RALPH HENSTOCK : RESEARCH SUMMARY

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Ralph Henstock is the author of 41 mathematics papers published between 1946 and 1998.

He was the author of four books on analysis:
"Theory of integration" (Butterworths, 1963);
"Linear analysis" (Butterworths, 1967);
"Lectures on the theory of integration" (World Scientific, 1988):
and the ponderous
"The general theory of integration"(Oxford University Press, 1991).

He also wrote 171 reviews for Math Reviews. In 1994 he was awarded the "Andy" prize at the XVIII Summer Symposium in Real Analysis.

Most of his work was concerned with integration. From initial studies of the Ward integral he formulated an integration process whereby the region of integration is partitioned and one forms suitable Riemann sums that approximate the integral. His methods led to an integral on the real line that was very similar in construction and simplicity to the Riemann integral but included the Lebesgue integral and allowed non-absolute convergence. These ideas were developed from the late 1950's. Independently, Jaroslav Kurzweil developed a similar Riemann-type integral on the real line. The resulting integral is now known as the Henstock-Kurzweil integral but has also been called the gauge, generalised Riemann and Riemann complete integral. On the real line, it is equivalent to the Denjoy-Perron integral, but has a much simpler definition and is generally much easier to work with.

Henstock took the integral well beyond the setting of the real line and developed very general integration processes in spaces that have a subdivision property (division spaces). A metric or topological structure is generally sufficient to define the Henstock-Kurzweil integral, which can also be defined in infinite dimensional spaces. Whereas Lebesgue integrals can be defined in measure spaces, which need have no additional structure, some structure is needed for Henstock's theory but the resulting integrals always include the Lebesgue integral as a special case and allow non-absolute convergence. As well, functions which take their values in a Banach space can be integrated with this theory.

The simplicity of the Henstock-Kurzweil integral has revivified the theory of integration and the theory now has many adherents. Numerous monographs and texts have appeared since 1980 and there have been several conferences devoted to the theory.

A personal note. I met Ralph Henstock in December 2000 when I gave a talk at the University of Ulster. At that time Henstock was somewhat frail but still mathematically active. He did not visit the campus all that often in those days but was clearly a respected figure. Our walk across campus involved many stops for friendly chats as different people

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spied us. My talk put Henstock to sleep three times but it also woke him up three times. I suppose that is what he meant when he described the talk as "most stimulating".

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