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CLASSIFICATION OF QUASI UNION HYPER K-ALGEBRAS OF ORDER LESS THAN 6

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Abstract. In this manuscript, at first we have given some theorems about non-isomorphic quasi union hyper K-algebra. Then we proceed with the study of classification quasi union hyper K-algebra of order less than or equal to 5. At the end, we give a conjecture about number of non-isomorphic quasi union hyper K-algebra of order n.

1 Introduction The study of BCK-algebra was initiated by Imai and Iséki [2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculi. The hyper structure theory (called also multi algebras) was introduced in 1934 by Marty [4] at the 8th congress of Scandinavian Mathematicians. Hyper structures have many applications to several sectors of both pure and applied sciences. Borzooei, et.al. [3, 1] applied the hyper structure to BCK-algebras and introduced the concept of hyper BCK-algebra and hyper K-algebra in which, each of them is a generalization of BCK-algebra. Nasr-Azadani and Zahedi [5] introduced quasi union hyper K-algebras. Now we study classification of them.

2 Preliminaries. Let H be a non-empty set, the set of all non-empty subset of H is denoted by $\mathcal{P}^*(H)$. A *hyperoperation* on H is a map $\circ : H \times H \to \mathcal{P}^*(H)$, where $(a,b) \to a \circ b, \forall a,b \in H$. A set H, endowed with a hyperoperation, " $\circ$ ", is called a *hyperstructure*. If $A,B \subseteq H$, then $A \circ B = \bigcup_{a\in A, b\in B} a \circ b$.

Definition 2.1. [1, 3] Let H be a non-empty set containing a constant " 0 " and " $\circ$ " be a hyperoperation on H. Then H is called a hyper K-algebra (hyper BCK-algebra) if it satisfies K1-K5(HK1-HK4).

K1: $(x \circ z) \circ (y \circ z) < x \circ y$,	HK1: $(x \circ z) \circ (y \circ z) \ll x \circ y$,
K2: $(x \circ y) \circ z = (x \circ z) \circ y$,	HK2: $(x \circ y) \circ z = (x \circ z) \circ y$,
K3: $x < x$,	HK3: $x \circ H \ll x$,
K4: $x < y,\ y < x$, then $x = y$,	HK4: $x \ll y,\ y \ll x$, then $x = y$,
K5: $0 < x$,	

for all $x, y, z \in H$, where $x < y (x \ll y)$ means $0 \in x \circ y$. Moreover for any $A, B \subseteq H$, $A < B$ if $\exists\, a \in A$, $\exists\, b \in B$ such that $a < b$ and $A \ll B$ if $\forall\, a \in A$, $\exists\, b \in B$ such that $a \ll b$.

For briefly the readers could see some definitions and results about hyper K-algebra and hyper BCK-algebra in [1, 3].

Note: Let H be a set containing "0", $\mathcal{P}_0(H) = \{A \subseteq H : 0 \in A\}$ and $\mathcal{S} = \{f | f : H \to \mathcal{P}_0(H)$ is a function$\}$. For convenience we use F^x instead of $f(x)$ for any $f \in \mathcal{S}$.

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Theorem 2.2. *[5] Let X be a set and $H = X \cup \{0\}$. Then for any $f \in \mathcal{S}$, $\circ_f : H \times H \to \mathcal{P}^*(H)$ which is defined by:*

$$x \circ_f y := \begin{cases} F^x & \text{if } x = y, \\ \{x\} & \text{otherwise,} \end{cases}$$

is a hyperoperation. Moreover the following statements are equivalent.
(i) $(H, \circ_f, 0)$ is a hyper K-algebra.
(ii) $F^x \circ_f y = F^x$ for all $y \neq x, y \in H$.
(iii) if $x \neq y$ and $y \in F^x$, then $y \in F^y$ and $F^y \subseteq F^x$.
This hyper K-algebra is called quasi union hyper K-algebra with respect to f and denoted by H_f.

Corollary 2.3. *[5] Let H_f be a quasi union hyper K-algebra and $x \neq y$. If $y \in F^x$ and $x \in F^y$, then $F^y = F^x$.*

Definition 2.4. [1] Let H_1 and H_2 are two hyper K-algebras. A mapping $f : H_1 \to H_2$ is said to be a homomorphism if $f(0) = 0$ and $f(x \circ y) = f(x) \circ f(y)$, for all $x, y \in H_1$, $Kerf = \{x \in H_1 : f(x) = 0\}$. Moreover if f is bijection this homomorphism is called an isomorphism.

3 classification quasi union hyper K-algebra of order ≤ 5. In this section we give some theorems about isomorphisms on quasi union hyper K-algebra and classification quasi union hyper K-algebras of order 1, 2, 3 , 4 and 5 .

Note: In this paper we let $\mathcal{H}_n = \{H_f : f \in \mathcal{S}\}$, where $H = \{x_0 = 0, x_1, ..., x_n\}$ and $n \in \mathbb{N}$, $N_i(H_f) = |\{F^{x_k} : x_k \in F^{x_k}, 0 \leq k \leq i, i \leq n\}|$. Further $t^f(m) = |\{x_i \in H \setminus \{0\} : |F^{x_i}| = m\}|$.

Definition 3.1. Let $H_f, H_g \in \mathcal{H}_n$. Then we say that H_f is of type $(l_0^f, l_1^f, \cdots, l_n^f)$ and denote it by $t(H_f)$, if $l_i^f = |F^{x_i}|$, $0 \leq i \leq n$. Also H_f and H_g are co-type, if the type of H_f is a permutation of type H_g

Example 3.2. Consider two quasi union hyper K-algebra H_f and H_g on $H = \{0, 1, 2, 3, 4\}$ as follows:

$\circ_f$	0	1	2	3	4
0	{0}	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}	{1}
2	{2}	{2}	{0,2}	{2}	{2}
3	{3}	{3}	{3}	{0,1,2}	{3}
4	{4}	{4}	{4}	{4}	{0,1,4}

$\circ_g$	0	1	2	3	4
0	{0}	{0}	{0}	{0}	{0}
1	{1}	{0,1,3}	{1}	{1}	{1}
2	{2}	{2}	{0,2,3}	{2}	{2}
3	{3}	{3}	{3}	{0,3}	{3}
4	{4}	{4}	{4}	{4}	{0,3}

It is clear that $N_0(H_f) = 1, N_1(H_f) = 2, N_2(H_f) = 3, N_3(H_f) = 3, N_4(H_f) = 4$ and $t^f(1) = t^g(1) = 0, t^f(2) = t^g(2) = 2$ and $t^f(3) = t^g(3) = 2$. Also $t(H_f) = (1, 2, 2, 3, 3)$, $t(H_g) = (1, 3, 3, 2, 2)$ and they are co-type.

Theorem 3.3. *Let H_f and H_g be quasi union hyper K-algebras and v be an isomorphism from H_f to H_g. Then $|F^x| = |v(F^x)| = |G^{v(x)}|$.*

Proof. By Definition 2.4, since $v(F^x) = v(x \circ_f x) = v(x) \circ_g v(x) = G^{v(x)}$ and v is a 1-1 correspondence, the proof immediately follows.

Corollary 3.4. *Let H_f and H_g be hyper K-algebras. Then H_f is not isomorphic to H_g, if one of the following statement holds.*
i) $|0 \circ_f 0| \neq |0 \circ_g 0|$.
ii) $F^{x_i} = G^{x_i}$ *for all* $1 \leq i \leq n$ *except for some* i.
iii) $t^f(m) \neq t^g(m)$ *for some* m, $1 \leq m \leq n$.
iv) $N_n(H_f) \neq N_n(H_g)$.
v) H_f *and* H_g *are not co-type.*

The following example shows that if $N_n(H_f) = N_n(H_g)$, or $t^f(m) = t^g(m)$ for all possible m or they are co-type, then H_f and H_g may be non-isomorphic in general.

Example 3.5. Consider two quasi union hyper K-algebra H_f and H_g on $H = \{0, 1, 2, 3, 4\}$ as follows:

$\circ_f$	0	1	2	3	4
0	{0}	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}	{1}
2	{2}	{2}	{0,2}	{2}	{2}
3	{3}	{3}	{3}	{0,1,2}	{3}
4	{4}	{4}	{4}	{4}	{0,1,4}

$\circ_g$	0	1	2	3	4
0	{0}	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}	{2}
3	{3}	{3}	{3}	{0,1,3}	{3}
4	{4}	{4}	{4}	{4}	{0,1,4}

It is clear that $N_n(H_f) = N_n(H_g) = 4$, $t(H_f) = t(H_g) = (1, 2, 2, 3, 3)$, $t^f(1) = t^g(1) = 0, t^f(2) = t^g(2) = 2$ and $t^f(3) = t^g(3) = 2$. But they are not isomorphic. Because if v ia an isomorphism map between H_f and H_g, then $v = (1), v = (12), v = (34)$ or $v = (12)(34)$. In all cases $v(2) \notin v(F^2)$, hence by Theorem 3.3, H_f and H_g are not isomorphic.

In remainder of this section we follow some conditions to recognize non-isomorphic quasi union hyper K-algebras.

Theorem 3.6. *Co-type relation on $\mathcal{H}_n$ is an equivalence relation.*

Proof. The proof is easy.

Theorem 3.7. *(Key Theorem) Let H_f be a quasi union hyper K-algebra of type $t(H_f) = (l_0^f, l_1^f, \cdots, l_n^f)$. Then, there is a union hyper K-algebra H_g isomorphic to H_f such that $t(H_g) = (l_0^g, l_1^g, \cdots, l_n^g)$ where $l_0^g \leq l_1^g \leq l_2^g, \cdots, \leq l_n^g$.*

Proof. At first by Theorem 2.2(iii) we know that $F^0 \subseteq F^x$ for all $x \in H$. Hence for any $f \in \mathcal{S}(H)$ we have $l_0^f \leq l_i^f, 0 \leq i \leq n$. Now suppose v be a permutation on $\{1, 2, \cdots, n\}$

such that $l^f_{v^{-1}(i)} \leq l^f_{v^{-1}(j)}$ for $1 \leq i \leq j \leq n$. Now we define a bijection map $u : H \to H$ where

$$u(x_i) = \begin{cases} x_{v^{-1}(i)}, & i \neq 0; \\ 0, & i = 0, \end{cases}$$

and hyperoperation $\circ_g : H \times H \to \mathcal{P}^*(H)$ which is defined by:

$$x_i \circ_g x_j := \begin{cases} u^{-1}(u(x_i) \circ_f u(x_i)) & \text{if } i = j \\ \{x_i\} & \text{otherwise,} \end{cases} \quad \forall x_i, x_j \in H.$$

At first we show that H_g is a union hyper K-algebra, $|x_i \circ_g x_i| = l^f_{v^{-1}(i)}$ and u is an isomorphism.

It is clear that $g \in \mathcal{S}(H)$. Also for all $i \neq j$ we have

$$\begin{aligned}
&(x_i \circ_g x_i) \circ_g x_j \\
&= u^{-1}(u(x_i) \circ_f u(x_i)) \circ_g x_j && \text{by definition of } \circ_g. \\
&= u^{-1}(u(u^{-1}((u(x_i) \circ_f u(x_i))) \circ_f u(x_j)) \\
&= u^{-1}((u(x_i) \circ_f u(x_i)) \circ_f u(x_j)) \\
&= u^{-1}((u(x_i) \circ_f u(x_j)) \circ_f u(x_i)) && \text{since } H_f \text{ is hyper K-algebra.} \\
&= u^{-1}(u(x_i) \circ_f u(x_i)) && \text{as } H_f \text{ is a quasi union hyper K-algebra.} \\
&= (x_i \circ_g x_i) && \text{by definition of } \circ_g.
\end{aligned}$$

Hence by Theorem 2.2(ii) we conclude that H_g is a quasi union hyper K-algebra. Moreover since u is a bijection map we have:

$$\begin{aligned}
l^g_i &= |x_i \circ_g x_i| = |u^{-1}(u(x_i) \circ_f u(x_i))| \\
&= |u(x_i) \circ_f u(x_i)| && \text{since } u \text{ is bijection} \\
&= |x_{v^{-1}(i)} \circ_f x_{v^{-1}(i)}| \\
&= l^f_{v^{-1}(i)}.
\end{aligned}$$

Finally, we show that H_g is isomorphic to H_f. To do this, since the map u is a bijection, it is sufficient to show that the map $u : H_g \to H_f$, with $x_i \to u(x_i)$ is a homomorphism. If $i \neq j$, then $u(x_i) \neq u(x_j)$ and $u(x_i \circ_g x_j) = u(x_i) = u(x_i) \circ_f u(x_j)$. If $i = j$, then

$$u(x_i \circ_g x_i) = u(u^{-1}((u(x_i) \circ_f u(x_i)))) = u(x_i) \circ_f u(x_i).$$

Remark 3.8. By Theorem 3.7, for any class of co-type relation on $\mathcal{H}_n$, we can choose a representative H_f of type $(l^f_0, l^f_1, \cdots, l^f_n)$, where $l^f_0 \leq l^f_1 \cdots \leq l^f_n$. From now on, for any quasi union hyper K-algebra of order n, we make the assumption: $l_0 \leq l_1 \cdots \leq l_n$.

Theorem 3.9. *Let H_f be a quasi union hyper K-algebra. Then there exist at most $t^f(1)!t^f(2)! \cdots t^f(n)!$ quasi union hyper K-algebra isomorphic to H_f.*

Let w_m be a permutation on $T^f(m) = \{x \in H \setminus \{0\} : |F^x| = m\}$ where $1 \leq m \leq n$ and v_m be an extension of w_m to H as follows:

$$v_m(x_i) = \begin{cases} w_m(x_i), & \text{if } x_i \in T^f(m); \\ x_i, & \text{otherewise.} \end{cases}$$

Then $v = v_1 o v_2 o \cdots o v_n$ is a permutation on H.

Now we define a bijection map $u : H \to H$ where

$$u(x_i) = \begin{cases} x_{v(i)}, & x_i \neq 0; \\ 0, & x_i = 0. \end{cases}$$

Further, we define hyper operation $\circ_g : H \times H \to \mathcal{P}^*(H)$ as follows:

$$x_i \circ_g x_j := \begin{cases} u(u^{-1}(x_i) \circ_f u^{-1}(x_i)) & \text{if } i = j \\ \{x_i\} & \text{otherwise,} \end{cases} \quad \forall x_i, x_j \in H.$$

As similar to the proof of Theorem 3.7 we can show that H_f and H_g are isomorphic. From this we conclude that there are $t^f(1)!t^f(2)!\cdots t^f(n)!$ quasi union hyper K-algebras where isomorphic to H_f.

Example 3.10. Consider two quasi union hyper K-algebra H_f and H_g on $H = \{0, 1, 2, 3, 4\}$ as follows:

$\circ_f$	0	1	2	3	4
0	{0}	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}	{1}
2	{2}	{2}	{0,2}	{2}	{2}
3	{3}	{3}	{3}	{0,1,2}	{3}
4	{4}	{4}	{4}	{4}	{0,1,4}

$\circ_g$	0	1	2	3	4
0	{0}	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}	{1}
2	{2}	{2}	{0,2}	{2}	{2}
3	{3}	{3}	{3}	{0,1,3,4}	{3}
4	{4}	{4}	{4}	{4}	{0,1,3,4}

Then $T^f(1) = \phi,\ T^f(2) = \{1, 2\},\ T^f(3) = \{3, 4\},\ T^f(4) = T^f(5) = \phi$ and $(1), (12), (34)$ and $(12)(34)$ are 4 permutation on H (i.e., $t^f(1)!t^f(2)!t^f(3)!t^f(4) = 0!2!2!0! = 4$). The following table shows 4 quasi union hyper K-algebras isomorphic to H_f where each one is denoted by their $F^x, x = 0, 1, 2, 3, 4$.

Nu.	F^0	F^1	F^2	F^3	F^4	Permutation
1	{0}	{0,1}	{0,2}	{0,1,2}	{0,1,4}	(1)
2	{0}	{0,1}	{0,2}	{0,1,2}	{0,2,4}	(12)
3	{0}	{0,1}	{0,2}	{0,1,3}	{0,1,2}	(34)
4	{0}	{0,1}	{0,2}	{0,2,3}	{0,1,2}	(12)(34)

Also $T^g(1) = \phi,\ T^g(2) = \{1, 2\},\ T^g(3) = \phi,\ T^g(4) = \{3, 4\}, T^g(5) = \phi$ and there are 2 union hyper K-algebra isomorphic to H_g as follows:

Nu.	G^0	G^1	G^2	G^3	G^4	Permutation
1	{0}	{0,1}	{0,2}	{0,1,3,4}	{0,1,3,4}	(1)
2	{0}	{0,1}	{0,2}	{0,2,3,4}	{0,2,3,4}	(12)
3	{0}	{0,1}	{0,2}	{0,1,3,4}	{0,1,3,4}	(34)
4	{0}	{0,1}	{0,2}	{0,2,3,4}	{0,2,3,4}	(12)(34)

Theorem 3.11. *Let H_f be a quasi union hyper K-algebra of type $(l_0, l_1, l_2, \cdots, l_n)$. Then $l_i \leq N_i(H_f)$ for all $i, 0 \leq i \leq n$, or $F^{x_i} = F^{x_{i+1}} = \cdots = F^{x_{i+m}}$ and so $l_i = l_{i+1} = \cdots = l_{i+m}$, where $m = l_i - N_i(H_f) > 0$.*

Proof. Let H_f be a quasi union hyper K-algebra and there is an $i, 0 \leq i \leq n$, such that $l_i > N_i(H_f)$. Then $m = l_i - N_i(H_f) > 0$ and by Theorem 2.2(iii), there are m elements, $x_{i+k} \in F^{x_i}$, $1 \leq k \leq m$, where $x_{i+k} \in F^{x_{i+k}}$ and $F^{x_{i+k}} \subseteq F^{x_i}$. Since $l_i \leq l_{i+k}$, we get that $F^{x_{i+k}} = F^{x_i}$, for all $1 \leq k \leq m$, and $l_i = l_{i+1} = \cdots = l_{i+m}$.

Corollary 3.12. *If H_f be a quasi union hyper K-algebra of type $(l_0, l_1, l_2, \cdots, l_n)$, then $l_0 = l_1 = \cdots = l_{l_0-1}$.*

Proof. Since $N_0(H_f) = 1$, it follows by Theorem 3.11.

Corollary 3.13. *If H_f be a quasi union hyperstructure, and $l_i \neq l_{i+k}, 0 \leq i \leq (n-1)$, for some k, $1 \leq k \leq l_i - (i+1)$. Then H_f is not quasi union hyper K-algebra.*

Proof. Since $N_i(H_f) \leq i + 1$, it follows by Theorem 3.11.

Theorem 3.14. *If H_f be a quasi union hyperstructure, and $l_n > N_n(H_f)$. Then H_f is not quasi union hyper K-algebra.*

Proof. If H_f is a quasi union hyper K-algebra, then by Theorem 2.2(iii) form $x_i \in F^{x_n}$ we get that $x_i \in F^{x_i}$. Hence $l_n = N_n(H_f)$, which is a contradiction to hypothesis.

Example 3.15. There are not any quasi union hyper K-algebras on $H_4 = \{0, 1, 2, 3, 4\}$ of types
i) (1,1,1,2,4), because by Theorem 3.14, $l_4 = 4 > N_4(H_f) \leq 3$.
ii) (3,3,4,5,5), because by Corollary 3.12, $l_2 = 4 \neq l_3 = 3$.
(iii) (1,3,4,4,4), because by Corollary 3.13, $l_1 - 2 = 3 - 2 = 1$ and $l_1 \neq l_2$.

Theorem 3.16. *Let $\sim$ be a co-type relation on $\mathcal{H}_n$. Then* $|\mathcal{H}_n/\sim| \leq \begin{pmatrix} 2n-1 \\ n-1 \end{pmatrix}$.

Proof. Since any type $(l_0, \cdots, l_{n-1})$ is a selection n objects from $\{1, \cdots, n\}$ in which repetitions are permitted, then the number of these combinations is $\begin{pmatrix} 2n-1 \\ n-1 \end{pmatrix}$. Since by Example 3.15, there is at least a selection such that it is not a representative of a quasi union hyper K-algebra, hence $|\mathcal{H}_n/\sim| \leq \begin{pmatrix} 2n-1 \\ n-1 \end{pmatrix}$.

4 classification of quasi union hyper K-algebras of order $\leq$ 5.

In this section we determine non-isomorphic quasi union hyper K-algebra of order less than or equal to 5 . Finally, we give a conjecture about the number of non-isomorphic quasi union hyper k-algebra of order n.

Theorem 4.1. *Let $H_i = \{0, 1, 2, \cdots, i\}, 0 \leq i \leq 4$. Then there are*
(i) 1 non-isomorphic quasi union hyper K-algebra on H_0,
(ii) 3 non-isomorphic quasi union hyper K-algebras on H_1,
(iii) 9 non-isomorphic quasi union hyper K-algebras on H_2,
(iv) 30 non-isomorphic quasi union hyper K-algebras on H_3,
(v) 107 non-isomorphic quasi union hyper K-algebras on H_4 .

Proof. (i): It is clear that the only hyper K-algebra on $H_0 = \{0\}$ is as follows:

$\circ$	0
0	$\{0\}$

.

(ii): By using Theorems 2.2 and Remark 3.8, we conclude that there are 3 non-isomorphic quasi union hyper K-algebras of type (1,1),(1,2) and (2,2) on $H_1 = \{0, 1\}$ as follows:

$\circ_1$	0	1
0	{0}	{0}
1	{1}	{0}

$\circ_2$	0	1
0	{0}	{0}
1	{1}	{0,1}

$\circ_3$	0	1
0	{0,1}	{0}
1	{1}	{0,1}.

(iii): By using Remark 3.8, there are 9 non-isomorphic quasi union hyper K-algebras on $H_2 = \{0, 1, 2\}$ as follows:

1 -Type (1,1,1). There is one quasi union hyper K-algebra of type (1,1,1) as follows:

$\circ_1$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{1}
2	{2}	{2}	{0}

2 -Type (1,1,2). There is one quasi union hyper K-algebra of type (1,1,2) as follows:

$\circ_2$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{1}
2	{2}	{2}	{0,2}

3,4- Type (1,2,2). There are two non-isomorphic quasi union hyper K-algebras of type (1,2,2). Since

$\circ_3$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0,1}	{1}
2	{2}	{2}	{0,1}

and

$\circ_3'$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0,2}	{1}
2	{2}	{2}	{0,2}

are isomorphic and by Corollary 3.4(iv)the following quasi union hyper K-algebra is not isomorphic to them.

$\circ_4$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0,1}	{1}
2	{2}	{2}	{0,2}

5- Type (1,2,3). By Theorem 2.2(iii), there is one quasi union hyper K-algebra of type (1,2,3) as follows:

$\circ_5$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0,1}	{1}
2	{2}	{2}	{0,1,2}.

6- Type (1,3,3). By Theorem 2.2(iii), there is one quasi union hyper K-algebra of type (1,3,3) as follows:

$\circ_6$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0,1,2}	{1}
2	{2}	{2}	{0,1,2}.

7- Type (2,2,2). Since $F^0 \subseteq F^x, x = 1, 2$, then $F^0 = F^1 = F^2$. Thus we have two quasi union hyper K-algebras of type (2,2,2) as follows:

$\circ_7$	0	1	2
0	{0,1}	{0}	{0}
1	{1}	{0,1}	{1}
2	{2}	{2}	{0,1}

and

$\circ_7'$	0	1	2
0	{0,2}	{0}	{0}
1	{1}	{0,2}	{1}
2	{2}	{2}	{0,2}.

But by Theorem 3.9 they are isomorphic together. So there is only one non-isomorphic quasi union hyper K-algebra of type (2,2,2).

8- Type (2,2,3). Since $F^0 \subseteq F^x, x = 1, 2$, by Theorem 2.2(iii), there is one quasi union hyper K-algebra of type (2,2,3) as follows:

$\circ_9$	0	1	2
0	{0,1}	{0}	{0}
1	{1}	{0,1}	{1}
2	{2}	{2}	{0,1,2}.

9- Type (3,3,3). Since $F^0 \subseteq F^x, x = 1, 2$, then $F^0 = F^1 = F^2$ and there is one quasi union hyper K-algebra of type (3,3,3) as follows:

$\circ_9$	0	1	2
0	{0,1,2}	{0}	{0}
1	{1}	{0,1,2}	{1}
2	{2}	{2}	{0,1,2}.

(iv): **Note**: As definition of quasi union hyper K-algebra, any quasi union hyper K-algebra H is determined completely if and only if $A^x = x \circ x$ are determined for all $x \in H$. Therefore in the remainder of this section, for simplicity of notation, we denote any quasi union hyper K-algebra H by $\{A^x : x \in H\}$. For example, a quasi union hyper K-algebra of $\mathcal{H}_3$ is denoted by$\{A^0, A^1, A^2, A^3\}$.

By considering Theorem 2.2(iii) and Corollary 3.4, we show that there are 30 non-isomorphic quasi union hyper K-algebras of order 4 as follows:

Table of 30 non-isomorphic quasi union hyper K-algebras of order 4

Type	Number	A^0	A^1	A^2	A^3
(1,1,1,1)	1	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
(1,1,1,2)	2	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,3\}$
(1,1,1,3)	By Theorem 3.14, there is not of this type				
(1,1,1,4)	By Theorem 3.14, there is not of this type				
(1,1,2,2)	3	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2\}$
	4	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3\}$
(1,1,2,3)	5	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2,3\}$
(1,1,2,4)	By Theorem 3.14, there is not of this type				
(1,1,3,3)	6	$\{0\}$	$\{0\}$	$\{0,2,3\}$	$\{0,2,3\}$
(1,1,3,4)	By Theorem 3.11, there is not of this type.				
(1,1,4,4)	By Theorem 3.11, there is not of this type.				

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Table of 30 non-isomorphic quasi union hyper K-algebras of order4

Type	Number	A^0	A^1	A^2	A^3
(1,2,2,2)	7	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
	8	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,3\}$
	9	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$
(1,2,2,3)	10	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$
	11	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,2\}$
	12	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,2,3\}$
(1,2,2,4)	13	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,2,3\}$
(1,2,3,3)	14	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$
	15	$\{0\}$	$\{0,1\}$	$\{0,2,3\}$	$\{0,2,3\}$
	16	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$
(1,2,3,4)	17	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$
(1,2,4,4)	18	$\{0\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
(1,3,3,3)	19	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
(1,3,3,4)	20	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$
(1,3,4,4)	By Corollary 3.13, there is not of this type.				
(1,4,4,4)	21	$\{0\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
(2,2,2,2)	22	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
(2,2,2,3)	23	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$
(2,2,2,4)	By Theorem 3.11, there is not of this type.				
(2,2,3,3)	24	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$
	25	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$
(2,2,3,4)	26	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$
(2,2,4,4)	27	$\{0,1\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
(2,3,3,3)	By Corollary 3.12, there is not of this type.				
(2,3,3,4)	By Corollary 3.12, there is not of this type.				
(2,3,4,4)	By Corollary 3.12, there is not of this type.				
(2,4,4,4)	By Corollary 3.12, there is not of this type.				
(3,3,3,3)	28	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
(3,3,3,4)	29	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$
(3,3,4,4)	By Corollary 3.12, there is not of this type.				
(3,4,4,4)	By Corollary 3.12, there is not of this type.				
(4,4,4,4)	30	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$

(v): By a similar arguments as above we can show that there are 107 non-isomorphic quasi union hyper K-algebra of order 5.

Table of 107 non-isomorphic quasi union hyper K-algebras of order 5

Type	Number	A^0	A^1	A^2	A^3	A^4
(1,1,1,1,1)	1	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$
(1,1,1,1,2)	2	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,4\}$
(1,1,1,1,3)	By Theorem 3.11, there is not of this type					
(1,1,1,1,4)	By Theorem 3.11, there is not of this type					

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Table of 107 non-isomorphic quasi union hyper K-algebras of order 5

Type	Number	A^0	A^1	A^2	A^3	A^4
(1,1,1,2,2)	3	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,3\}$	$\{0,3\}$
	4	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,3\}$	$\{0,4\}$
(1,1,1,2,3)	5	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,3\}$	$\{0,3,4\}$
(1,1,1,2,4)	By Theorem 3.11, there is not of this type					
(1,1,1,3,3)	6	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,3,4\}$	$\{0,3,4\}$
(1,1,1,3,4)	By Theorem 3.11, there is not of this type.					
(1,1,1,4,4)	By Theorem 3.11, there is not of this type.					
(1,1,2,2,2)	7	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2\}$	$\{0,2\}$
	8	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2\}$	$\{0,4\}$
	9	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3\}$	$\{0,4\}$
(1,1,2,2,3)	10	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2\}$	$\{0,2,4\}$
	11	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3\}$	$\{0,2,3\}$
	12	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3\}$	$\{0,2,4\}$
(1,1,2,2,4)	13	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3\}$	$\{0,2,3,4\}$
(1,1,2,3,3)	14	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2,3\}$	$\{0,2,3\}$
	15	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3,4\}$	$\{0,3,4\}$
	16	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2,3\}$	$\{0,2,4\}$
(1,1,2,3,4)	17	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,3\}$	$\{0,2,3,4\}$
(1,1,2,4,4)	18	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0,2,3,4\}$	$\{0,2,3,4\}$
(1,1,3,3,3)	19	$\{0\}$	$\{0\}$	$\{0,2,3\}$	$\{0,2,3\}$	$\{0,2,3\}$
(1,1,3,3,4)	20	$\{0\}$	$\{0\}$	$\{0,2,3\}$	$\{0,2,3\}$	$\{0,2,3,4\}$
(1,1,3,4,4)	By Theorem 3.11, there is not of this type.					
(1,1,4,4,4)	21	$\{0\}$	$\{0\}$	$\{0,2,3,4\}$	$\{0,2,3,4\}$	$\{0,2,3,4\}$
(1,2,2,2,2)	22	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
	23	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1\}$	$\{0,1\}$
	24	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,2\}$	$\{0,1\}$
	25	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,1\}$
	26	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,4\}$
(1,2,2,2,3)	27	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,4\}$
	28	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
	29	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1\}$	$\{0,1,4\}$
	30	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,1,2\}$
	31	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,1,4\}$
(1,2,2,2,4)	32	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,1,2,3\}$
	33	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,1,2,4\}$
	34	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1\}$	$\{0,1,2,4\}$
(1,2,2,2,5)	35	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,3\}$	$\{0,1,2,3,4\}$
(1,2,2,3,3)	36	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
	37	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,3\}$	$\{0,1,3\}$
	38	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,3\}$	$\{0,1,4\}$
	39	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$	$\{0,1,3\}$
	40	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$	$\{0,1,4\}$
(1,2,2,3,4)	41	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,3\}$	$\{0,1,2,3\}$
	42	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$	$\{0,1,3,4\}$

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Table of 107 non-isomorphic quasi union hyper K-algebras of order 5

Type	Number	A^0	A^1	A^2	A^3	A^4
	43	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,3\}$	$\{0,1,2,4\}$
(1,2,2,3,5)	44	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,3\}$	$\{0,1,2,3,4\}$
(1,2,2,4,4)	45	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	46	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3,4\}$	$\{0,1,3,4\}$
	47	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,3,4\}$	$\{0,1,3,4\}$
(1,2,2,4,5)	48	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(1,2,2,5,5)	49	$\{0\}$	$\{0,1\}$	$\{0,2\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(1,2,3,3,3)	50	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
	51	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,4\}$
	52	$\{0\}$	$\{0,1\}$	$\{0,2,3\}$	$\{0,2,3\}$	$\{0,2,3\}$
	53	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,4\}$
(1,2,3,3,4)	54	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,2,3\}$
	55	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,2,4\}$
	56	$\{0\}$	$\{0,1\}$	$\{0,2,3\}$	$\{0,2,3\}$	$\{0,2,3,4\}$
(1,2,3,3,5)	57	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,2,3,4\}$
	58	$\{0\}$	$\{0,1\}$	$\{0,2,3\}$	$\{0,2,3\}$	$\{0,1,2,3,4\}$
(1,2,3,4,4)	59	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	60	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,4\}$
(1,2,3,4,5)	61	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(1,2,3,5,5)	62	$\{0\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(1,2,4,4,4)	63	$\{0\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	64	$\{0\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,4\}$
(1,2,4,4,5)	65	$\{0\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(1,2,4,5,5)	By Corollary 3.13, there is not of this type.					
(1,2,5,5,5)	66	$\{0\}$	$\{0,1\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(1,3,3,3,3)	67	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
	68	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,3,4\}$	$\{0,3,4\}$
(1,3,3,3,4)	69	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,4\}$
(1,3,3,3,5)	By Theorem 2.2(iii), there is not of this type.					
(1,3,3,4,4)	70	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	71	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,4\}$
(1,3,3,4,5)	72	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(1,3,3,5,5)	73	$\{0\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(1,4,4,4,4)	74	$\{0\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	75	$\{0\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,4\}$
(1,4,4,4,5)	76	$\{0\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(1,4,4,5,5)	By Corollary 3.13, there is not of this type.					
(1,4,5,5,5)	By Corollary 3.13, there is not of this type.					
(1,5,5,5,5)	77	$\{0\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(2,2,2,2,2)	78	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
(2,2,2,2,3)	79	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,4\}$
(2,2,2,2,4)	By Corollary 3.13, there is not of this type.					
(2,2,2,2,5)	By Corollary 3.13, there is not of this type.					
(2,2,2,3,2)	80	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$	$\{0,1,3\}$

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Table of 107 non-isomorphic quasi union hyper K-algebras of order 5						
Type	Number	A^0	A^1	A^2	A^3	A^4
	81	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$	$\{0,1,4\}$
(2,2,2,3,4)	82	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3\}$	$\{0,1,3,4\}$
(2,2,2,3,5)	By Corollary 3.13, there is not of this type.					
(2,2,2,4,4)	83	$\{0\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1,3,4\}$	$\{0,1,3,4\}$
(2,2,2,4,5)	By Corollary 3.13, there is not of this type.					
(2,2,3,3,3)	84	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
(2,2,3,3,3)	85	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,4\}$
	86	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,4\}$
(2,2,3,3,4)	87	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,4\}$
	88	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,2,3\}$
	89	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,2,4\}$
(2,2,3,3,5)	90	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3\}$	$\{0,1,2,3,4\}$
(2,2,3,4,4)	91	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	92	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,4\}$
	93	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,3,4\}$	$\{0,1,3,4\}$
(2,2,3,4,5)	94	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(2,2,3,5,5)	95	$\{0,1\}$	$\{0,1\}$	$\{0,1,2\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(2,2,4,4,4)	96	$\{0,1\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
(2,2,4,4,5)	97	$\{0,1\}$	$\{0,1\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(2,2,4,5,5)	By Corollary 3.13, there is not of this type.					
(2,2,5,5,5)	98	$\{0,1\}$	$\{0,1\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(3,3,3,3,3)	99	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
(3,3,3,3,4)	100	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,4\}$
(3,3,3,3,5)	By Corollary 3.13, there is not of this type.					
(3,3,3,4,4)	101	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
	102	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,4\}$
(3,3,3,4,5)	103	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(3,3,3,5,5)	104	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$
(3,3,4,4,4)	By Corollary 3.13, there is not of this type.					
(3,3,4,4,5)	By Corollary 3.13, there is not of this type.					
(3,3,4,5,5)	By Corollary 3.13, there is not of this type.					
(3,3,5,5,5)	By Corollary 3.13, there is not of this type.					
(3,4,4,4,4)	By Corollary 3.13, there is not of this type.					
(3,4,4,4,5)	By Corollary 3.13, there is not of this type.					
(3,4,4,5,5)	By Corollary 3.13, there is not of this type.					
(3,4,5,5,5)	By Corollary 3.13, there is not of this type.					
(3,5,5,5,5)	By Corollary 3.13, there is not of this type.					
(4,4,4,4,4)	105	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$
(4,4,4,4,5)	106	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$
(4,4,4,5,5)	By Corollary 3.13, there is not of this type.					
(4,4,5,5,5)	By Corollary 3.13, there is not of this type.					
(4,5,5,5,5)	By Corollary 3.13, there is not of this type.					
(5,5,5,5,5)	107	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$	$\{0,1,2,3,4\}$

Now we give a formula, as a guess, for the number of non-isomorphic quasi union hyper

K-algebras of order n. Let $T_n, n \in \mathbb{N}$, be the number of non-isomorphic quasi union hyper K-algebras of order n. Then by Theorem 4.1, we have

$$T_1 = 1,$$

$$T_2 = 3,$$

$$T_3 = 9 = 2T_2 - T_1 + 4 = 2T_2 - T_1 + \binom{4}{1},$$

$$T_4 = 30 = 2T_3 - T_2 + 15 = 2T_3 - T_2 + \binom{6}{2},$$

$$T_5 = 107 = 2T_4 - T_3 + 56 = 2T_4 - T_3 + \binom{8}{3}.$$

These relations lead us to the following conjecture:

Conjecture 4.2. *Let $T_n, n \in \mathbb{N}$, be the number of non-isomorphic quasi union hyper K-algebras of order n. Then $T_1 = 1, T_2 = 3$ and for $n > 2$*

$$T_n = 2T_{n-1} - T_{n-2} + \binom{2n-2}{n-2}.$$

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