MULTIPLY POSITIVE IMPLICATIVE HYPER K-IDEALS

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Abstract. Multiplicities of sever types of positive implicative hyper K-ideals are considered, and relations among them are discussed.

1. Introduction.

The study of BCK-algebras was initiated by K. Iséki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Since then many researches worked in this area. The hyperstructure theory (called also multialgebras) is introduced in 1934 by F. Marty [7] at the 8th congress of Scandinavian Mathematiciens. Around the 40's, several authors worked on hypergroups, especially in France and in the United States, but also in Italy, Russia, Japan and Iran. Hyperstructures have many applications to several sectors of both pure and applied sciences. Recently in [6] Y. B. Jun et al. introduced and studied hyperBCK-algebra which is a generalization of a BCK-algebra. In [1] and [2] R. A. Borzooei et al. constructed the hyper K-algebras, and studied (positive implicative) hyper K-ideals in hyper K-algebras. In this paper, we consider the multiplicity of sever types of positive implicative hyper K-ideals, and discuss relations among them.

2. Preliminaries

Let H be a non-empty set endowed with a hyperoperation "$\circ$". For two subsets A and B of H, denote by $A \circ B$ the set $\bigcup_{a\in A, b\in B} a \circ b$.

By a *hyper I-algebra* we mean a nonempty set H endowed with a hyperoperation "$\circ$" and a constant 0 satisfying the following axioms:

(H1) $(\forall x, y, z \in H)\ ((x \circ z) \circ (y \circ z) < x \circ y)$,
(H2) $(\forall x, y, z \in H)\ ((x \circ y) \circ z = (x \circ z) \circ y)$,
(H3) $(\forall x \in H)\ (x < x)$,
(H4) $(\forall x, y \in H)\ (x < y,\ y < x \Rightarrow x = y)$,

where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a < b$. If a hyper I-algebra $(H, \circ, 0)$ satisfies

(H5) $(\forall x \in H)\ (0 < x)$,

then $(H, \circ, 0)$ is called a *hyper K-algebra*.

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In a hyper I-algebra H, the following are true(see [1]).

(a1) $(A \circ B) \circ C = (A \circ C) \circ B$	(a2) $x \circ (x \circ y) < y$
(a3) $x \circ y < z \Leftrightarrow x \circ z < y$	(a4) $A \circ B < C \Leftrightarrow A \circ C < B$
(a5) $(x \circ z) \circ (x \circ y) < y \circ z$	(a6) $(A \circ C) \circ (B \circ C) < A \circ B$
(a7) $(A \circ C) \circ (A \circ B) < B \circ C$	(a8) $A \circ (A \circ B) < B$
(a9) $A < A$	(a10) $A \subseteq B \Rightarrow A < B$

for all $x, y, z \in H$ and for all nonempty subsets A, B and C of H.

In a hyper K-algebra H, the following are true(see [1]).

(a11) $x \circ y < x$	(a12) $A \circ B < A$
(a13) $A \circ A < A$	(a14) $0 \in x \circ (x \circ 0)$
(a15) $x < x \circ 0$	(a16) $A < A \circ 0$
(a17) $A < A \circ B$ if $0 \in B$	

for all $x, y \in H$ and for all nonempty subsets A and B of H.

An element a of a hyper K-algebra H is said to be *left* (resp. *right*) *scalar* if $|a \circ x| = 1$ (resp. $|x \circ a| = 1$) for all $x \in H$.

A nonempty subset I of a hyper K-algebra $(H, \circ, 0)$ is called a *hyper K-ideal* of H if it satisfies

(I1) $0 \in I$,
(I2) $(\forall x, y \in H)\ (x \circ y < I,\ y \in I \Rightarrow x \in I)$.

3. Multiply positive implicative hyper K-ideals

In what follows, let H denote a hyper K-algebra unless otherwise specified.

Definition 3.1. Let k be natural numbers. A nonempty subset I of H is called a *k-multiply hyper K-ideal* of H if it satisfies (I1) and

$$(1) \qquad (\forall x \in H)\,(\forall y \in I)\,(x \circ y^k < I \Rightarrow x \in I).$$

Definition 3.2. Let k be natural numbers. A nonempty subset I of H is called a *k-multiply weak hyper K-ideal* of H if it satisfies (I1) and

$$(2) \qquad (\forall x \in H)\,(\forall y \in I)\,(x \circ y^k \subseteq I \Rightarrow x \in I).$$

Every k-multiply hyper K-ideal is a k-multiply weak hyper K-ideal, but the converse is not true. To see this, consider the following example:

Example 3.3. Let $H = \{0, a, b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Then $I := \{0, b\}$ is a k-multiply weak hyper K-ideal of H for every natural number k, but not a k-multiply hyper K-ideal of H for all natural number k since $a \circ 0^k = \{a\} < I$ and $a \notin I$.

Definition 3.4. Let k, m and n be natural numbers. A nonempty subset I of H is called a *$(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal* of H if it satisfies (I1) and

$$(3) \qquad (\forall x, y, z \in H)\,((x \circ y) \circ z^k \subseteq I,\ y \circ z^m \subseteq I \Rightarrow x \circ z^n \subseteq I).$$

Example 3.5. Let $H = \{0, a, b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{0, b\}$

Then $\{0, a\}$ is a 2-multiply weak hyper K-ideal of H, and it is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal of H.

Theorem 3.6. *Let k, m and n be natural numbers. If $0 \in H$ is a right scalar element, then every $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal is an n-multiply weak hyper K-ideal.*

Proof. Let I be a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal of H. Let $x, y \in H$ be such that $x \circ y^n \subseteq I$ and $y \in I$. Since $0 \in H$ is right scalar element, we have

$$(x \circ y) \circ 0^n = x \circ y \subseteq I \quad \text{and} \quad y \circ 0^m = \{y\} \subseteq I.$$

It follows from (3) that $x = x \circ 0^n \subseteq I$ so that I is an n–multiply weak hyper K-ideal of H. □

Note that in Theorem 3.6, the condition "$0 \in H$ to be a right scalar element" can not be omitted. Consider a hyper K-algebra $H = \{0, a, b\}$ having the following Cayley table:

$\circ$	0	a	b
0	$\{0, a\}$	$\{0\}$	$\{0, a\}$
a	$\{a, b\}$	$\{0, a\}$	$\{0, b\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Note that $0 \in H$ is not a right scalar element. The set $I := \{0, b\}$ is a $(k, m; 1)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal of H. But it is not an 1-multiply weak hyper K-ideal of H since $a \circ b \subseteq I$ and $a \notin I$.

The following example shows that the converse of Theorem 3.6 is not true in general.

Example 3.7. Let $H = \{0, a, b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{a\}$
b	$\{b\}$	$\{0, a\}$	$\{0, a, b\}$

Obviously $0 \in H$ is a right scalar element. Moreover $I := \{0, b\}$ is an n-multiply weak hyper K-ideal of H for every natural number n, but not a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal of H for all natural numbers k, m and n since $(b \circ a) \circ a^k \subseteq I$ and $a \circ a^m \subseteq I$, but $b \circ a = \{0, a\} \nsubseteq I$.

Definition 3.8. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k, m; n)$*-multiply $PI(<, \subseteq, \subseteq)_K$-ideal* of H if it satisfies (I1) and

$$(4) \qquad (\forall x, y, z \in H)\,((x \circ y) \circ z^k < I,\ y \circ z^m \subseteq I \Rightarrow x \circ z^n \subseteq I).$$

Example 3.9. In Example 3.5, $\{0, b\}$ is a $(k, m; n)$-multiply $PI(<, \subseteq, \subseteq)_K$-ideal of H.

Using (a10), we know that every $(k, m; n)$-multiply $PI(<, \subseteq, \subseteq)_K$-ideal is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal, but the converse may not be true as seen in the following example.

Example 3.10. In Example 3.5, $\{0, a\}$ is not a $(k, m; n)$-multiply $PI(<, \subseteq, \subseteq)_K$-ideal of H since $(b \circ 0) \circ b^3 = \{0, b\} < \{0, a\}$ and $0 \circ b^2 = \{0\} \subseteq \{0, a\}$ but $b \circ b = \{0, b\} \nsubseteq \{0, a\}$.

Definition 3.11. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k,m;n)$*-multiply* $PI(<,<,\subseteq)_K$*-ideal* of H if it satisfies (I1) and

$$(5) \qquad (\forall x,y,z\in H)\,((x\circ y)\circ z^k < I,\ y\circ z^m < I \Rightarrow x\circ z^n \subseteq I).$$

Example 3.12. In Example 3.5, $\{0,b\}$ is a $(k,m;n)$-multiply $PI(<,<,\subseteq)_K$-ideal of H.

Let I be a $(k,m;n)$-multiply $PI(<,<,\subseteq)_K$-ideal of H and let $x,y,z\in H$ be such that $(x\circ y)\circ z^k < I$ and $y\circ z^m \subseteq I$. Then $y\circ z^m < I$ by (a10), and so $x\circ z^n \subseteq I$ by (5). Hence we have the following theorem.

Theorem 3.13. *Every* $(k,m;n)$*-multiply* $PI(<,<,\subseteq)_K$*-ideal is a* $(k,m;n)$*-multiply* $PI(<,\subseteq,\subseteq)_K$*-ideal.*

The converse of Theorem 3.13 is not true in general. To see this, consider the following example:

Example 3.14. Let $H=\{0,a,b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0,a\}$	$\{0,a\}$	$\{0,a\}$
a	$\{a\}$	$\{0,a\}$	$\{a\}$
b	$\{a,b\}$	$\{0,a\}$	$\{0,a,b\}$

Then $I:=\{0,b\}$ is a $(k,m;n)$-multiply $PI(<,\subseteq,\subseteq)_K$-ideal of H, but is not a $(k,m;n)$-multiply $PI(<,<,\subseteq)_K$-ideal of H since $(0\circ a)\circ a^k=\{0,a\}<\{0,b\}$ and $a\circ a^m=\{0,a\}<\{0,b\}$ but $0\circ a^n \not\subseteq \{0,b\}$.

Definition 3.15. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k,m;n)$*-multiply* $PI(<,<,<)_K$*-ideal* of H if it satisfies (I1) and

$$(6) \qquad (\forall x,y,z\in H)\,((x\circ y)\circ z^k < I,\ y\circ z^m < I \Rightarrow x\circ z^n < I).$$

Example 3.16. Let $H=\{0,a,b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0\}$	$\{0\}$
b	$\{b\}$	$\{a,b\}$	$\{0\}$

Then $\{0,b\}$ is a $(k,m;n)$-multiply $PI(<,<,<)_K$-ideal of H.

Theorem 3.17. *Every* $(k,m;n)$*-multiply* $PI(<,<,\subseteq)_K$*-ideal is a* $(k,m;n)$*-multiply* $PI(<,<,<)_K$*-ideal.*

Proof. Let I be a $(k,m;n)$-multiply $PI(<,<,\subseteq)_K$-ideal of H. Let $x,y,z\in H$ be such that $(x\circ y)\circ z^k<I$ and $y\circ z^m<I$. Then $x\circ z^n\subseteq I$ by (5), and so $x\circ z^n<I$ by (a10). Thus I is a $(k,m;n)$-multiply $PI(<,<,<)_K$-ideal of H. □

The converse of Theorem 3.17 is not true in general. For example, in Example 3.5, $I:=\{0,a\}$ $(k,m;n)$-multiply $PI(<,<,<)_K$-ideal of H for every natural number k,m and n. But I is not a $(k,m;n)$-multiply $PI(<,<,\subseteq)_K$-ideal of H since $(b\circ 0)\circ b^k<I$ and $0\circ b^m<I$ but $b\circ b^n\not\subseteq I$.

Definition 3.18. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k,m;n)$*-multiply* $PI(\subseteq,<,\subseteq)_K$*-ideal* of H if it satisfies (I1) and

$$(7) \qquad (\forall x,y,z\in H)\,((x\circ y)\circ z^k \subseteq I,\ y\circ z^m < I \Rightarrow x\circ z^n \subseteq I).$$

Let I be a $(k, m; n)$-multiply $PI(<, <, \subseteq)_K$-ideal of H and let $x, y, z \in H$ be such that $(x \circ y) \circ z^k \subseteq I$ and $y \circ z^m < I$. Then $(x \circ y) \circ z^k < I$ by (a10), and so $x \circ z^n \subseteq I$ by (7). Hence we have the following theorem.

Theorem 3.19. *Every $(k, m; n)$-multiply $PI(<, <, \subseteq)_K$-ideal is a $(k, m; n)$-multiply $PI(<, \subseteq, \subseteq)_K$-ideal.*

The converse of Theorem 3.19 is not true in general. To see this, consider the following example:

Example 3.20. Let $H = \{0, a, b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0, b\}$	$\{0, a, b\}$
a	$\{a, b\}$	$\{0, a\}$	$\{0, b\}$
b	$\{b\}$	$\{a, b\}$	$\{0\}$

Then $I := \{0, a\}$ is a $(k, m; n)$-multiply $PI(\subseteq, <, \subseteq)_K$-ideal of H, but I is not a $(k, m; n)$-multiply $PI(<, <, \subseteq)_K$-ideal of H since $(a \circ 0) \circ 0^k = \{a, b\} < I$ and $0 \circ 0^m = \{0\} < I$ but $a \circ 0^n = \{a, b\} \not\subseteq I$.

Using (a10), we know that every $(k, m; n)$-multiply $PI(\subseteq, <, \subseteq)_K$-ideal is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal, but the converse may not be true as seen in the following example.

Example 3.21. In Example 3.20, $I := \{0, b\}$ is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal of H, but I is not a $(k, m; n)$-multiply $PI(\subseteq, <, \subseteq)_K$-ideal of H since $(a \circ b) \circ 0^k = \{0, b\} \subseteq I$ and $b \circ 0^m = \{b\} \subseteq I$ but $a \circ 0^n = \{a, b\} \not\subseteq I$.

Definition 3.22. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k, m; n)$-*multiply* $PI(\subseteq, \subseteq, <)_K$-*ideal* of H if it satisfies (I1) and

$$(\forall x, y, z \in H)\,((x \circ y) \circ z^k \subseteq I,\, y \circ z^m \subseteq I \;\Rightarrow\; x \circ z^n < I). \tag{8}$$

Example 3.23. Let $H = \{0, a, b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0, b\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, b\}$	$\{a\}$
b	$\{b\}$	$\{0, b\}$	$\{0, b\}$

Then $\{0, b\}$ is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, <)_K$-ideal of H.

Definition 3.24. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k, m; n)$-*multiply* $PI(\subseteq, <, <)_K$-*ideal* of H if it satisfies (I1) and

$$(\forall x, y, z \in H)\,((x \circ y) \circ z^k \subseteq I,\, y \circ z^m < I \;\Rightarrow\; x \circ z^n < I). \tag{9}$$

Example 3.25. In Example 3.23, $\{0, b\}$ is a $(k, m; n)$-multiply $PI(\subseteq, <, <)_K$-ideal of H.

Let I be a $(k, m; n)$-multiply $PI(<, <, <)_K$-ideal of H and let $x, y, z \in H$ be such that $(x \circ y) \circ z^k \subseteq I$ and $y \circ z^m < I$. Then $(x \circ y) \circ z^k < I$ by (a10), and so $x \circ z^n \subseteq I$ by (6). Hence we have the following theorem.

Theorem 3.26. *Every $(k, m; n)$-multiply $PI(<, <, <)_K$-ideal is a $(k, m; n)$-multiply $PI(\subseteq, <, <)_K$-ideal.*

The converse of Theorem 3.26 is not true in general. To see this, consider the following example:

Example 3.27. In Example 3.16, $I := \{0, a\}$ is a $(k, m; n)$-multiply $PI(\subseteq, <, <)_K$-ideal of H, but I is not a $(k, m; n)$-multiply $PI(<, <, <)_K$-ideal of H since $(b \circ a) \circ 0^k = \{a, b\} < I$ and $a \circ 0^m = \{a\} < I$ but $b \circ 0^n = \{b\} \not< I$.

Using (a10), we know that every $(k, m; n)$-multiply $PI(\subseteq, <, <)_K$-ideal is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, <)_K$-ideal.

Here, we guess that the converse may not be true, but we failed to find an appropriate example. So we pose this as a question.

Definition 3.28. Let k, m and n be natural numbers. A nonempty subset I of H is called a $(k, m; n)$-*multiply* $PI(<, \subseteq, <)_K$-*ideal* of H if it satisfies (I1) and

(10) $$(\forall x, y, z \in H)\,((x \circ y) \circ z^k < I,\ y \circ z^m \subseteq I \ \Rightarrow\ x \circ z^n < I).$$

Example 3.29. In Example 3.23, $\{0, a\}$ is a $(k, m; n)$-multiply $PI(<, \subseteq, <)_K$-ideal of H.

Using (a10), we know that every $(k, m; n)$-multiply $PI(<, <, <)_K$-ideal is a $(k, m; n)$-multiply $PI(<, \subseteq, <)_K$-ideal, and that every $(k, m; n)$-multiply $PI(<, \subseteq, <)_K$-ideal is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, <)_K$-ideal, but the converse may not be true as seen in the following example.

Example 3.30. (1) In Example 3.20, $I := \{0, a\}$ is a $(k, m; n)$-multiply $PI(<, \subseteq, <)_K$-ideal of H, but I is not a $(k, m; n)$-multiply $PI(<, <, <)_K$-ideal of H since $(b \circ a) \circ 0^k = \{a, b\} < I$ and $a \circ 0^m = \{a, b\} < I$ but $b \circ 0^n = \{b\} \not< I$.

(2) Let $H = \{0, a, b\}$ be a hyper K-algebra having the following Cayley table:

$\circ$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{a\}$	$\{0, a\}$	$\{0, a\}$
b	$\{b\}$	$\{a, b\}$	$\{0, a, b\}$

Then $I := \{0, a\}$ is a $(k, m; n)$-multiply $PI(\subseteq, \subseteq, <)_K$-ideal of H, but I is not a $(k, m; n)$-multiply $PI(<, \subseteq, <)_K$-ideal of H since $(b \circ a) \circ 0^k = \{a, b\} < I$ and $a \circ 0^m = \{a\} \subseteq I$ but $b \circ 0^n = \{b\} \not< I$.

Consider the special set
$$I_a = \{x \in H | x \circ a^k \subseteq I\}$$
where $a \in H$, k is a natural number and I is a nonempty subset of H. We give condition for the set I_a to be weak hyper K-ideal.

Theorem 3.31. *If I is a $(k, k; k)$-multiply $PI(\subseteq, <, \subseteq)_K$-ideal of H, then the set $I_a, a \in H$ is a weak hyper K-ideal of H.*

Proof. Let $x, y \in H$ be such that $x \circ y \subseteq I_a$ and $y \in I_a$ for $a \in H$. Then $(x \circ y) \circ a^k \subseteq I$ and $y \circ a^k \subseteq I$. Using (a10), $y \circ a^k \subseteq I$ implies $y \circ a^k < I$. It follows from (7) that $x \circ a^k \subseteq I$ so that $x \in I_a$. □

Using Theorem 3.31 and (a10), we have the following results.

Corollary 3.32. *If I is a $(k, k; k)$-multiply $PI(\subseteq, \subseteq, \subseteq)_K$-ideal of H, then the set $I_a, a \in H$ is a weak hyper K-ideal of H.*

Corollary 3.33. *If I is a $(k, k; k)$-multiply $PI(<, <, \subseteq)_K$-ideal of H, then the set $I_a, a \in H$ is a weak hyper K-ideal of H.*

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