THE BEST CONSTANT OF L^p SOBOLEV INEQUALITY CORRESPONDING TO DIRICHLET BOUNDARY VALUE PROBLEM FOR $(d/dx)^{4m}$

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Abstract. For $M = 2m$ $(m = 1, 2, 3, \cdots)$ and $p > 1$, the best constant of L^p Sobolev inequality

$$\sup_{|y|\le 1} |\, u(y)\,| \le C \left(\int_{-1}^{1} \left| u^{(M)}(x) \right|^p dx \right)^{1/p}$$

for $u(x)$ satisfying $u(x), u^{(M)}(x) \in L^p(-1,1)$ and $u^{(2i)}(\pm 1) = 0$ $(0 \le i \le m-1)$ is obtained. The best constant is the L^q norm $(1/p+1/q = 1)$ of some special polynomial relating to Bernoulli polynomial of order M. The special case of $p = 2$ is treated completely in [1].

1 Conclusion

In this paper $M = 2m$ $(m = 1, 2, 3, \cdots)$ is an even natural number. $p > 1$ is a real number and its conjugate $q > 1$ satisfies $1/p + 1/q = 1$.

For function $u(x)$ on an interval $-1 < x < 1$, we use notations

$$\|\, u \,\|_\infty = \sup_{|y|\le 1} |\, u(y)\,|, \qquad \|\, u \,\|_p = \left(\int_{-1}^{1} |\, u(x)\,|^p \, dx \right)^{1/p}$$

We introduce Sobolev space

$$H = H(M) = \left\{ u(x) \,\middle|\, u(x),\ u^{(M)}(x) \in L^p(-1,1), \quad u^{(2i)}(\pm 1) = 0 \quad (0 \le i \le m-1) \right\} \tag{1.1}$$

and its conjugate

$$H^* = H^*(M) = \left\{ v(x) \,\middle|\, v(x),\ v^{(M)}(x) \in L^q(-1,1), \quad v^{(2j)}(\pm 1) = 0 \quad (m \le j \le M-1) \right\} \tag{1.2}$$

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Sobolev energy form

$$(u,v)_M \;=\; \int_{-1}^{1} u^{(M)}(x)\,\overline{v}^{(M)}(x)\,dx \tag{1.3}$$

for $u \in H$ and $v \in H^*$, and Sobolev functional

$$S(u) \;=\; S(M;u) \;=\; \|\,u\,\|_\infty \Big/ \Big\|\,u^{(M)}\,\Big\|_p \tag{1.4}$$

for $u \in H$.

Throughout this paper Bernoulli polynomials $b_n(x)$ defined by the relation

$$\begin{cases} b_0(x) \;=\; 1 \\ b_n'(x) \;=\; b_{n-1}(x), \quad \displaystyle\int_0^1 b_n(x)\,dx \;=\; 0 \qquad (n=1,2,3,\cdots) \end{cases}$$

play an important role.

$$\begin{aligned}
& b_0(x) \;=\; 1, \qquad b_1(x) \;=\; x-\frac{1}{2}, \qquad b_2(x) \;=\; \frac{1}{2}\,x^2-\frac{1}{2}\,x+\frac{1}{12}, \\
& b_3(x) \;=\; \frac{1}{6}\,x^3-\frac{1}{4}\,x^2+\frac{1}{12}\,x, \qquad b_4(x) \;=\; \frac{1}{24}\,x^4-\frac{1}{12}\,x^3+\frac{1}{24}\,x^2-\frac{1}{720} \\
& b_5(x) \;=\; \frac{1}{120}\,x^5-\frac{1}{48}\,x^4+\frac{1}{72}\,x^3-\frac{1}{720}\,x \\
& b_6(x) \;=\; \frac{1}{720}\,x^6-\frac{1}{240}\,x^5+\frac{1}{288}\,x^4-\frac{1}{1440}\,x^2+\frac{1}{30240} \\
& b_7(x) \;=\; \frac{1}{5040}\,x^7-\frac{1}{1440}\,x^6+\frac{1}{1440}\,x^5-\frac{1}{4320}\,x^3+\frac{1}{30240}\,x \\
& b_8(x) \;=\; \frac{1}{40320}\,x^8-\frac{1}{10080}\,x^7+\frac{1}{8640}\,x^6-\frac{1}{17280}\,x^4+\frac{1}{60480}\,x^2-\frac{1}{1209600} \\
& \cdots
\end{aligned}$$

Our conclusion is as follows.

Theorem 1.1 *Using Green function*

$$G(x,y) \;=\; G(M;x,y) \;=\; (-1)^{M+1}\,4^{2M-1}\left[\;b_{2M}\left(\frac{|x-y|}{4}\right)\;-\;b_{2M}\left(\frac{2-x-y}{4}\right)\;\right]$$

$$(-1<x,y<1) \tag{1.5}$$

which will be explained later in Theorem 2.1*, we can assert*

(1) $\displaystyle\sup_{u\in H,\ u\not\equiv 0} S(u) = C_0$ *is given by*

$$C_0 \,=\, C(M) \,=\, \left\| \left(\partial_x^M G(M;x,y)\right)\Big|_{y=0} \right\|_q \,=\, \|\, G(m;x,0)\,\|_q \,=$$

$$\left\| 4^{M-1} \left[\, b_M\left(\frac{|x|}{4}\right) \,-\, b_M\left(\frac{2-x}{4}\right)\,\right] \right\|_q \tag{1.6}$$

$$C(2) \,=\, \frac{1}{2}\left(\frac{2}{q+1}\right)^{1/q}, \qquad C(4) \,=\, \frac{1}{12}\,\|\, 2-3x^2+|x|^3\,\|_q$$

$$C(6) \,=\, \frac{1}{240}\,\|\, 16-20x^2+5x^4-|x|^5\,\|_q$$

$$C(8) \,=\, \frac{1}{10080}\,\|\, 272-336x^2+70x^4-7x^6+|x|^7\,\|_q\,, \qquad \cdots$$

(2) $$S(\,U(x)\,) \,=\, C_0 \tag{1.7}$$

where

$$U(x) \,=\, \int_{-1}^{1} G(m;x,y)\left(G(m;y,0)\right)^{q-1} dy \qquad (-1<x<1) \tag{1.8}$$

This paper is organized as follows. In section 2, we consider a boundary value problem for $(-1)^M(d/dx)^{2M}$ with Dirichlet boundary condition. In section 3, we show that Green function $G(x,y)$ is a reproducing kernel for H, H^* and $(\cdot,\cdot)_M$. Finally, section 4 is devoted to the proof of Theorem 1.1.

2 Dirichlet boundary value problem

In the previous work [1], we proved the following two theorems.

Theorem 2.1 *For any bounded continuous function $f(x)$ on an interval $-1<x<1$, Dirichlet boundary value problem*

BVP (M)

$$\begin{cases} (-1)^M u^{(2M)} \,=\, f(x) & (-1<x<1) \qquad (2.1)\\ u^{(2i)}(\pm 1) \,=\, 0 & (0\le i\le M-1) \qquad (2.2)\end{cases}$$

has a unique classical solution $u(x)$ expressed as follows.

$$u(x) \,=\, \int_{-1}^{1} G(x,y)\,f(y)\,dy \qquad (-1<x<1) \tag{2.3}$$

Green function $G(x,y)=G(M;x,y)$ is given by (1.5).

Theorem 2.2 *Green function $G(x,y) = G(M;x,y)$ satisfies the following conditions.*

(1) $\partial_x^{2M} G(x,y) \,=\, 0 \qquad (-1<x,y<1, \quad x\neq y)$ (2.4)

(2) $\partial_x^{2i} G(x,y)\Big|_{x=\pm 1} \,=\, 0 \qquad (0\le i\le M-1, \quad -1<y<1)$ (2.5)

(3) $$\partial_x^{i} G(x,y)\Big|_{y=x-0} - \partial_x^{i} G(x,y)\Big|_{y=x+0} = \begin{cases} 0 & (0\le i\le 2M-2) \\ (-1)^M & (i=2M-1) \quad (-1<x<1) \end{cases} \tag{2.6}$$

(4) $$\partial_x^{i} G(x,y)\Big|_{x=y+0} - \partial_x^{i} G(x,y)\Big|_{x=y-0} = \begin{cases} 0 & (0\le i\le 2M-2) \\ (-1)^M & (i=2M-1) \quad (-1<y<1) \end{cases} \tag{2.7}$$

(5) $G(M;x,y) \,>\, 0 \qquad (-1<x,y<1)$ (2.8)

Differentiating $G(x,y)$ with respect to x, we have

$$\partial_x^i G(x,y) \,=\,$$
$$(-1)^{M+1}4^{2M-1-i}\left[\Big(\mathrm{sgn}(x-y)\Big)^i b_{2M-i}\Big(\frac{|x-y|}{4}\Big) - (-1)^i b_{2M-i}\Big(\frac{2-x-y}{4}\Big)\right]$$
$$(-1<x,y<1, \quad x\neq y, \quad 0\le i\le 2M)$$

Putting $i=M=2m$ we have the following lemma.

Lemma 2.1

$$\partial_x^M G(M;x,y) \,=\, (-1)^m G(m;x,y) \qquad (-1<x,y<1, \quad x\neq y) \tag{2.9}$$

3 Reproducing kernel

For the sake of convenience, we use the notation

$$g(x,y) \,=\, G(m;x,y) \,=\, (-1)^m\, \partial_x^M G(M;x,y) \,=\, \beta\Big(\frac{|x-y|}{4}\Big) - \beta\Big(\frac{2-x-y}{4}\Big)$$
$$(-1<x,y<1) \tag{3.1}$$

where

$$\beta(x) \,=\, (-1)^{m+1}4^{M-1}b_M(x) \qquad (0<x<1) \tag{3.2}$$

Green function $G(x,y)$ is a reproducing kernel for H, H^* and $(\cdot,\cdot)_M$.

Theorem 3.1 *For any fixed y $(-1\le y\le 1)$ $G(x,y)=G(M;x,y)$ belongs to H^*. For any $u(x)\in H$, we have the following reproducing relation.*

$$u(y) \,=\, (\,u(x), G(x,y)\,)_M \,=\, \int_{-1}^{1} u^{(M)}(x)\,\partial_x^M G(M;x,y)\,dx \,=$$
$$\int_{-1}^{1} u^{(M)}(x)\,(-1)^m g(x,y)\,dx \qquad (-1\le y\le 1) \tag{3.3}$$

Proof of Theorem 3.1 For functions $u = u(x)$ and $v = v(x) = G(x, y)$ with y arbitrarily fixed in $-1 \le y \le 1$, we have

$$u^{(M)}\, v^{(M)} \;-\; u\,(-1)^M v^{(2M)} \;=\; \left(\sum_{j=0}^{M-1} (-1)^{M-1-j}\, u^{(j)}\, v^{(2M-1-j)} \right)'$$

Integrating this with respect to x on intervals $-1 < x < y$ and $y < x < 1$, we have

$$\int_{-1}^{1} u^{(M)}(x)\, v^{(M)}(x)\, dx \;-\; \int_{-1}^{1} u(x)\,(-1)^M v^{(2M)}(x)\, dx \;=$$
$$\left[\sum_{j=0}^{M-1} (-1)^{M-1-j}\, u^{(j)}(x)\, v^{(2M-1-j)}(x) \right] \left\{ \Big|_{x=-1}^{x=y-0} + \Big|_{x=y+0}^{x=1} \right\} \;=$$
$$\sum_{j=0}^{M-1} (-1)^{M-1-j} \Big[u^{(j)}(1)\, v^{(2M-1-j)}(1) \;-\; u^{(j)}(-1)\, v^{(2M-1-j)}(-1) \Big] \;+$$
$$\sum_{j=0}^{M-1} (-1)^{M-1-j}\, u^{(j)}(y) \Big[v^{(2M-1-j)}(y-0) \;-\; v^{(2M-1-j)}(y+0) \Big]$$

The first term on the right-hand side is rewritten as follows.

$$\sum_{j=0}^{M-1} (-1)^{M-1-j} \Big[u^{(j)}(1)\, v^{(2M-1-j)}(1) \;-\; u^{(j)}(-1)\, v^{(2M-1-j)}(-1) \Big] \;=$$
$$\sum_{j=0}^{m-1} (-1)^{M-1} \Big[u^{(2j)}(1)\, v^{(2(M-1-j)+1)}(1) \;-\; u^{(2j)}(-1)\, v^{(2(M-1-j)+1)}(-1) \Big] \;+$$
$$\sum_{j=0}^{m-1} (-1)^{M} \Big[u^{(2j+1)}(1)\, v^{(2(M-1-j))}(1) \;-\; u^{(2j+1)}(-1)\, v^{(2(M-1-j))}(-1) \Big]$$

Using (2.4), (2.5), (2.7) and (2.9), we have (3.3). ■

4 L^p Sobolev inequality

The most delicate point of this paper is the following theorem.

Theorem 4.1

$$\sup_{|y| \le 1} \| \, g(\cdot, y) \, \|_q \;=\; \| \, g(\cdot, 0) \, \|_q \tag{4.1}$$

We will prove this later and will prove our main theorem using this.

Proof of Theorem 1.1 Applying Hölder inequality to the reproducing identity (3.3), we have the following inequality

$$\| \, u \, \|_\infty \;\le\; \sup_{|y| \le 1} \| \, g(\cdot, y) \, \|_q \left\| \, u^{(M)} \, \right\|_p \;=\; \| \, g(\cdot, 0) \, \|_q \left\| \, u^{(M)} \, \right\|_p \tag{4.2}$$

Now we will show that the equality holds for special $u(x)$. For the function

$$f(x) \;=\; g^{q-1}(x, 0) \qquad (-1 < x < 1) \tag{4.3}$$

Dirichlet boundary value problem

$$\begin{cases} (-1)^m u^{(2m)} = f(x) & (-1 < x < 1) \\ u^{(2i)}(\pm 1) = 0 & (0 \le i \le m-1) \end{cases} \tag{4.4, 4.5}$$

has a unique classical solution $u(x) = U(x)$ expressed as follows.

$$U(x) = \int_{-1}^{1} g(x,y)\, f(y)\, dy \qquad (-1 < x < 1) \tag{4.6}$$

This is the same as (1.8). Applying the inequality (4.2) to $u(x) = U(x) \in H$ and have

$$\| U \|_\infty \le \| g(\cdot,0) \|_q \left\| U^{(M)} \right\|_p = \| g(\cdot,0) \|_q \left\| g^{q-1}(\cdot,0) \right\|_p = \| g(\cdot,0) \|_q^q \tag{4.7}$$

We used a fact $\left\| g^{q-1}(\cdot,0) \right\|_p = \| g(\cdot,0) \|_q^{q-1}$. On the other hand, we have

$$U(0) = \int_{-1}^{1} g(0,y)\, f(y)\, dy = \int_{-1}^{1} g(y,0) \Big(g(y,0) \Big)^{q-1} dy = \| g(\cdot,0) \|_q^q \tag{4.8}$$

Combining (4.7) and trivial inequality $\| g(\cdot,0) \|_q^q = | U(0) | \le \| U \|_\infty$, we have

$$\| g(\cdot,0) \|_q^q = | U(0) | \le \| U \|_\infty \le \| g(\cdot,0) \|_q \left\| U^{(M)} \right\|_p = \| g(\cdot,0) \|_q^q \tag{4.9}$$

That is to say

$$\| U \|_\infty = \| g(\cdot,0) \|_q \left\| U^{(M)} \right\|_p \tag{4.10}$$

This completes the proof of Theorem 1.1. ■

To prove Theorem 4.1, we need the following lemma concerning Bernoulli polynomial.

Lemma 4.1 *Bernoulli polynomial*

$$\beta(x) = (-1)^{m+1} 4^{M-1} b_M(x) \qquad (0 < x < 1) \tag{4.11}$$

satisfies the following properties.

(1) $\beta(1/2 - x) = \beta(1/2 + x) \qquad (0 \le x \le 1/2)$ (4.12)

(2) $\beta'(x) < 0 \qquad (0 < x < 1/2)$ (4.13)

(3) $\beta(0) = (-1)^{m+1} 4^{M-1} b_M(0) = 2^{M-1} \pi^{-M} \zeta(M) > 0$ (4.14)

(4) $\beta(1/2) = (-1)^{m+1} 4^{M-1} b_M(1/2) = -(-1)^{m+1} 2^{M-1} \left(2^{M-1} - 1\right) b_M(0) =$
$- \left(2^{M-1} - 1\right) \pi^{-M} \zeta(M) < 0$ (4.15)

(5) $|\beta(0)| > |\beta(1/2)|$ (4.16)

where $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ $(\mathrm{Re}\ s > 1)$ *is Riemann zeta function.*

We omit to proof this lemma. (See [1] or [2]).

Now we proceed to prove Theorem 4.1.

Proof of Theorem 4.1 First of all, we note the important fact

$$g(x,y) \,>\, 0 \qquad (-1<x,y<1) \tag{4.17}$$

by Theorem 2.2 (5). $\|\, g(\cdot,y)\,\|_q$ is an even function of y because of the symmetry

$$g(-x,-y) \,=\, g(x,y) \qquad (-1<x,y<1) \tag{4.18}$$

It is enough to show

$$h(y) \,=\, -\frac{2}{q}\,\frac{d}{dy}\,\|\, g(\cdot,y)\,\|_q^q \,>\, 0 \qquad (0<y<1) \tag{4.19}$$

Using simple facts

$$(\partial_x \,-\, \partial_y)\, g(x,y) \,=\, \frac{1}{2}\,\mathrm{sgn}(x-y)\,\beta'\Big(\frac{|x-y|}{4}\Big) \qquad (-1<x,y<1, \quad x\neq y) \tag{4.20}$$

and

$$\int_{-1}^{1} \partial_x\Big(g^q(x,y)\Big)dx \,=\, 0 \qquad (-1<y<1) \tag{4.21}$$

we have

$$\begin{aligned}
&h(y) \,=\, -\frac{2}{q}\,\partial_y \int_{-1}^{1} g^q(x,y)\,dx \,=\, -2\int_{-1}^{1} g^{q-1}(x,y)\,\partial_y g(x,y)\,dx \,= \\
&-\,2\int_{-1}^{1} g^{q-1}(x,y)\,\partial_x g(x,y)\,dx \,+\, \int_{-1}^{1} g^{q-1}(x,y)\,\mathrm{sgn}(x-y)\,\beta'\Big(\frac{|x-y|}{4}\Big)\,dx \,= \\
&\int_{-1}^{1} \mathrm{sgn}(x-y)\,\beta'\Big(\frac{|x-y|}{4}\Big)\Big[\beta\Big(\frac{|x-y|}{4}\Big) - \beta\Big(\frac{2-x-y}{4}\Big)\Big]^{q-1}\,dx \,= \\
&\int_{-1}^{y} -\beta'\Big(\frac{y-x}{4}\Big)\Big[\beta\Big(\frac{y-x}{4}\Big) - \beta\Big(\frac{2-x-y}{4}\Big)\Big]^{q-1}\,dx \,+ \\
&\int_{y}^{1} \beta'\Big(\frac{x-y}{4}\Big)\Big[\beta\Big(\frac{x-y}{4}\Big) - \beta\Big(\frac{2-x-y}{4}\Big)\Big]^{q-1}\,dx \,= \\
&\int_{0}^{1+y} \Big(-\beta'\Big(\frac{\xi}{4}\Big)\Big)\Big[\beta\Big(\frac{\xi}{4}\Big) - \beta\Big(\frac{2-2y+\xi}{4}\Big)\Big]^{q-1}\,d\xi \,- \\
&\int_{0}^{1-y} \Big(-\beta'\Big(\frac{\xi}{4}\Big)\Big)\Big[\beta\Big(\frac{\xi}{4}\Big) - \beta\Big(\frac{2-2y-\xi}{4}\Big)\Big]^{q-1}\,d\xi \qquad (0<y<1)
\end{aligned} \tag{4.22}$$

Here we divide $h(y)$ into the two parts

$$h(y) \,=\, h_0(y) \,+\, h_1(y) \qquad (0<y<1) \tag{4.23}$$

where

$$h_0(y) \;=\; \int_{1-y}^{1+y} \left(-\beta'\left(\frac{\xi}{4}\right)\right) \left[\beta\left(\frac{\xi}{4}\right) - \beta\left(\frac{2-2y+\xi}{4}\right)\right]^{q-1} d\xi \tag{4.24}$$

$$h_1(y) \;=\; \int_{0}^{1-y} \left(-\beta'\left(\frac{\xi}{4}\right)\right) \Bigg[\left[\beta\left(\frac{\xi}{4}\right) - \beta\left(\frac{2-2y+\xi}{4}\right)\right]^{q-1} - \left[\beta\left(\frac{\xi}{4}\right) - \beta\left(\frac{2-2y-\xi}{4}\right)\right]^{q-1} \Bigg] d\xi \tag{4.25}$$

For $0 < 1-y < \xi < 1+y < 2$ we have $0 < \xi/4 < 1/2$. From the inequality (4.17), it follows the inequality

$$\beta\left(\frac{\xi}{4}\right) - \beta\left(\frac{2-2y+\xi}{4}\right) \;>\; 0 \qquad (0 < \xi/4 < 1/2, \quad 0 < y < 1) \tag{4.26}$$

We also have

$$-\beta'(\xi/4) \;>\; 0 \qquad (0 < \xi/4 < 1/2) \tag{4.27}$$

from Lemma 4.1 (2). From these two inequalities it follows $h_0(y) > 0$ $(0 < y < 1)$.

An inequality $h_1(y) > 0$ $(0 < y < 1)$ follows from the following lemma.

Lemma 4.2 *For every fixed y $(0 < y < 1)$ we have*

$$\beta\left(\frac{2-2y-\xi}{4}\right) \;>\; \beta\left(\frac{2-2y+\xi}{4}\right) \qquad (0 < \xi < 1-y) \tag{4.28}$$

Proof of Lemma 4.2 For every fixed y $(0 < y < 1)$, we treat the first case $0 < \xi < (2y) \wedge (1-y)$. It follows

$$0 \;<\; \frac{2-2y-\xi}{4} \;<\; \frac{2-2y+\xi}{4} \;<\; \frac{1}{2}$$

so we have

$$\beta\left(\frac{2-2y-\xi}{4}\right) \;>\; \beta\left(\frac{2-2y+\xi}{4}\right) \qquad (0 < \xi < (2y) \wedge (1-y))$$

by Lemma 4.1 (2).

In the second case $(2y) \wedge (1-y) < \xi < 1-y$, we have $(2y) \le \xi < 1-y$, hence

$$0 \;<\; \frac{2-2y-\xi}{4} \;<\; \frac{2+2y-\xi}{4} \;<\; \frac{1}{2}$$

By Lemma 4.1 (2), it follows

$$\beta\left(\frac{2-2y-\xi}{4}\right) \;>\; \beta\left(\frac{2+2y-\xi}{4}\right) \;=\; \beta\left(\frac{2-2y+\xi}{4}\right) \quad ((2y) \wedge (1-y) < \xi < 1-y)$$

The last equality follows from Lemma 4.1 (1). This completes the proof of Lemma 4.2. ■

From Lemma 4.2, it follows $h_1(y) > 0$ $(0 < y < 1)$. We proved $h(y) > 0$ $(0 < y < 1)$. This completes the proof of Theorem 4.1. ■

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References

[1] Y.Kametaka, H.Yamagishi, K.Watanabe, A.Nagai and K.Takemura, *The best constant of Sobolev inequality corresponding to Dirichlet boundary value problem for* $(-1)^M(d/dx)^{2M}$, to appear in Sci. Math. Jpn.

[2] Y.Kametaka, H.Yamagishi, K.Watanabe, A.Nagai and K.Takemura, *Riemann zeta function, Bernoulli polynomials and the best constant of Sobolev inequality*, Sci. Math. Jpn., **e-2007** (2007), 63–89.

[3] Y.Kametaka, Y.Oshime, K.Watanabe, H.Yamagishi, A.Nagai and K.Takemura, *The best constant of L^p Sobolev inequality corresponding to the periodic boundary value problem for* $(-1)^M(d/dx)^{2M}$, Sci. Math. Jpn. **e-2007** (2007), 269–281.

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