then $G(K_1)$ is topologically isomorphic with $G(K_2)$.*

Proof. Let $x \in G(K_1)$. Then $T(\delta_x)T(\delta_{\check{x}}) = T(\delta_{\check{x}})T(\delta_x) = e_2$ (the identity of K_2). So by Lemma 5 there exist $\alpha(x) \in \mathbb{C}$ with $\mid \alpha(x) \mid = 1$ and $\tau(x) \in K_2$ such that $T(\delta_x) = \alpha(x)\delta_{\tau(x)}$. It is also obvious that α defines a character on K_1, that is $\alpha(\delta_x * \delta_y) = \alpha(x)\alpha(y)$ and $\mid \alpha(x) \mid = 1$ $(x, y \in K_1)$, and τ defines an isomorphism of K_1 onto K_2. Let (x_i) be a net in K_1 which converges to x in K_1; then $\delta_{x_i} \longrightarrow \delta_x$ strictly. So by Lemma 7, $T(\delta_{x_i}) \longrightarrow T(\delta_x)$ in the weak * - topology of $LUC(K_2)^*$. Consequently, $\alpha(x_i) \longrightarrow \alpha(x)$ and $\tau(x_i) \longrightarrow \tau(x)$. This proves the continuity of α and τ. The proof is complete. ■

Let $\tau : K_1 \to K_2$ be a (topological) isomorphism of K_1 onto K_2 and let $\alpha : K_1 \to \mathbb{C}$ be a continuous character. Define $\tau_\alpha : C_0(K_2) \to C_0(K_1)$ by $(\tau_\alpha f)(x) = \alpha(x)f(\tau(x))$ for all $x \in K_1$ and $f \in C_0(K_2)$. Then τ_α is an isometric isomorphism of $C_0(K_2)$ onto $C_0(K_1)$. Furthermore, $T_{\tau,\alpha} = \tau_\alpha^*$ is an isometric algebra isomorphism from $M(K_1)$ onto $M(K_2)$, where

$$\langle \tau_\alpha^*, f\rangle = \int_{K_1} \alpha(x)f(\tau(x))d\mu(x) \quad (f \in C_0(K_2),\ \mu \in M(K_1)).$$

For each $\mu \in M(K_1)$, let $\mu^\tau \in M(K_2)$ be defined by

$$\langle \mu^\tau, f\rangle = \int_{K_1} f(\tau(x))d\mu(x) \quad (f \in C_0(K_2)).$$

Lemma 9. *Let K_1 and K_2 be two locally compact hypergroups. Let τ be a topological isomorphism of K_1 onto K_2 and T be an isometric isomorphism of $LUC(K_1)^*$ onto $LUC(K_2)^*$ such that $T(\delta_x) = \tau_\alpha^*(x)(x \in K_1)$. Then*

$$T(\mu) = \alpha\mu^\tau \quad (\mu \in M(K_1)). \tag{2}$$

In particular, T maps $M(K_1)$ onto $M(K_2)$ and $M_a(K_1)$ onto $M_a(K_2)$.

Proof. It is clear that (2) holds for $\mu = \delta_x (x \in K_1)$, and hence for all convex combinations of such measures. Let $\mu \in M(K_1)$ be a positive measure with $\| \mu \|= 1$. There exists a net $\mu_\beta = \sum_{i=1}^{n_\beta} \lambda_i^\beta \delta_{x_i}$ of convex combinations of δ_x's, $x \in K_1$ such that μ_β converges to μ in the weak * - topology of $M(K_1)$. Therefore

$$\| \mu_\beta \|= \langle \mu_\beta, 1\rangle \longrightarrow \langle \mu, 1\rangle =\| \mu \| .$$

From Corollary 3 it follows that (μ_β) converges to μ strictly. So $(T\mu_\beta)$ converges to $T\mu$ in the weak * - topology of $LUC(K_2)^*$, by Lemma 7. Hence, the net $\alpha\mu_\beta^\tau \longrightarrow \alpha\mu^\tau$ in the weak * - topology. That is, (2) holds for all positive measures μ with $\| \mu \|= 1$, and hence it must hold for all $\mu \in M(K_1)$.

In order to prove the next assertion, we assume that (z_β) is a net in K_2 which converges to $z \in K_2$. Then $y_\beta \longrightarrow y$ in K_1, where $y_\beta = \tau^{-1}(z_\beta)$ and $y = \tau^{-1}(z)$. Thus for every $\mu \in M_a(K_1)$ we have

$$\begin{aligned} \| (T\mu) * \delta_{y_\beta} - (T\mu) * \delta_y \| &= \sup\{| \int_{K_1} (f \circ \tau)(x)\alpha(x) d(\mu * \delta_{z_\beta} - \mu * \delta_z)(x) | \; : \\ & \quad f \in C_0(K_2), \;\; \| f \|_u \leq 1\} \\ &\leq \| \mu * \delta_{z_\beta} - \mu * \delta_z \| \longrightarrow 0. \end{aligned}$$

Similarly, $\| \delta_{y_\beta} * (T\mu) - \delta_y * (T\mu) \| \longrightarrow 0$. Hence $T\mu \in M_a(K_2)$. ■

In the case of join hypergroups we present the following result. For the definition of the join hypergroup $G \vee J$ of a compact group G and a discrete hypergroup J we refer the interested reader to 10.5 of [**4**]. Note that every join hypergroup has a left Haar measure, by Proposition 1.1 of [**9**].

Theorem 10. *Let $K_1 = G_1 \vee J_1$ and $K_2 = G_2 \vee J_2$ be two join hypergroups. If J_1 is isomorphic with J_2 and $LUC(K_1)^*$ is topologically isomorphic with $LUC(K_2)^*$, then the following are valid:*
i) K_1 is topologically isomorphic with K_2;
ii) $M_a(K_1)$ is isometrically isomorphic with $M_a(K_2)$;
iii) $M(K_1)$ is isometrically isomorphic with $M(K_2)$.

Proof. Since $G(K_i) = G_i (i = 1, 2)$, from Theorem 8 it follows that G_1 is topologically isomorphic with G_2. From the fact that J_1 is isomorphic with J_2, we conclude that K_1 is topologically isomorphic with K_2. Now (ii) and (iii) follow from Theorem 9. ■

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