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ON (COMPLETE) NORMALITY OF FUZZY d-IDEALS IN d-ALGEBRAS

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Abstract. In this paper, we introduce the notion of normal fuzzy d-ideals, maximal fuzzy d-ideals and completely normal fuzzy d-ideals in d-transitive d-algebras. We investigate some interesting properties of normal (resp. maximal, completely normal) d-ideals. We show that every non-constant normal fuzzy d-ideal which is a maximal element of $(\mathcal{N}(X), \subseteq)$, where X is a d-transitive d-algebra, takes only the values 0 and 1, and every maximal fuzzy d-ideal in a d-transitive d-algebra is completely normal. Moreover, we state a fuzzy characteristic d-ideal in d-algebras.

1. Introduction

Y. Imai and K. Iséki introduced two classes of abstract algebras : BCK-algebras and BCI-algebras ([5,6]). It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [2,3] Q. P. Hu and X. Li introduced a wide class of abstract algebras : BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. J. Neggers and H. S. Kim ([11]) introduced the notion of d-algebras which is another useful generalization of BCK-algebras, and investigated several relations between d-algebras and BCK-algebras, and then investigated other relations between d-algebras and oriented digraphs. J. Neggers, Y. B. Jun and H. S. Kim ([10]) discussed ideal theory in d-algebras, and introduced the notions of d-subalgebra, d-ideal, $d^{\#}$-ideal and d^*-ideal, and investigated some relations among them. Y. C. Lee and H. S. Kim ([8]) introduced the notion of d-transitive d^*-algebra which is a generalization of BCK-algebras.

In this paper, we introduce the notion of normal fuzzy d-ideals, maximal fuzzy d-ideals and completely normal fuzzy d-ideals in d-transitive d-algebras. We investigate some interesting properties of normal (resp., maximal, completely normal) d-ideals. We show that every non-constant normal fuzzy d-ideal which is a maximal element of $(\mathcal{N}(X), \subseteq)$, where X is a d-transitive d-algebra, takes only the values 0 and 1, and every maximal fuzzy d-ideal in a d-transitive d-algebra is completely normal. Moreover, we state a fuzzy characteristic d-ideal in d-algebras.

2. Preliminaries

A *d-algebra* ([11]) is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying axioms :

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(I) $x * x = 0$,

(II) $0 * x = x$,

(III) $x * y = 0$ and $y * x = 0$ imply $x = y$ for all $x, y \in X$.

A d-algebra $(X; *, 0)$ is said to be d-transitive if $x * y = 0$ and $y * z = 0$ imply $x * z = 0$.

A BCK-algebra is a d-algebra $(X; *, 0)$ satisfying additional axioms:

(IV) $((x * y) * (x * z)) * (z * y) = 0$,

(V) $(x * (x * y)) * y = 0$, for all $x, y, z \in X$.

For brevity we also call X a *d-algebra*. In X we can define a binary relation " $\leq$ " by $x \leq y$ if and only if $x * y = 0$.

Now we review some fuzzy logic concepts. A *fuzzy set* in a set X is a function $\mu : X \to [0, 1]$. By X_μ we denote the set $\{x \in X | \mu(x) = \mu(0)\}$. For any fuzzy sets μ and ν in a set X, we define

$$\mu \subseteq \nu \quad \Leftrightarrow \quad \mu(x) \leq \nu(x) \quad \text{for} \quad \text{all} \quad x \in X.$$

Definition 2.1. ([10]) Let $(X; *, 0)$ be a d-algebra and $\emptyset \neq I \subseteq X$. I is a d-subalgebra of X if $x * y \in I$ whenever $x \in I$ and $y \in I$. I is called a BCK-ideal of X if it satisfies:

(D_0) $0 \in I$,

(D_1) $x * y \in I$ and $y \in I$ imply $x \in I$.

I is a called a d-ideal of X if it satisfies (D_1) and

(D_2) $x \in I$ and $y \in X$ imply $x * y \in I$, i.e., $I * X \subseteq I$.

3. Normality of fuzzy d-ideals

In this section, X will denote a d-algebra, unless otherwise specified.

Definition 3.1. ([7]) Let μ be a fuzzy set in a d-algebra X. Then μ is called a *fuzzy d-subalgebra* of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, $\forall x, y \in X$. μ is called a *fuzzy BCK-ideal* of X if

(F_0) $\mu(0) \geq \mu(x)$,

(F_1) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$, for all $x, y \in X$.

μ is called a *fuzzy d-ideal* of X if it satisfies (F_1) and

(F_2) $\mu(x * y) \geq \mu(x)$ for all $x, y \in X$.

Lemma 3.2. ([7]) *If μ is a fuzzy d-ideal of a d-algebra X, then $\mu(0) \geq \mu(x)$ for all $x \in X$.*

Let μ be a fuzzy d-ideal of X and let $t \in Im(\mu)$. The level subset $\mu_t := \{x \in X | \mu(x) \geq t\}$ is a fuzzy d-ideal of X ([7]).

Example 3.3. Let $X = \{0, a, b, c, d, e\}$ be a set with the following Cayley table:

$*$	0	a	b	c	d	e
0	0	0	0	0	0	0
a	a	0	0	0	0	a
b	b	b	0	0	a	a
c	c	b	a	0	a	a
d	d	d	d	d	0	a
e	e	e	e	e	e	0

Then X is a d-algebra. Define a fuzzy set $\mu : X \to [0,1]$ by $\mu(e) = 0.3$ and $\mu(x) = 0.5$ for all $x \neq e$. Then μ is a fuzzy d-ideal of X.

Proposition 3.4. *Let A be a non-empty subset of X and let μ_A be defined by*

$$\mu_A := \begin{cases} s & \text{if} \quad x \in A \\ t & \text{otherwise,} \end{cases}$$

for all $x \in X$ and some $s, t \in [0,1]$ with $s > t$. Then μ_A is a fuzzy d-ideal of X if and only if A is a d-ideal of X. Moreover,

$$X_{\mu_A} := \{x \in X | \mu_A(x) = \mu_A(0)\} = A.$$

Proof. Straightforward. □

Definition 3.5. A fuzzy d-ideal μ of X is said to be *normal* if there exists $x \in X$ such that $\mu(x) = 1$.

Example 3.6. Let X be the d-algebra of Example 3.3. Then a fuzzy set μ in X defined by $\mu(e) = t < 1$ and $\mu(x) = 1$ for all $x \neq e$ is a normal fuzzy d-ideal of X.

We note that if μ is a normal fuzzy d-ideal of X, then clearly $\mu(0) = 1$, and hence μ is normal if and only if $\mu(0) = 1$.

Proposition 3.7. *Given a fuzzy d-ideal μ of X let μ^+ be the fuzzy set in X defined by $\mu^+(x) = \mu(x) + 1 - \mu(0)$ for all $x \in X$. Then μ^+ is a normal fuzzy d-ideal of X which contains μ.*

Proof. Let $x, y \in X$. Then we have

$$\begin{aligned} \min\{\mu^+(x*y), \mu^+(y)\} &= \min\{\mu(x*y) + 1 - \mu(0), \mu(y) + 1 - \mu(0)\} \\ &= \min\{\mu(x*y), \mu(y)\} + 1 - \mu(0) \\ &\leq \mu(x) + 1 - \mu(0) = \mu^+(x) \end{aligned}$$

and

$$\mu^+(x*y) = \mu(x*y) + 1 - \mu(0) \geq \mu(x) + 1 - \mu(0) = \mu^+(x).$$

Also we obtain $\mu^+(0) = \mu(0) + 1 - \mu(0) = 1$. This shows that μ^+ is a normal fuzzy d-ideal of X. Clearly $\mu \subseteq \mu^+$, completing the proof. □

Corollary 3.8. *Let μ and μ^+ be as in Proposition 3.7. If there is $x \in X$ such that $\mu^+(x) = 0$, then $\mu(x) = 0$.*

Proof. Since $\mu \subseteq \mu^+$, it is straightforward. □

Using Proposition 3.4, we know that for any d-ideal A of X the characteristic function χ_A of A is a normal fuzzy d-ideal of X. It is clear that μ is normal if and only if $\mu^+ = \mu$.

Proposition 3.9. *If μ is a fuzzy d-ideal of X, then $(\mu^+)^+ = \mu^+$. Moreover if μ is normal, then $(\mu^+)^+ = \mu$.*

Proof. Straightforward. □

Proposition 3.10. *If μ and ν are fuzzy d-ideals of X such that $\mu \subseteq \nu$ and $\mu(0) = \nu(0)$, then $X_\mu \subseteq X_\nu$.*

Proof. Let $x \in X_\mu$. Then $\nu(x) \geq \mu(x) = \mu(0) = \nu(0)$ and so $\nu(x) \geq \nu(0)$. By Lemma 3.2, we have $\nu(0) \geq \nu(x)$. Hence $\nu(x) = \nu(0)$, i.e., $x \in X_\nu$, proving $X_\mu \subseteq X_\nu$. □

Corollary 3.11. *If μ and ν are normal fuzzy d-ideals of X such that $\mu \subseteq \nu$, then $X_\mu \subseteq X_\nu$.*

Proposition 3.12. *Let μ be a fuzzy d-ideal of X. If there exists a fuzzy d-ideal ν of X such that $\nu^+ \subseteq \mu$, then μ is normal.*

Proof. Assume that there exists a fuzzy d-ideal ν of X such that $\nu^+ \subseteq \mu$. Then $1 = \nu^+(0) \leq \mu(0)$, and so $\mu(0) = 1$ and we are done. □

Proposition 3.13. *Let μ be a fuzzy d-ideal of X and let $f : [0, \mu(0)] \to [0, 1]$ be an increasing function. Then a fuzzy set $\mu_f : X \to [0, 1]$ defined by $\mu_f := f(\mu(x))$ for all $x \in X$ is a fuzzy d-ideal of X. In particular, if $f(\mu(0)) = 1$ then μ_f is normal; and if $f(t) \geq t$ for all $t \in [0, \mu(0)]$, then μ is contained in μ_f.*

Proof. For any $x, y \in X$, we have

$$\begin{aligned}\min\{\mu_f(x * y), \mu_f(y)\} &= \min\{f(\mu(x * y)), f(\mu(y))\} \\ &= f(\min\{\mu(x * y), \mu(y)\}) \\ &\leq f(\mu(x)) = \mu_f(x)\end{aligned}$$

and

$$\mu_f(x * y) = f(\mu(x * y)) \geq f(\mu(x)) = \mu_f(x).$$

Hence μ_f is a fuzzy d-ideal of X. If $f(\mu(0)) = 1$, then clearly μ_f is normal. Assume that $f(t) \geq t$ for all $t \in [0, \mu(0)]$. Then $\mu_f(x) = f(\mu(x)) \geq \mu(x)$ for all $x \in X$, which proves that μ is contained in μ_f. □

Denote by $\mathcal{N}(X)$ the set of all normal fuzzy d-ideals of a d-transitive d-algebra X. Note that $\mathcal{N}(X)$ is a poset under set inclusion.

Theorem 3.14. *Let $\mu \in \mathcal{N}(X)$ be a non-constant function such that it is a maximal element of $(\mathcal{N}(X), \subseteq)$. Then μ takes only the values 0 and 1.*

Proof. Note that $\mu(0) = 1$ since μ is normal. Let $x \in X$ be such that $\mu(x) \neq 1$. We claim that $\mu(x) = 0$. If not, then there exists $a \in X$ such that $0 < \mu(a) < 1$. Let ν be a fuzzy set in X defined by $\nu(x) := \frac{1}{2}(\mu(x) + \mu(a))$ for all $x \in X$. Then clearly ν is well-defined. For all $x, y \in X$, we have

$$\nu(x * y) = \frac{1}{2}(\mu(x * y) + \mu(a)) \geq \frac{1}{2}(\mu(x) + \mu(a)) = \nu(x)$$

and

$$\begin{aligned}\nu(x) = \frac{1}{2}(\mu(x) + \mu(a)) \geq & \frac{1}{2}[\min\{\mu(x * y), \mu(y)\} + \mu(a)] \\ = & \min\{\frac{1}{2}(\mu(x * y) + \mu(a)), \frac{1}{2}(\mu(y) + \mu(a))\} \\ = & \min\{\nu(x * y), \nu(y)\}.\end{aligned}$$

Hence ν is a fuzzy d-ideal of X. It follows from Proposition 3.7 that $\nu^+ \in \mathcal{N}(X)$ where ν^+ is defined by $\nu^+(x) = \nu(x) + 1 - \nu(0)$ for all $x \in X$. Clearly $\nu^+(x) \geq \mu(x)$ for all $x \in X$. Note that

$$\begin{aligned}\nu^+(a) = & \nu(a) + 1 - \nu(0) \\ = & \frac{1}{2}(\mu(a) + \mu(a)) + 1 - \frac{1}{2}(\mu(0) + \mu(a)) \\ = & \frac{1}{2}(\mu(a) + 1) > \mu(a)\end{aligned}$$

and $\nu(a) < 1 = \nu^+(0)$. Hence ν^+ is non-constant, and μ is not a maximal element of $\mathcal{N}(X)$. This is a contradiction. □

We construct a new fuzzy d-ideal from an old one. Let $t > 0$ be a real number. If $\alpha \in [0, 1]$, α^t shall mean the positive root in case $t < 1$. We define $\mu^t : X \to [0, 1]$ by $\mu^t(x) := (\mu(x))^t$ for all $x \in X$.

Proposition 3.15. *If μ is a fuzzy d-ideal of X, then so is μ^t and $X_{\mu^t} = X_\mu$.*

Proof. For any $x, y \in X$, we have $\mu^t(x * y) = (\mu(x * y))^t \geq (\mu(x))^t = \mu^t(x)$ and

$$\begin{aligned}\mu^t(x) = (\mu(x))^t \geq & (\min\{\mu(x * y), \mu(y)\})^t \\ = & \min\{(\mu(x * y))^t, (\mu(y))^t\} \\ = & \min\{\mu^t(x * y), \mu^t(y)\}.\end{aligned}$$

Hence μ^t is a fuzzy d-ideal of X. Now

$$\begin{aligned}X_{\mu^t} = & \{x \in X | \mu^t(x) = \mu^t(0)\} \\ = & \{x \in X | (\mu(x))^t = (\mu(0))^t\} \\ = & \{x \in X | \mu(x) = \mu(0)\} = X_\mu.\end{aligned}$$

This completes the proof. □

Corollary 3.16. *If $\mu \in \mathcal{N}(X)$, then so is μ^t.*

Proof. Straightforward. □

Definition 3.17. Let μ be a fuzzy d-ideal of a d-transitive d-algebra X. Then μ is said to be *maximal* if

(i) μ is non-constant,

(ii) μ^+ is a maximal element of the poset $(\mathcal{N}(X), \subseteq)$.

Theorem 3.18. *If μ is a maximal fuzzy d-ideal of a d-transitive d-algebra, then*

(i) *μ is normal,*

(ii) *μ takes only the values 0 and 1,*

(iii) *$\mu_{X_\mu} = \mu$,*

(iv) *X_μ is a maximal d-ideal of X.*

Proof. Let μ be a maximal fuzzy d-ideal of X. Then μ^+ is a non-constant maximal element of the poset $(\mathcal{N}(X), \subseteq)$. It follows from Theorem 3.14 that μ^+ takes only the values 0 and 1. Note that $\mu^+(x) = 1$ if and only if $\mu(x) = \mu(0)$, and $\mu^+(x) = 0$ if and only if $\mu(x) = \mu(0) - 1$. By Corollary 3.8, we have $\mu(x) = 0$, i.e., $\mu(0) = 1$. Hence μ is normal, and clearly $\mu^+ = \mu$. This proves (i) and (ii).
(iii) Clearly $\mu_{X_\mu} \subseteq \mu$ and μ_{X_μ} takes the values 0 and 1. Let $x \in X$. If $\mu(x) = 0$, then obviously $\mu \subseteq \mu_{X_\mu}$. If $\mu(x) = 1$, then $x \in X_\mu$, and so $\mu_{X_\mu}(x) = 1$. This shows that $\mu \subseteq \mu_{X_\mu}$.
(iv) X_μ is a proper d-ideal of X because μ is non-constant. Let A be a d-ideal of X such that $X_\mu \subseteq A$. Noticing that for any d-ideals A and B of X, $A \subseteq B$ if and only if $\mu_A \subseteq \mu_B$, then we obtain $\mu = \mu_{X_\mu} \subseteq \mu_A$. Since μ and μ_A are normal and since $\mu = \mu^+$ is a maximal element of $\mathcal{N}(X)$, we have that either $\mu = \mu_A$ or $\mu_A = \mathbf{1}$ where $\mathbf{1} : X \to [0, 1]$ is a fuzzy set defined by $\mathbf{1}(x) := 1$ for all $x \in X$. The other case implies that $A = X$. If $\mu = \mu_A$, then $X_\mu = X_{\mu_A} = A$ by proposition 3.4. This proves that X_μ is a maximal d-ideal of X, completing the proof. □

Definition 3.19. A normal fuzzy d-ideal μ of X is said to be *completely normal* if there exists $x \in X$ such that $\mu(x) = 0$.

Denote by $\mathcal{C}(X)$ the set of all completely normal fuzzy d-ideals of a d-transitive d-algebra X. We note that $\mathcal{C}(X) \subseteq \mathcal{N}(X)$ and there the restriction of the partial ordering $\subseteq$ of $\mathcal{N}(X)$ gives a partial ordering of $\mathcal{C}(X)$.

Example 3.20 Let X be the d-algebra of Example 3.3. Then a fuzzy set μ in defined by $\mu(e) = 0 < 1$ and $\mu(x) = 1$ for all $x \neq e$ is a normal fuzzy d-ideal of X. Since $e \in X$ such that $\mu(e) = 0$, μ is completely normal.

Proposition 3.21. *Any non-constant maximal element of $(\mathcal{N}(X), \subseteq)$ is also a maximal element of $(\mathcal{C}(X), \subseteq)$.*

Proof. Let μ be a non-constant maximal element of $(\mathcal{N}(X), \subseteq)$. By Theorem 3.14, μ takes only the values 0 and 1, and so $\mu(0) = 1$ and $\mu(x) = 0$ for some $x \in X$. Hence

$\mu \in \mathcal{C}(X)$. Assume that there exists $\nu \in \mathcal{C}(X)$ such that $\mu \subseteq \nu$. It follows that $\mu \subseteq \nu$ in $\mathcal{N}(X)$. Since μ is maximal in $(\mathcal{N}(X), \subseteq)$ and ν is non-constant, therefore $\mu = \nu$. Thus μ is maximal element of $(\mathcal{C}(X), \subseteq)$, ending the proof. □

Theorem 3.22. *Every maximal fuzzy d-ideal of a d-transitive d-algebra X is completely normal.*

Proof. Let μ be a maximal fuzzy d-ideal of X. Then by Theorem 3.18, μ is normal and $\mu = \mu^+$ takes the values 0 and 1. Since μ is non-constant, it follows that $\mu(0) = 1$ and $\mu(x) = 0$ for some $x \in X$. Hence μ is completely normal, ending the proof. □

4. Fuzzy characteristic d-ideals

For an endomorphism f of a d-algebra X and a fuzzy set μ in X, a new fuzzy set μ^f in X is defined by $\mu^f(x) := \mu(f(x))$ ([1]) for all $x \in X$.

Proposition 4.1. ([1]) *If μ is a fuzzy d-ideal of a d-algebra X, then so is μ^f.*

Example 4.2. Let $X = \{0, a, b, c\}$ be a set with the following Cayley table:

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	0
c	c	c	a	0

Then X is a d-algebra ([8]). Define a fuzzy set $\mu : X \to [0, 1]$ by $\mu(0) = \mu(a) = t_1$ and $\mu(b) = \mu(c) = t_2$ with $t_1 > t_2$. Then μ is a fuzzy d-ideal of X. There are 5 endomorphisms of a d-algebra X as follows:

$$f_1 : 0 \to 0, a \to 0, b \to 0, c \to 0$$
$$f_2 : 0 \to 0, a \to 0, b \to a, c \to a$$
$$f_3 : 0 \to 0, a \to 0, b \to b, c \to b$$
$$f_4 : 0 \to 0, a \to 0, b \to c, c \to c$$
$$f_5 : 0 \to 0, a \to a, b \to b, c \to c.$$

By Proposition 4.1, $\mu^{f_i} (i = 1, 2, 3, 4, 5)$ are fuzzy d-ideals of X.

Definition 4.3. A fuzzy d-ideal A of a d-algebra X is said to be *characteristic* if $f(A) = A$ for all $f \in Aut(X)$, where $Aut(X)$ is the set of all automorphisms of X.

Definition 4.4. A fuzzy d-ideal of a d-algebra X is said to be *fuzzy characteristic* if $\mu^f(x) = \mu(x)$ for all $x \in X$ and $f \in Aut(X)$.

Example 4.5. In Example 4.2, f_5 is an automorphism of X. It is easy to check that $f_5(\mu(x)) = \mu(x)$ for all $x \in X$. Hence μ is characteristic. Also $\mu^{f_5}(x) = \mu(f_5(x)) = \mu(x)$ for all $x \in X$. Therefore μ is fuzzy characteristic.

Lemma 4.6. *Let μ be a fuzzy d-ideal of a d-algebra X and let $x \in X$. Then $\mu(x) = t$ if and only if $x \in \mu_t$ and $x \notin \mu_s$ for all $s > t$.*

Proof. Straightforward. □

Theorem 4.7. *For a fuzzy d-ideal μ of a d-algebra X, the following are equivalent:*

(i) *μ is fuzzy characteristic,*

(ii) *Each level d-ideal $\mu_t := \{x \in X | \mu(x) \geq t\}$ is characteristic.*

Proof. Assume that μ is a fuzzy characteristic and let $t \in Im(\mu), f \in Aut(X)$ and $x \in \mu_t$. Then $\mu^f(x) = \mu(x) \geq t$, i.e., $\mu(x) \geq t$. Hence $f(x) \in \mu_t$, i.e., $f(\mu_t) \subseteq \mu_t$. Now let $x \in \mu_t$ and $y \in X$ be such that $f(y) = x$. Then $\mu(y) = \mu^f(y) = \mu(f(y)) = \mu(x) \geq t$, whence $y \in \mu_t$, so that $x = f(y) \in f(\mu_t)$. Consequently, $\mu_t \subseteq f(\mu_t)$. Therefore $f(\mu_t) = \mu_t$ and μ_t is characteristic.

Conversely, suppose that each level d-ideal μ_t of μ is characteristic and let $x \in X, f \in AutX$ and $\mu(x) = t$. Then by Lemma 4.6, $x \in \mu_t$ and $x \notin \mu_s$ for all $s > t$. It follows from hypothesis that $f(x) \in f(\mu_t) = \mu_t$, so that $\mu^f(x) = \mu(f(x)) \geq t$. Let $s = \mu^f(x)$ and assume that $s > t$. Then $f(x) \in \mu_s = f(\mu_s)$, which implies from the injectivity of f that $x \in \mu_s$, a contradiction. Hence $\mu^f(x) = \mu(f(x)) = t = \mu(x)$. Therefore μ^f is fuzzy characteristic. □

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