TRIANGULAR NORMED FUZZY SUBALGEBRAS OF BCK-ALGEBRAS

YOUNG BAE JUN, MEHMET ALİ ÖZTÜRK AND EUN HWAN ROH*

Received September 24, 2004

Abstract. Using a t-norm T, the notion of T-fuzzy topological subalgebras in BCK-algebras is introduced, and the fact that T-fuzzy subalgebras of a BCK-algebra X form a complete lattice is proved. Using a chain of subalgebras, a T-fuzzy subalgebra is established. Some of Foster's results on homomorphic image and inverse image to T-fuzzy topological subalgebras are considered.

1. Introduction

A BCK-algebra is an important class of logical algebras introduced by Iséki and was extensively investigated by several researchers. The concept of fuzzy sets, which was introduced in [9], provides a natural framework for generalizing many of the concepts of general topology to what might be called fuzzy topological spaces. Foster [1] combined the structure of a fuzzy topological spaces with that of a fuzzy group, introduced by Rosenfeld [6], to formulate the elements of a theory of fuzzy topological groups. In [2], Jun, one of the present authors, introduced the concept of fuzzy topological BCK-algebras. Jun and Zhang [4] redefined the fuzzy subalgebra of a BCK-algebra with respect to a t-norm, and Jun [3] considered the direct product and t-normed product of fuzzy subalgebras of a BCK-algebra with respect to a t-norm. In the present paper, using triangular norm, we discuss the concept of fuzzy topological subalgebras in BCK-algebras. We verify T-fuzzy subalgebras of a BCK-algebra X form a complete lattice. Using a chain of subalgebras, we establish a T-fuzzy subalgebra. We apply some of Foster's results on the homomorphic images and inverse images to a T-fuzzy subalgebra.

2. Preliminaries

An algebra $(X;*,0)$ of type $(2,0)$ is said to be a *BCK-algebra* if it satisfies: for all $x,y,z \in X$,

(I) $((x*y)*(x*z))*(z*y)=0$,
(II) $(x*(x*y))*y=0$,
(III) $x*x=0$,
(IV) $0*x=0$,
(V) $x*y=0$ and $y*x=0$ imply $x=y$.

Define a binary relation $\leq$ on X by letting $x \leq y$ if and only if $x*y=0$. Then $(X;\leq)$ is a partially ordered set with the least element 0. A subset S of a BCK-algebra X is called a *subalgebra* of X if $x*y \in S$ whenever $x,y \in S$. A mapping $f: X \to X'$ of BCK-algebras is called a *homomorphism* if $f(x*y)=f(x)*f(y)$ for all $x,y \in X$. In any BCK-algebra X, the following hold:

(P1) $(x*y)*z=(x*z)*y$,

2000 *Mathematics Subject Classification.* 06F35, 03G25, 54A40, 03E72.
Key words and phrases. T-fuzzy subalgebra, T-fuzzy topological subalgebra.
*Corresponding author: Tel. +82 55 740 1232.

(P2) $x * y \leq x$,
(P3) $x * 0 = x$,
(P4) $(x * z) * (y * z) \leq x * y$,
(P5) $x * (x * (x * y)) = x * y$,
(P6) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$,

for all $x, y, z \in X$. Generally, an aggregation operator is a mapping $F : I \to I^n$ $(n \geq 2)$, where $I = [0, 1]$. Essentially it takes a collection of arguments and provides an aggregated value. An important class of aggregation operators are the triangular norm operators, *t*-norm and *t*-conorm. These operators play a significant role in the theory of fuzzy subsets by generalizing the intersection (*and*) and union (*or*) operators, respectively. By a *t-norm* T (see [7]) we mean a function $T : I \times I \to I$ satisfying the following conditions:

(T1) $T(x, 1) = x$,
(T2) $T(x, y) \leq T(x, z)$ whenever $y \leq z$,
(T3) $T(x, y) = T(y, x)$,
(T4) $T(x, T(y, z)) = T(T(x, y), z)$,

for all $x, y, z \in I$. For a *t*-norm T, let Δ_T denote the set of elements $\alpha \in I$ such that $T(\alpha, \alpha) = \alpha$, that is,

$$\Delta_T := \{\alpha \in I \mid T(\alpha, \alpha) = \alpha\}.$$

Note that every *t*-norm T has a useful property:

(p7) $T(\alpha, \beta) \leq \min\{\alpha, \beta\}$ for all $\alpha, \beta \in I$.

A *t*-norm T on I is said to be *continuous* if T is a continuous function from $I \times I$ to I with respect to the usual topology. A fuzzy set μ in a set L is said to satisfy *imaginable property* if $\mathrm{Im}(\mu) \subseteq \Delta_T$.

3. Triangular normed fuzzy subalgebras

Definition 3.1 [4] A fuzzy set μ in a BCK-algebra X is called a *fuzzy subalgebra* of X with respect to a *t*-norm T (briefly, a *T-fuzzy subalgebra* of X) if it satisfies the inequality $\mu(x * y) \geq T(\mu(x), \mu(y))$ for all $x, y \in X$.

Example 3.2 Let $X = \{0, a, b, c\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Let T_m be a *t*-norm defined by $T_m(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$ for all $\alpha, \beta \in [0, 1]$.

(1) Define a fuzzy set $\mu : X \to [0, 1]$ by

$$\mu(x) := \begin{cases} 0.7 & \text{if } x \in \{0, b\}, \\ 0.07 & \text{otherwise.} \end{cases}$$

Then μ is a T_m-fuzzy subalgebra of X which is not imaginable.

(2) Let ν be a fuzzy set in X defined by

$$\nu(x) := \begin{cases} 1 & \text{if } x \in \{0, b\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then ν is an imaginable T_m-fuzzy subalgebra of X.

Proposition 3.3 Let T be a t-norm on I. If a fuzzy set μ in a BCK-algebra X is an imaginable T-fuzzy subalgebra of X, then $\mu(0) \geq \mu(x)$ for all $x \in X$.

Proof. For every $x \in X$, we have $\mu(0) = \mu(x * x) \geq T(\mu(x), \mu(x)) = \mu(x)$.

Proposition 3.4 Let S be a subalgebra of a BCK-algebra X and let μ be a fuzzy set in X defined by

$$\mu(x) := \begin{cases} \alpha & \text{if } x \in S, \\ \beta & \text{otherwise,} \end{cases}$$

for all $x \in X$, where $\alpha, \beta \in [0, 1]$ with $\alpha > \beta$. Then μ is a T_m-fuzzy subalgebra of X. If $\alpha = 1$ and $\beta = 0$, then μ is imaginable, where T_m is the t-norm in Example 3.2.

Proof. Let $x, y \in X$. If $x, y \in S$, then

$$\begin{aligned} T_m(\mu(x), \mu(y)) &= T_m(\alpha, \alpha) = \max\{2\alpha - 1, 0\} \\ &= \begin{cases} 2\alpha - 1 & \text{if } \alpha \geq \frac{1}{2} \\ 0 & \text{if } \alpha < \frac{1}{2} \end{cases} \\ &\leq \alpha = \mu(x * y). \end{aligned}$$

If $x \in S$ and $y \notin S$ (or, $x \notin S$ and $y \in S$), then

$$\begin{aligned} T_m(\mu(x), \mu(y)) &= T_m(\alpha, \beta) = \max\{\alpha + \beta - 1, 0\} \\ &= \begin{cases} \alpha + \beta - 1 & \text{if } \alpha + \beta \geq 1 \\ 0 & \text{otherwise} \end{cases} \\ &\leq \beta = \mu(x * y). \end{aligned}$$

If $x \notin S$ and $y \notin S$, then

$$\begin{aligned} T_m(\mu(x), \mu(y)) &= T_m(\beta, \beta) = \max\{2\beta - 1, 0\} \\ &= \begin{cases} 2\beta - 1 & \text{if } \beta \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \\ &\leq \beta = \mu(x * y). \end{aligned}$$

Hence μ is a T_m-fuzzy subalgebra. Assume that $\alpha = 1$ and $\beta = 0$. Then

$$T_m(\alpha, \alpha) = \max\{2\alpha - 1, 0\} = 1 = \alpha,$$

$$T_m(\beta, \beta) = \max\{2\beta - 1, 0\} = 0 = \beta.$$

Thus $\alpha, \beta \in \Delta_{T_m}$, that is, $\mathrm{Im}(\mu) \subseteq \Delta_{T_m}$ and so μ is imaginable. This completes the proof.

Proposition 3.5 If μ_i, $i \in I$, is a T-fuzzy subalgebra of a BCK-algebra X, then $\bigcap_{i \in I} \mu_i$ is also a T-fuzzy subalgebra of X where $\bigcap_{i \in I} \mu_i$ is defined by $(\bigcap_{i \in I} \mu_i)(x) = \inf_{i \in I} \mu_i(x)$ for all $x \in X$.

Proof. For any $x, y \in X$, we have $\mu_i(x) \geq \inf_{i \in I} \mu_i(x)$ and $\mu_i(y) \geq \inf_{i \in I} \mu_i(y)$. Hence for every $i \in I$,

$$T(\mu_i(x), \mu_i(y)) \geq T(\inf_{i \in I} \mu_i(x), \inf_{i \in I} \mu_i(y)),$$

and so $\inf_{i\in I} T(\mu_i(x), \mu_i(y)) \geq T(\inf_{i\in I} \mu_i(x), \inf_{i\in I} \mu_i(y))$. It follows that

$$\begin{aligned}(\bigcap_{i\in I} \mu_i)(x*y) &= \inf_{i\in I} \mu_i(x*y) \\ &\geq \inf_{i\in I} T(\mu_i(x), \mu_i(y)) \\ &\geq T(\inf_{i\in I} \mu_i(x), \inf_{i\in I} \mu_i(y)) \\ &= T((\bigcap_{i\in I} \mu_i)(x), (\bigcap_{i\in I} \mu_i)(y)).\end{aligned}$$

Obviously, $(\bigcap_{i\in I} \mu_i)(0) = (\bigcap_{i\in I} \mu_i)(1)$. This completes the proof.

It follows that the T-fuzzy subalgebras of a BCK-algebra X form a complete lattice. In this lattice, the inf of a set of T-fuzzy subalgebras μ_i is just $\bigcap_{i\in I} \mu_i$, while their sup is the least μ, i.e., the intersection of μ's, which contains $\bigcup_{i\in I} \mu_i$, where $(\bigcup_{i\in I} \mu_i)(x) = \sup_{i\in I} \mu_i(x)$ for all $x \in L$.

Theorem 3.6 Let T be a t-norm on I and let μ be a fuzzy set in a BCK-algebra X with $\mathrm{Im}(\mu) = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$, where $\alpha_i < \alpha_j$ whenever $i > j$. Suppose that there exists a chain of subalgebras of X:

$$G_0 \subset G_1 \subset \cdots \subset G_n = X$$

such that $\mu(\tilde{G}_k) = \alpha_k$, where $\tilde{G}_k = G_k \setminus G_{k-1}$ and $G_{-1} = \emptyset$ for $k = 0, 1, \cdots, n$. Then μ is a T-fuzzy subalgebra of X.

Proof. Let $x, y \in X$. If x and y belong to the same $\tilde{G}_k$, then $\mu(x) = \mu(y) = \alpha_k$ and $x * y \in G_k$. Hence

$$\mu(x*y) \geq \alpha_k = \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y)).$$

Assume that $x \in \tilde{G}_i$ and $y \in \tilde{G}_j$ for every $i \neq j$. Without loss of generality we may assume that $i > j$. Then $\mu(x) = \alpha_i < \alpha_j = \mu(y)$ and $x * y \in G_i$. It follows that

$$\mu(x*y) \geq \alpha_i = \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y)).$$

Consequently, μ is a T-fuzzy subalgebra of X.

4. Fuzzy Topological Subalgebras

In this section, for the convenience of notation, we use symbols A, B, $\cdots$, etc. instead of fuzzy sets μ, ν, $\cdots$, etc., that is, A, B, $\cdots$ are fuzzy sets with membership functions μ_A, μ_B, $\cdots$, respectively. Let B be a fuzzy set in Y with membership function μ_B. The *inverse image* of B, denoted $f^{-1}(B)$, is the fuzzy set in X with membership function $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ for all $x \in X$. Conversely, let A be a fuzzy set in X with membership function μ_A. Then the *image* of A, denoted $f(A)$, is the fuzzy set in Y such that

$$\mu_{f(A)}(y) = \begin{cases} \sup\limits_{z \inf^{-1}(y)} \mu_A(z) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

A *fuzzy topology* on a set X is a family $\mathcal{T}$ of fuzzy sets in X which satisfies the following conditions:

(i) For all $c \in I$, $k_c \in \mathcal{T}$, where k_c have constant membership functions with the value c,
(ii) If $A, B \in \mathcal{T}$, then $A \cap B \in \mathcal{T}$,
(iii) If $A_j \in \mathcal{T}$ for all $j \in \Lambda$, then $\bigcup_{j\in\Lambda} A_j \in \mathcal{T}$.

The pair $(X, \mathcal{T})$ is called a *fuzzy topological space* and members of $\mathcal{T}$ are *open fuzzy sets*. Let A be a fuzzy set in X and $\mathcal{T}$ a fuzzy topology on X. Then the *induced fuzzy topology* on A is the family of fuzzy subsets of A which are the intersection with A of $\mathcal{T}$-open fuzzy sets in X. The induced fuzzy topology is denoted by $\mathcal{T}_A$, and the pair $(A, \mathcal{T}_A)$ is called a *fuzzy subspace* of $(X, \mathcal{T})$. Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be two fuzzy topological spaces. A mapping f of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$ is *fuzzy continuous* if for each open fuzzy set U in $\mathcal{U}$ the inverse image $f^{-1}(U)$ is in $\mathcal{T}$. Conversely, f is *fuzzy open* if for each open fuzzy set V in $\mathcal{T}$, the image $f(V)$ is in $\mathcal{U}$. Let $(A, \mathcal{T}_A)$ and $(B, \mathcal{U}_B)$ be fuzzy subspaces of fuzzy topological spaces $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ respectively, and let f be a mapping $(X, \mathcal{T}) \to (Y, \mathcal{U})$. Then f is a mapping of $(A, \mathcal{T}_A)$ into $(B, \mathcal{U}_B)$ if $f(A) \subset B$. Furthermore f is *relatively fuzzy continuous* if for each open fuzzy set V' in $\mathcal{U}_B$, the intersection $f^{-1}(V') \cap A$ is in $\mathcal{T}_A$. Conversely, f is *relatively fuzzy open* if for each open fuzzy set U' in $\mathcal{T}_A$, the image $f(U')$ is in $\mathcal{U}_B$.

Lemma 4.1 [1] Let $(A, \mathcal{T}_A)$, $(B, \mathcal{U}_B)$ be fuzzy subspaces of fuzzy topological spaces $(X, \mathcal{T})$, $(Y, \mathcal{U})$ respectively, and let f be a fuzzy continuous mapping of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$ such that $f(A) \subset B$. Then f is a relatively fuzzy continuous mapping of $(A, \mathcal{T}_A)$ into $(B, \mathcal{U}_B)$.

Proposition 4.2 Let T be a t-norm and let $f : X \to Y$ be a homomorphism of BCK-algebras. If G is a T-fuzzy subalgebra of Y with the membership function μ_G, then the inverse image $f^{-1}(G)$ of G under f with the membership function $\mu_{f^{-1}(G)}$ is a T-fuzzy subalgebra of X.

Proof. For any $x, y \in X$, we have

$$\begin{aligned} \mu_{f^{-1}(G)}(x * y) &= \mu_G(f(x * y)) = \mu_G(f(x) * f(y)) \\ &\geq T(\mu_G(f(x)),\, \mu_G(f(y))) \\ &= T(\mu_{f^{-1}(G)}(x),\, \mu_{f^{-1}(G)}(y)). \end{aligned}$$

Hence $f^{-1}(G)$ is a T-fuzzy subalgebra of X.

Definition 4.3 [8] A t-norm T on I is called a *continuous t-norm* if T is a continuous function from $I \times I$ to I with respect to the usual topology.

Proposition 4.4 [4] Let T be a continuous t-norm and let f be a homomorphism of a BCK-algebra X onto a BCK-algebra Y. If a fuzzy set F with the membership function μ_F is a T-fuzzy subalgebra of X, then the image $f(F)$ of F under f with the membership function $\mu_{f(F)}$ is a T-fuzzy subalgebra of Y.

For any BCK-algebra X and any element $a \in X$ we use a_r to denote the self-map of X defined by $a_r(x) = x * a$ for all $x \in X$.

Definition 4.5 Let $\mathcal{T}$ be a fuzzy topology on a BCK-algebra X. For a t-norm T, let F be a T-fuzzy subalgebra of X with the induced topology $\mathcal{T}_F$. Then F is called a *T-fuzzy topological subalgebra* of X if for each $a \in X$ the mapping $a_r : x \mapsto x * a$ of $(F, \mathcal{T}_F) \to (F, \mathcal{T}_F)$

is relatively fuzzy continuous.

Theorem 4.6 Let T be a t-norm. Given BCK-algebras X, Y and a homomorphism $f : XtoY$, let $\mathcal{U}$ and $\mathcal{T}$ be fuzzy topologies on Y and X respectively such that $\mathcal{T} = f^{-1}(\mathcal{U})$. Let G be a T-fuzzy topological subalgebra of Y with the membership function μ_G. Then $f^{-1}(G)$ is a T-fuzzy topological subalgebra of X with the membership function $\mu_{f^{-1}(G)}$.

Proof. Note from Proposition 4.2 that $f^{-1}(G)$ is a T-fuzzy subalgebra of X. It is sufficient to show that for each $a \in X$ the mapping

$$a_r : x \mapsto x * a \text{ of } (f^{-1}(G), \mathcal{T}_{f^{-1}(G)}) \to (f^{-1}(G), \mathcal{T}_{f^{-1}(G)})$$

is relatively fuzzy continuous. Let U be an open fuzzy set in $\mathcal{T}_{f^{-1}(G)}$ on $f^{-1}(G)$. Since f is a fuzzy continuous mapping of $(X, \mathcal{T})$ into $(Y, \mathcal{U})$, it follows from Lemma 4.1 that f is a relatively fuzzy continuous mapping of $(f^{-1}(G), \mathcal{T}_{f^{-1}(G)})$ into $(G, \mathcal{U}_G)$. Note that there exists an open fuzzy set $V \in \mathcal{U}_G$ such that $f^{-1}(V) = U$. The membership function of $a_r^{-1}(U)$ is given by

$$\begin{aligned} \mu_{a_r^{-1}(U)}(x) &= \mu_U(a_r(x)) = \mu_U(x * a) = \mu_{f^{-1}(V)}(x * a) \\ &= \mu_V(f(x * a)) = \mu_V(f(x) * f(a)). \end{aligned}$$

As G is a T-fuzzy topological subalgebra of Y, the mapping

$$b_r : y \mapsto y * b \text{ of } (G, \mathcal{U}_G) \to (G, \mathcal{U}_G)$$

is relatively fuzzy continuous for each $b \in Y$. Hence

$$\begin{aligned} \mu_{a_r^{-1}(U)}(x) &= \mu_V(f(x) * f(a)) = \mu_V(f(a)_r(f(x))) \\ &= \mu_{f(a)_r^{-1}(V)}(f(x)) = \mu_{f^{-1}(f(a)_r^{-1}(V))}(x), \end{aligned}$$

which implies that $a_r^{-1}(U) = f^{-1}(f(a)_r^{-1}(V))$ so that

$$a_r^{-1}(U) \cap f^{-1}(G) = f^{-1}(f(a)_r^{-1}(V)) \cap f^{-1}(G)$$

is open in the induced fuzzy topology on $f^{-1}(G)$. This completes the proof.

We say that the membership function μ_G of a T-fuzzy subalgebra G of a BCK-algebra X is f-*invariant* if, for all $x, y \in X$, $f(x) = f(y)$ implies $\mu_G(x) = \mu_G(y)$.

Theorem 4.7 Let T be a continuous t-norm. Given BCK-algebras X, Y and a homomorphism f of X onto Y, let $\mathcal{T}$ and $\mathcal{U}$ be fuzzy topologies on X and Y, respectively, such that $f(\mathcal{T}) = \mathcal{U}$. Let F be a T-fuzzy topological subalgebra of X. If the membership function μ_F of F is f-invariant, then $f(F)$ is a T-fuzzy topological subalgebra of Y.

Proof. By Proposition 4.4, $f(F)$ is a T-fuzzy subalgebra of Y. Hence it is sufficient to show that the mapping

$$b_r : y \mapsto y * b \text{ of } (f(F), \mathcal{U}_{f(F)}) \to (f(F), \mathcal{U}_{f(F)})$$

is relatively fuzzy continuous for each $b \in Y$. Note that f is relatively fuzzy open; for if $U' \in \mathcal{T}_F$, there exists $U \in \mathcal{T}$ such that $U' = U \cap F$ and by the f-invariance of μ_F,

$$f(U') = f(U) \cap f(F) \in \mathcal{U}_{f(F)}.$$

Let V' be an open fuzzy set in $\mathcal{U}_{f(F)}$. Since f is onto, for each $b \in Y$ there exists $a \in X$ such that $b = f(a)$. Hence

$$\begin{aligned}\mu_{f^{-1}(b_r^{-1}(V'))}(x) &= \mu_{f^{-1}(f(a)_r^{-1}(V'))}(x) = \mu_{f(a)_r^{-1}(V')}(f(x)) \\ &= \mu_{V'}(f(a)_r(f(x))) = \mu_{V'}(f(x) * f(a)) \\ &= \mu_{V'}(f(x * a)) = \mu_{f^{-1}(V')}(x * a) \\ &= \mu_{f^{-1}(V')}(a_r(x)) = \mu_{a_r^{-1}(f^{-1}(V'))}(x),\end{aligned}$$

which implies that $f^{-1}(b_r^{-1}(V')) = a_r^{-1}(f^{-1}(V'))$. By hypothesis, $a_r : x \mapsto x * a$ is a relatively fuzzy continuous mapping: $(F, \mathcal{T}_F) \to (F, \mathcal{T}_F)$ and f is a relatively fuzzy continuous mapping: $(F, \mathcal{T}_F) \to (f(F), \mathcal{U}_{f(F)})$. Hence

$$f^{-1}(b_r^{-1}(V')) \cap F = a_r^{-1}(f^{-1}(V')) \cap F$$

is open in $\mathcal{T}_F$. Since f is relatively fuzzy open,

$$f(f^{-1}(b_r^{-1}(V')) \cap F) = b_r^{-1}(V') \cap f(F)$$

is open in $\mathcal{U}_{f(F)}$. Consequently, $f(F)$ is a T-fuzzy topological subalgebra of Y.

References

[1] D. H. Foster, *Fuzzy topological groups*, J. Math. Anal. Appl. **67** (1979), 549-564.

[2] Y. B. Jun, *Fuzzy topological BCK-algberas,* Math. Japonica **38(6)** (1993), 1059-1063.

[3] Y. B. Jun, *Direct product and t-normed product of fuzzy subalgebras in BCK-algebras with respect to a t-norm,* Soochow J. Math. **27(1)** (2001), 83–88.

[4] Y. B. Jun and Q. Zhang, *Fuzzy subalgebras of BCK-algebras with respect to a t-norm,* Far East J. Math. Sci. (FJMS) **2(3)** (2000), 489-495.

[5] J. Meng and Y. B. Jun, *BCK-algebras*: Kyungmoonsa, Seoul, Korea 1994.

[6] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl. **35** (1971), 512-517.

[7] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific J. Math. **10** (1960), 313–334.

[8] Y. Yu, J. N. Mordeson and S. C. Cheng, *Elements of L-algebra*, Lecture Notes in Fuzzy Math. and Computer Science, Creighton Univ., Omaha, Nebraska 68178, USA, 1994.

[9] L. A. Zadeh, *Fuzzy sets*, Inform. Control **8** (1965), 338-353.

Y. B. Jun, Department of Mathematics Education, Gyeongsang National University, Jinju 660-701, Korea

E-mail address: ybjun@gsnu.ac.kr icedoor@hotmail.com

M. A. Öztürk, Department of Mathematics, Faculty of Arts and Sciences, Cumhuriyet University, 58140-Sivas, Turkey

E-mail address: maozturk@cumhuriyet.edu.tr

E. H. Roh, Department of Mathematics Education, Chinju National University of Education, Jinju 660-756, Korea

E-mail address: ehroh@cue.ac.kr