A 0-1 KNAPSACK PROBLEM CONSIDERING RANDOMNESS OF FUTURE RETURNS AND FLEXIBLE GOALS OF AVAILABLE BUDGET AND TOTAL RETURN

Takashi Hasuike and Hiroaki Ishii

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Abstract.

This paper proposes a 0-1 knapsack problem considering both the maximization of the total return including randomness and minimization of available budget, simultaneously. For the flexibility of setting each goal, the satisfaction function is introduced and the model maximizing minimum aspiration levels is proposed. Since this problem is not a well-defined problem due to including randomness and fuzziness, chance constraints are introduced, and a main problem is transformed into a nonlinear 0-1 programming problem. In previous researches, the solution method using a parametric dynamic programming approach is constructed. However, this solution method is not efficient due to using dynamic programming repeatedly. Therefore, in this paper, the efficient solution method is constructed. This means that the number of using dynamic programming is as small as possible.

1 Introduction

0-1 knapsack problem is one of basic problems in mathematical programming problems and applied to many practical problems such as project selection problems, capital budgeting and resource allocation which many companies and industrial firms are faced upon. Therefore, the 0-1 knapsack problem plays an important role in the real world. In recent investment fields, through the remarkable development of information technology and computers, many people can receive and transmit various types of information all over the world, and so the role of budget allocation problems becomes more and more important.

Therefore, many researches with respect to knapsack problems have been conducted until now (as a review, [7, 13]). A basic knapsack problem is solved using the strict solution method such as dynamic programming and branch-and-bound method, or approximate method such as greedy algorithm, genetic algorithm and many other heuristics. Recently, most of them have been mainly studied as the proposition of its efficient solution method (recent studies [2, 12]), the application to actual problem in the real world (for example, traveling salesman problem [3], bin-packing [4, 14], quadratic assignment problem [2, 5]) and the extension for multidimensional model (recent studies [1, 6, 15, 16]) or more complicate model to apply to various situations (for example, [8, 11]).

In most of previous studies, coefficients in the problem are assumed to be known and fixed values. However, in real world, it is hard that coefficients in the problem are considered as the fixed values because of randomness such as the prediction derived from historical data and fuzziness such as intuition of a decision maker. Particularly, in the cases that a decision maker must do a decision making without having all data or information, and that there are various types of efficient or inefficient information, the uncertain factors are more and more increasing. Consequently, with respect to knapsack problems, in order to consider problems

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in real world more widely and flexibly, we need to construct the model of knapsack problem considering such randomness and fuzziness, simultaneously. Therefore, in this paper, we propose a model considering random future returns, fuzzy coefficients of the constraint and flexibility of objective value and maximum value of constraint.

On the other hand, with respect to mathematical programming problems, the knapsack problem considering randomness and fuzziness is more complicate than the previous problems due to including both random and fuzzy numbers. Then, since this problem is not a well-defined knapsack problem, it is hard to solve it directly. Therefore, we need to construct its efficient solution method. In this paper, we transform main problems into deterministic equivalent integer programming problems using chance constraints, possibility measure and fuzzy goals based on both stochastic and fuzzy programming approaches.

Furthermore, through the development of information technology and improvement of computers, we solve the knapsack problem more quickly using not only approximate solution methods but also strict solution methods even if it is a little bit large scale problem. Therefore, we propose the efficient strict solution method based on dynamic programming. In previous researches considering only random variables, the solution method using a parametric dynamic programming approach is constructed. However, this is not efficient due to using dynamic programming repeatedly. In this paper, we construct the solution method to decrease the number of using dynamic programming as small as possible.

This paper is organized as follows. In Section 2, we formulate a basic 0-1 knapsack problem and introduce its probability fractile optimization model due to including random variables. Furthermore, considering ambiguous situations, we introduce the fuzzy numbers and fuzzy goals to ambiguous parameters and flexible target values. This problem is a nonlinear 0-1 knapsack problem and so it is hard to solve it directly. Therefore, in Section 3, by the deterministic equivalent transformation of the problem and introducing the 0-1 relaxation problem, we construct the efficient solution method. Finally, in Section 4, we conclude this paper and discuss the future studies.

2 Formulation of project selection problem under the random and ambiguous situation

First of all, each notation in this paper means as follows:

n**:** Total number of decision variables

r_j**:** Future return of decision variable j

c_j**:** Capital budgeting of decision variable j

f**:** Goal of total future returns

b**:** Upper limited value of total capital budgeting

x_j Decision variable satisfying $x_j = \begin{cases} 1 & \text{select decision variable } j \\ 0 & \text{not select decision variable } j \end{cases}$

A basic 0-1 knapsack problem maximizing the total profit is generally formulated as follows:

$$
\text{(1)} \qquad \begin{array}{ll} \text{Maximize} & \sum_{j=1}^{n} r_j x_j \\ \text{subject to} & \sum_{j=1}^{n} c_j x_j \leq b, \\ & x_j \in \{0,1\}, \; j = 1, 2, \cdots, n \end{array}
$$

With respect to this problem, we obtain a strict optimal solution using dynamic programming method or branch-bound method. However, in the case that we assume each return r_j as a random variable, problem (1) is not a well-defined problem since the objective function also becomes a random variable. Therefore, in this paper, introducing a chance constraint with respect to the objective function, we consider a probability fractile optimization model for the total profit.

2.1 Formulation of probability fractile optimization model

We apply probability fractile optimization model to problem (1). This problem is formulated as the following form using the chance constraint and its probability level β:

$$
\begin{aligned}
&\text{Maximize} \quad f \\
&\text{subject to} \quad \Pr\left\{\sum_{j=1}^{n} r_j x_j \geq f\right\} \geq \beta, \\
&\qquad \sum_{j=1}^{n} c_j x_j \leq b,\ x_j \in \{0,1\},\ j = 1,2,\cdots,n
\end{aligned} \tag{2}
$$

In this problem, we assume each future return r_j occurs according to a normal distribution $N\left(\bar{r}_j, \sigma_j^2\right)$ where $\bar{r}_j$ is the mean value of r_j and σ_j^2 is its variance. In this paper, since each coefficient of the objective function is assumed to be independent of other variables, i.e.

$$
\sigma_{ij} = \left\{ \begin{array}{ll} \sigma_j^2, & i = j \\ 0, & i \neq j \end{array} \right. , \quad i,j = 1,2,\cdots,n \tag{3}
$$

Under these assumptions, its stochastic constraint is transformed into the following inequality:

$$
\begin{aligned}
&\Pr\left\{\sum_{j=1}^{n} r_j x_j \geq f\right\} \geq \beta \Leftrightarrow \Pr\left\{ \frac{\sum_{j=1}^{n} r_j x_j - \sum_{j=1}^{n} \bar{r}_j x_j}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j}} \geq \frac{f - \sum_{j=1}^{n} \bar{r}_j x_j}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j}} \right\} \geq \beta \\
&\Leftrightarrow \frac{\sum_{j=1}^{n} \bar{r}_j x_j - f}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j}} \geq K_\beta \Leftrightarrow \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j} \geq f
\end{aligned} \tag{4}
$$

where $F(y)$ is the distribution function of the standard normal distribution and $K_\beta = F^{-1}(\beta)$. Therefore, problem (2) is transformed into the following problem:

$$
\begin{aligned}
&\text{Maximize} \quad f \\
&\text{subject to} \quad \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j^2} \geq f, \\
&\qquad \sum_{j=1}^{n} c_j x_j \leq b,\ x_j \in \{0,1\},\ j = 1,2,\cdots,n
\end{aligned} \tag{5}
$$

Since each decision variable x_j satisfies $x_j \in \{0,1\}$, we obtain $x_j^2 = x_j$, and so problem (5) is equivalently transformed into the following problem:

$$\text{(6)} \quad \begin{array}{ll} \text{Maximize} & f \\ \text{subject to} & \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j} \geq f, \\ & \sum_{j=1}^{n} c_j x_j \leq b,\ x_j \in \{0,1\},\ j = 1,2,\cdots,n \end{array}$$

2.2 Introduction of fuzzy numbers and fuzzy goals

In this subsection, we consider the upper limited value of capital budgeting constraint. Since there is a lack of information, we assume each coefficient of the constraint to be the following L-fuzzy number:

$$\text{(7)} \quad \mu_{\tilde{c}_j}(\omega) = L\left(\frac{\omega - \bar{c}_j}{d_j}\right), \quad j = 1,2,\ldots,n$$

where $L(x)$ is a continuous nonincreasing nonnegative function satisfying $L(0) = 1, L(1) = 0$. Therefore, $\sum_{j=1}^{n} \tilde{c}_j x_j$ is also a fuzzy variable, and so the constraint of problem (6) is not a well-defined constraint. Hence, for the transformation into the deterministic equivalent constraint, we introduce the following chance constraint. The membership function with respect to $\sum_{j=1}^{n} \tilde{c}_j x_j$ is as follows:

$$\text{(8)} \quad \mu_Y(y) = L\left(\frac{y - \sum_{j=1}^{n} \bar{c}_j x_j}{\sum_{j=1}^{n} d_j x_j}\right)$$

Furthermore, we assume that the upper limited value b of total capital budgeting includes flexibility. Generally speaking, it is possible to increase maximum capital budget b a little in order to increase the goal of total future profits. On the other hand, if b is assumed to be too large value, the aspiration level of decision maker has to be too small. Considering these situations, we introduce a fuzzy goal with respect to b as the following membership function:

$$\text{(9)} \quad \mu_G(\omega) = \begin{cases} 1, & \omega \leq b_1 \\ g(\omega), & b_1 \leq \omega \leq b_0 \\ 0, & b_0 \leq \omega \end{cases}$$

where $g(\omega)$ is a monotonically decreasing function. Then, we consider the following possibility measure:

$$\text{(10)} \quad \Pi_Y(G) = \sup_y \min\{\mu_Y(y), \mu_G(y)\}$$

In a way similar to b , we also introduce a fuzzy goal with respect to the total profit as the following membership function:

$$\mu_F(z) = \begin{cases} 1, & f_1 \le z \\ f(z), & f_0 \le z \le f_1 \\ 0, & z \le f_0 \end{cases}, \quad \left(z = \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j} \right) \tag{11}$$

where $f(z)$ is a monotonically increasing function. Using these possibility measure and fuzzy goal, we propose the following maximization model of minimum aspiration levels as the reformulation of problem (6):

$$\begin{array}{ll} \text{Maximize} & \min\{\mu_F(z), \Pi_Y(G)\} \\ \text{subject to} & x_j \in \{0,1\},\ j = 1, 2, \cdots, n \end{array} \tag{12}$$

Then, by introducing a marameter h, this problem is transformed into the following problem:

$$\begin{array}{ll} \text{Maximize} & h \\ \text{subject to} & \mu_F(z) \ge h,\ \Pi_Y(G) \ge h, \\ & x_j \in \{0,1\},\ j = 1, 2, \cdots, n \end{array} \tag{13}$$

In this problem, constraint $\Pi_Y(G) \ge h$ is equivalently transformed into the following form:

$$\begin{array}{ll} \Pi_Y(G) \ge h & \\ \Leftrightarrow & \sup_y \min\{\mu_Y(y), \mu_G(y)\} \ge h \\ \Leftrightarrow & \mu_Y(y) \ge h,\ \mu_G(y) \ge h \\ \Leftrightarrow & \sum_{j=1}^{n} \bar{c}_j x_j - L^*(h) \sum_{j=1}^{n} d_j x_j \le y,\ y \le g_b^{-1}(h) \\ \Leftrightarrow & \sum_{j=1}^{n} \bar{c}_j x_j - L^*(h) \sum_{j=1}^{n} d_j x_j \le g_b^{-1}(h) \end{array} \tag{14}$$

Then, in a way similar to transformation (14), constraint $\mu_F(z) \ge h$ is transformed into the following form:

$$\mu_F(z) \ge h \quad \Leftrightarrow \quad \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j} \ge f^{-1}(h) \tag{15}$$

Therefore, we equivalently transform problem (13) into the following problem:

$$\begin{array}{ll} \text{Maximize} & h \\ \text{subject to} & \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j} \ge f^{-1}(h), \\ & \sum_{j=1}^{n} \bar{c}_j x_j - L^*(h) \sum_{j=1}^{n} d_j x_j \le g^{-1}(h), \\ & x_j \in \{0,1\},\ j = 1, 2, \cdots, n \end{array} \tag{16}$$

3 The efficient solution method of proposed 0-1 knapsack problem

Main problem (16) in Subsection 2.2 is a nonlinear 0-1 knapsack problem, and so it is hard to solve it directly using the standard solution method to solve discrete mathematical programming problems. However, in the case that parameter h is fixed, constraint $\sum_{j=1}^{n} \bar{c}_j x_j - L^*(h) \sum_{j=1}^{n} d_j x_j \leq g^{-1}(h)$ is equivalent to a linear constraint with respect to x. Furthermore, to solve problem (16) efficiently, we introduce the following subproblem:

$$
\begin{array}{ll}
\text{Maximize} & \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j} \\
\text{subject to} & \sum_{j=1}^{n} \bar{c}_j x_j - L^*(\bar{h}) \sum_{j=1}^{n} d_j x_j \leq g^{-1}(\bar{h}), \\
& x_j \in \{0,1\},\ j = 1, 2, \cdots, n
\end{array}
\tag{17}
$$

where $\bar{h}$ is a fixed value of parameter h satisfying $0 \leq \bar{h} \leq 1$. With respect to the relation between subproblem (17) and problem (16), the following theorem holds based on the result of previous research [11].

Theorem 1

Let an optimal solution of subproblem (17) be $x_{\bar{h}}$ and its optimal value $Z(x_{\bar{h}})$. Furthermore, let the optimal value of problem (16) be h^*. Then the following relation holds:

$$
\begin{cases}
Z(x_{\bar{h}}) > f^{-1}(\bar{h}) & \Leftrightarrow \quad \bar{h} < h^* \\
Z(x_{\bar{h}}) = f^{-1}(\bar{h}) & \Leftrightarrow \quad \bar{h} = h^* \\
Z(x_{\bar{h}}) < f^{-1}(\bar{h}) & \Leftrightarrow \quad \bar{h} > h^*
\end{cases}
\tag{18}
$$

¿From this theorem, we obtain that the optimal solution $x_{\bar{h}}$ of subproblem (17) is equivalent to that of problem (16) in the case that $Z(x_{\bar{h}}) = f^{-1}(\bar{h})$. Furthermore, we consider the following auxiliary problem to subproblem (17) introducing a parameter R :

$$
\begin{array}{ll}
\text{Maximize} & R \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \left(\sum_{j=1}^{n} \sigma_j^2 x_j \right) \\
\text{subject to} & \sum_{j=1}^{n} \bar{c}_j x_j - L^*(\bar{h}) \sum_{j=1}^{n} d_j x_j \leq g^{-1}(\bar{h}), \\
& x_j \in \{0,1\},\ j = 1, 2, \cdots, n
\end{array}
\tag{19}
$$

With respect to the relation between problems (17) and (19), the following theorem holds based on previous research [10].

Theorem 2 ([10])

Let an optimal solution of problem (19) be x^*. If $R = 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j^*}$ is satisfied, x^* is also an optimal solution of problem (17).

Problem (19) is a parametric 0-1 knapsack problem. In previous researches, a solution method based on the parametric dynamic programming approach has been proposed. However, in this solution method, a dynamic programming is repeatedly used. Therefore, this solution method is not efficient. We introduce the following 0-1 relaxation problems with respect to problem (19);

$$\begin{array}{lll} & \text{Maximize} & \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \sqrt{\sum_{j=1}^{n} \sigma_j^2 (x_j)^2} \\ (20) & \text{subject to} & \sum_{j=1}^{n} \bar{c}_j x_j - L^* \left(\bar{h}\right) \sum_{j=1}^{n} d_j x_j \leq g^{-1}\left(\bar{h}\right), \\ & & 0 \leq x_j \leq 1,\ j = 1, 2, \cdots, n \end{array}$$

and its auxiliary problem:

$$\begin{array}{lll} & \text{Maximize} & R \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \left(\sum_{j=1}^{n} \sigma_j^2 (x_j)^2 \right) \\ (21) & \text{subject to} & \sum_{j=1}^{n} \bar{c}_j x_j - L^* \left(\bar{h}\right) \sum_{j=1}^{n} d_j x_j \leq g^{-1}\left(\bar{h}\right), \\ & & 0 \leq x_j \leq 1,\ j = 1, 2, \cdots, n \end{array}$$

Then, the following theorem holds with respect to the relation between problems (20) and (21) based on previous research [9].

Theorem 3 ([9])

For $g(R) = R - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 (x_j^*)^2}$, the following relation holds:

$$\begin{array}{llcl} (22) & R^* > R & \Leftrightarrow & g(R) > 0 \\ & R^* = R & \Leftrightarrow & g(R) = 0 \\ & R^* < R & \Leftrightarrow & g(R) < 0 \end{array}$$

¿From this theorem, the optimal solution of problem (21) becomes equal to that of problem (20). In previous studies, parameter R is repeatedly modified using bisection algorithm in order to solve problem (19) using dynamic programming. However, this solution method is not efficient. Therefore, we propose a new solution method introducing the 0-1 relaxation problem and its optimal solution. In order to construct the efficient solution method, the following lemmas are derived.

Lemma 1

With respect to problem (19), there exists the ranges $[R_k, R_{k+1}]$,$(k = 1, 2, ..., K)$ that the optimal solution of problem (19) is unique in the case of R including in $[R_k, R_{k+1}]$.

Proof

¿From the discreteness of decision variable, it is obvious that this theorem holds.

Lemma 2

We set a range $[R_L, R_U]$ satisfying $R^* \in [R_L, R_U]$. Let the optimal solution of problem (19) be $\bar{x}$. Then, $\left(R_L - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)\left(R_U - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right) \leq 0$ holds.

Proof

Let $R^* = 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 x_j^*}$ and $\bar{R} = 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}$. In the case that $\bar{R} < R^*$, with respect to $\left(R_U - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)$,

$$\text{(23)} \qquad \left(R_U - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right) = \left(R_U - 2\bar{R}\right) > (R_U - 2R^*) > 0$$

holds. Furthermore, from the monotonous function $R - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}$ with repect to R and $R^* - \bar{R} > 0$, it is obvious that $\left(R_L - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right) < 0$. Therefore,

$$\text{(24)} \qquad \left(R_L - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)\left(R_U - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right) \le 0$$

holds. In the case that $\bar{R} > R^*$, with respect $\left(R_L - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)\left(R_U - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)$,

$$\text{(25)} \qquad \begin{aligned} \left(R_L - \sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)\left(R_U - \sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right) &= \left(R_L - \bar{R}\right)\left(R_U - \bar{R}\right) \\ &\le (R_L - R^*)(R_U - R^*) \end{aligned}$$

holds. Then,since we obtain $(R_L - R^*) < 0$ and $(R_U - R^*) > 0$, $(R_L - R^*)(R_U - R^*) < 0$ and $\left(R_L - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right)\left(R_U - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j}\right) \le 0$ hold. Consequently, this theorem holds.

Lemma 3

In the case that $T(R) = R - 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 \bar{x}_j} = 0$, $\bar{x}_j$ is an optimal solution of main problem (19).

Proof

¿From Theorem 2, it is obvious that this lemma holds.

Using these lemmas, the following holds.

Theorem 4

Let the optimal solution of problem (21) be x^* and $R^* = 2\sqrt{\sum_{j=1}^{n} \sigma_j^2 (x_j^*)^2}$. Then, the optimal solution of the following problem;

$$(26)\quad \begin{array}{ll} \text{Maximize} & R^* \sum_{j=1}^{n} \bar{r}_j x_j - K_\beta \left(\sum_{j=1}^{n} \sigma_j^2 x_j \right) \\ \text{subject to} & \sum_{j=1}^{n} \bar{c}_j x_j - L^* \left(\bar{h} \right) \sum_{j=1}^{n} d_j x_j \leq g^{-1} \left(\bar{h} \right), \\ & x_j \in \{0,1\},\ j = 1, 2, \cdots, n \end{array}$$

is also optimal for problem (19).

Consequently, in the case that we fix the parameter h of main problem (16), introducing 0-1 relaxation problem and finding its optimal solution, we obtain an optimal solution without using dynamic programming repeatedly. Therefore, this solution method is more efficient than previous parametric dynamic programming approaches in that the number of using dynamic programming is significantly decreasing. Then, we construct the following efficient solution method to solve main 0-1 nonlinear knapsack problem (16).

Solution Method

STEP1: Elicit the membership function of a fuzzy goal with respect to the total return and the maximum budget.

STEP2: Set $h \leftarrow 1$ and solve problem (17). If the optimal objective value $Z(h)$ of problem (17) satisfies $Z(h) \geq f^{-1}(h)$ and its optimal solution are included in constraints, then terminate. In this case, the obtained current solution is an optimal solution of main problem.Otherwise go to STEP3.

STEP3: Set $h \leftarrow 0$ and solve problem (17). If the optimal objective value $Z(h)$ of problem (17) satisfies $Z(h) < f^{-1}(h)$ or the feasible solution including in constraints does not exist, then terminate. In this case, there is no feasible solution and it is necessary to reset a fuzzy goal for the aspiration level f or maximum budget d. Otherwise go to STEP4.

STEP4: Set $U_h \leftarrow 1$ and $L_h \leftarrow 0$.

STEP5: Set $h \leftarrow \frac{U_h + L_h}{2}$.

STEP6: Solve problem (17) and calculate the optimal objective value $Z(h)$ of problem (17). If $Z(h) > f^{-1}(h)$, then set $L_h \leftarrow h$ and return to Step 5. If $Z(h) \leq f^{-1}(h)$, then set $U_h \leftarrow h$ and return to Step 5. If $Z(h) = f^{-1}(h)$, then terminate the algorithm. In this case, $x^*(h)$ is equal to a global optimal solution of main problem.

4 Conclusion

In this paper, we have proposed a new model of 0-1 knapsack problem considering randomness of future returns and flexible goals of available budget and total return. Since our proposed model has been a nonlinear 0-1 knapsack problem by introducing the chance constraint and doing the transformation into the deterministic equivalent problems, we have constructed the efficient solution method. We have dealt with the 0-1 relaxation problem and its optimal value and found that the number of using dynamic programming in our proposed method is much less than that of previous parametric dynamic programming. This solution method is applicable to the general integer programming problems, particularly portfolio selection problem. As the future studies, we consider the multidimensional random 0-1 knapsack problem and construct its efficient solution method using not only dynamic programming approach but also approximation methods such as genetic algorithm and heuristic approaches.

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Corresponding Author:
Takashi Hasuike
Department of Information Physical and Science
Graduate School of Information Science and Technology, Osaka University
2-1 Yamadaoka, Suita, Osaka 565-0871, JAPAN
Tel.+Fax: +81-(0)6-6879-7868
E-mail: thasuike@ist.osaka-u.ac.jp