Admissibility under the LINEX loss function in non-regular case

Hidekazu Tanaka

Received November 5, 2009; revised September 2, 2010

Abstract. In the statistical estimation theory, sufficient conditions for admissibility of estimators have been discussed mostly under quadratic loss function. Recently, under the LINEX loss function, sufficient conditions were developed in some families of distributions. However, they do not include some typical non-regular distributions so that the dimension of the minimal sufficient statistic is two. The purpose of this paper is to give sufficient conditions for admissibility of estimators in such non-regular distributions under the LINEX loss function. Some examples are also given.

1 Introduction We consider an estimation problem of a real-valued function of unknown parameter θ based on samples obtained from a probability distribution with unknown parameter. Under quadratic loss function, sufficient conditions for linear estimators to be admissible has been investigated by many authors (Karlin [3], Ghosh and Meeden [1], Ralescu and Ralescu [10], Hoffmann [2]). Further, the general theory for admissibility was derived by Pulskamp and Ralescu [9] for regular case, and Sinha and Gupta [15], Kim and Meeden [4] for non-regular case.

On the other hand, under the LINEX loss function, admissibility results were limited to whether linear estimator is admissible or not (Rojo [11], Sadooghi-Alvandi and Nematollahi [14], Kuo and Dey [5], Sadooghi-Alvandi [13], Pandey [6], Parsian and Farsipour [7]). Here, the LINEX loss function is defined by

$$L(\theta,\delta) = b\left\{e^{a(\delta-g(\theta))} - a(\delta-g(\theta)) - 1\right\}, \tag{1}$$

for an estimator δ of function $g(\theta)$ to be estimated, where $a \neq 0$ and $b > 0$ (Varian [18], Zellner [19]). It should be noted that the LINEX loss function is regarded as an extension of quadratic loss function. Recently, sufficient conditions for admissibility has been developed for regular case by Tanaka [16]. Similar result was obtained by Tanaka [17] for the non-regular case when the dimension of the minimal sufficient statistic is one, which includes a uniform distribution $U(0,\theta)$. However, it does not include some typical distributions such as $U(\theta,\theta+1)$, which will be treated in Section 4. The purpose of this paper is to give sufficient conditions for generalized Bayes estimator (GBE) to be admissible under the LINEX loss function when the dimension of the minimal sufficient statistic is two.

This paper is organized as follow. In Section 2, we give preliminaries. In Section 3, we give sufficient conditions for GBE to be admissible under the LINEX loss function and its corollaries. In Section 4, we give some examples. Finally, we give proof of the theorem in Section 5.

2000 *Mathematics Subject Classification.* Primary 62C15, 62F10; Secondary 62F15.
Key words and phrases. LINEX loss function, admissibility, non-regular distribution.

2 Preliminaries Suppose that a random vector $X := (X_1, X_2)$ is distributed according to a probability distribution, and its probability density function (p.d.f.) with respect to the Lebesgue measure is given by

$$f(x,\theta) = \begin{cases} q(\theta)r(x) & (x \in \mathcal{X}_\theta), \\ 0 & (\text{otherwise}), \end{cases} \tag{2}$$

where $x := (x_1, x_2)$, $q(\theta) > 0$, $r(x) > 0$ and $\mathcal{X}_\theta := \{x \in \mathbb{R}^2 | \alpha(\theta) < x_1 < x_2 < \beta(\theta)\}$ for some functions $\alpha(\theta)$ and $\beta(\theta)$. Here, we assume that both $\alpha(\theta)$ and $\beta(\theta)$ are strictly increasing. This setup is motivated by the case when the dimension of sufficient statistic is two. Then we consider the admissibilities of estimators of function $g(\theta)$ based on X under the LINEX loss function (1). We can assume $b = 1$ without loss of generality. By the general theory about admissibility (Sacks [12]), it is enough to focus on GBE w.r.t. improper priors. Let $\pi(\theta)$ be an improper prior density of θ. Here, we assume $\pi(\theta) > 0$ for all $\theta \in \Theta$. We further restrict estimators to the class

$$\Delta := \{\delta | \text{(A1) and (A2) are satisfied}\}, \tag{3}$$

where

(A1) $E_\theta|\delta(X)| < \infty$ and $E_\theta[e^{a\delta(X)}] < \infty$ for all $\theta \in \Theta$,

(A2) $\int_u^v E_\theta|\delta(X) - g(\theta)|\pi(\theta)d\theta < \infty$ and $\int_u^v E_\theta[e^{a(\delta(X)-g(\theta))}]\pi(\theta)d\theta < \infty$ for all $u < v$ $(u, v \in \Theta)$.

Under these conditions, the GBE of $g(\theta)$ w.r.t. $\pi(\theta)$ is given by $\delta_\pi(X)$, where

$$\delta_\pi(x) = -\frac{1}{a} \log \frac{\int_\Theta e^{-ag(\theta)} q(\theta)\pi(\theta) I_{\Theta_x}(\theta) d\theta}{\int_\Theta q(\theta)\pi(\theta) I_{\Theta_x}(\theta) d\theta} \tag{4}$$

for all $x \in \mathcal{X} := \{x \in \mathbb{R}^2 | x_1 < x_2\}$, provided that the integrals exist and are finite, where $\Theta_x := \{\theta \in \Theta | \beta^{-1}(x_2) < \theta < \alpha^{-1}(x_1)\}$.

3 Sufficient conditions for generalized Bayes estimator to be admissible In this section, we give sufficient conditions for GBE (4) to be admissible under the LINEX loss function. The following is the main result.

Theorem 1 Suppose that $\delta_\pi(X) \in \Delta$ which is defined in (3). For $\theta \in \Theta$ and $x \in \mathcal{X}_\theta$, put

$$F(x,\theta) := \int_{\underline{\theta}}^{\theta} (e^{-a\delta_\pi(x)} - e^{-ag(t)}) q(t)\pi(t) I_{\Theta_x}(t) dt,$$

and

$$\gamma(\theta) := \frac{e^{ag(\theta)}}{q(\theta)\pi(\theta)} \int_{\mathbb{R}^2} F^2(x,\theta) e^{a\delta_\pi(x)} r(x) I_{\mathcal{X}_\theta}(x) dx,$$

where $I_A(x)$ is the indicator function of set A. If $\gamma(\theta) < \infty$ for all $\theta \in \Theta$ and there exists $d \in \Theta$ such that

$$\lim_{c \to \overline{\theta}} \int_d^c \frac{d\theta}{\gamma(\theta)} = \lim_{c \to \underline{\theta}} \int_c^d \frac{d\theta}{\gamma(\theta)} = \infty, \tag{5}$$

then $\delta_\pi(X)$ is Δ-admissible for $g(\theta)$ under the LINEX loss function.

The proof is given in Section 5.

The sufficient condition (5) in Theorem 1 is somewhat complicated. By adding some conditions on $g(\theta)$, we give two corollaries. Now we note from (4) that $F(x,\theta)$ is expressed as

$$F(x,\theta)=\frac{1}{\int_{\underline{\theta}}^{\overline{\theta}} q(u)\pi(u)I_{\Theta_x}(u)du}\int_{\theta}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta}(e^{-ag(s)}-e^{-ag(t)})q(s)\pi(s)I_{\Theta_x}(s)q(t)\pi(t)I_{\Theta_x}(t)dtds.$$

Corollary 1 Suppose that $g(\theta)$ is bounded and $\delta_\pi(X)\in\Delta$. For $\theta\in\Theta$ and $x\in\mathcal{X}_\theta$, put

$$\tilde{F}(x,\theta):=\frac{1}{\int_{\underline{\theta}}^{\overline{\theta}} q(u)\pi(u)I_{\Theta_x}(u)du}\int_{\theta}^{\overline{\theta}} q(s)\pi(s)I_{\Theta_x}(s)ds\int_{\underline{\theta}}^{\theta} q(t)\pi(t)I_{\Theta_x}(t)dt,$$

and

$$\tilde{\gamma}(\theta):=\frac{1}{q(\theta)\pi(\theta)}\int_{\mathbb{R}^2}\tilde{F}^2(x,\theta)r(x)I_{\mathcal{X}_\theta}(x)dx.$$

If $\tilde{\gamma}(\theta)<\infty$ for all $\theta\in\Theta$ and there exists $d\in\Theta$ such that

$$\lim_{c\to\overline{\theta}}\int_d^c\frac{d\theta}{\tilde{\gamma}(\theta)}=\lim_{c\to\underline{\theta}}\int_c^d\frac{d\theta}{\tilde{\gamma}(\theta)}=\infty,$$

then $\delta_\pi(X)$ is Δ-admissible for $g(\theta)$ under the LINEX loss function.

The proof is omitted since it can be easily obtained.

Next, we consider the class of prior density functions treated by Sinha and Gupta [15] and Kim and Meeden [4]. Suppose that $g(\theta)$ is strictly increasing and differentiable. Then, put

$$\pi_h(\theta)=\frac{g'(\theta)}{q(\theta)}h(g(\theta)) \tag{6}$$

for positive function $h(\cdot)$. In this case, the GBE of $g(\theta)$ w.r.t. $\pi_h(\theta)$ is given by $\delta_{\pi_h}(X)$, where

$$\delta_{\pi_h}(x)=-\frac{1}{a}\log\frac{\int_{g(\Theta)}e^{-az}h(z)I_{g(\Theta_x)}(z)dz}{\int_{g(\Theta)}h(z)I_{g(\Theta_x)}(z)dz}.$$

The next corollary is immediately obtained from Theorem 1.

Corollary 2 Suppose that $g(\theta)$ is strictly increasing and differentiable, and $\delta_{\pi_h}(X)\in\Delta$. For $\theta\in\Theta$ and $x\in\mathcal{X}_\theta$, put

$$F_h(x,\theta):=\int_{g(\underline{\theta})}^{g(\theta)}(e^{-a\delta_{\pi_h}(x)}-e^{-az})h(z)I_{g(\Theta_x)}(z)dz,$$

and

$$\gamma_h(\theta):=\frac{e^{ag(\theta)}}{g'(\theta)h(g(\theta))}\int_{\mathbb{R}^2}F_h^2(x,\theta)e^{a\delta_{\pi_h}(x)}r(x)I_{\mathcal{X}_\theta}(x)dx.$$

If $\gamma_h(\theta)<\infty$ for all $\theta\in\Theta$ and there exists $d\in\Theta$ such that

$$\lim_{c\to\overline{\theta}}\int_d^c\frac{d\theta}{\gamma_h(\theta)}=\lim_{c\to\underline{\theta}}\int_c^d\frac{d\theta}{\gamma_h(\theta)}=\infty,$$

then $\delta_{\pi_h}(X)$ is Δ-admissible for $g(\theta)$ under the LINEX loss function.

Of course, same discussion for strictly decreasing function $g(\theta)$ is possible. But we omit it since it is obtained by similar argument. Also, it can be shown that these results are essentially regarded as extensions of Kim and Meeden [4] under suitable conditions (see Tanaka [16]). In this paper, we do not discuss the detail of the relation.

4 Examples In this section, we give some typical examples. In Examples 1,2 and 3, we treat a location parameter family of distributions. In Example 4, we treat a scale parameter family of distributions.

Example 1 Suppose that $Y_1, \ldots, Y_n$ are i.i.d. random variables according to a uniform distribution $U(\theta, \theta+1)$, where $\theta (\in \mathbb{R})$ is unknown. Then, the p.d.f. of the sufficient statistic $(X_1, X_2) = (\min_{1 \le i \le n} Y_i, \max_{1 \le i \le n} Y_i)$ is given by (2), where $q(\theta) = 1$, $r(x) = n(n-1)(x_2 - x_1)^{n-2}$ and $\mathcal{X}_\theta = \{x \in \mathbb{R}^2 | \theta < x_1 < x_2 < \theta + 1\}$. The GBE of θ w.r.t. $\pi_\eta(\theta) = e^{\eta\theta}$ $(\eta \in \mathbb{R})$ is given by

$$\delta_{\pi_\eta}(x) = \begin{cases} -\frac{1}{a} \log \left[\frac{e^{-ax_1} - e^{-a(x_2-1)}}{a(x_2 - x_1 - 1)} \right] & (\eta = 0), \\ -\frac{1}{a} \log \left[\frac{a(x_1 - x_2 + 1)}{e^{ax_1} - e^{a(x_2-1)}} \right] & (\eta = a), \\ -\frac{1}{a} \log \left[\frac{\eta}{\eta - a} \frac{e^{(\eta-a)x_1} - e^{(\eta-a)(x_2-1)}}{e^{\eta x_1} - e^{\eta(x_2-1)}} \right] & (\eta \neq 0, a). \end{cases}$$

Here, we note that these estimators are location invariant, that is, $\delta_{\pi_\eta}(x_1 + \theta, x_2 + \theta) = \delta_{\pi_\eta}(x_1, x_2)$ for all $(x_1, x_2) \in \mathbb{R}^2$ and for all $\theta \in \mathbb{R}$. Further, we can easily obtain

$$r(x_1 + \theta, x_2 + \theta) = r(x_1, x_2), \quad F(x_1 + \theta, x_2 + \theta, \theta) = e^{(\eta - a)\theta} F(x_1, x_2, 0).$$

Hence, we see that

$$\gamma(\theta) = e^{\eta\theta} \int_{\mathbb{R}^2} F^2(x, 0) e^{a\delta_{\pi_\eta}(x)} r(x) I_{\mathcal{X}_0}(x) dx < \infty,$$

and consequently, $\delta_{\pi_\eta}(X)$ is Δ-admissible for $\eta = 0$ from Theorem 1. We remark that for $\eta \neq 0$ Theorem 1 can not tell whether $\delta_{\pi_\eta}(X)$ is Δ-admissible or not. However, we see that $\delta_{\pi_0}(X)$ dominates $\delta_{\pi_\eta}(X)$ for $\eta \neq 0$, since $\delta_{\pi_0}(X)$ is the best invariant estimator (Parsian, Farsipour and Nematollahi [8], page 103).

Example 2 In Example 1, consider the estimation problem of

$$g(\theta) = P_\theta(Y_1 \le 1) = \begin{cases} 1 & (\theta < 0), \\ 1 - \theta & (0 < \theta < 1), \\ 0 & (1 < \theta). \end{cases}$$

Suppose that the prior distribution of θ is given by $\pi_\eta(\theta) = e^{\eta\theta}$ $(\eta \in \mathbb{R})$. Of course, in this case, $g(\theta)$ is bounded. So, we use Corollary 1. It is easy to derive that

$$\tilde{\gamma}(\theta) = \begin{cases} \int_{\mathcal{X}_0} \tilde{F}^2(x, 0) r(x) dx & (\eta = 0), \\ \eta^{-2} e^{-\eta\theta} & (\eta \neq 0). \end{cases}$$

¿From Corollary 1, we see that $\delta_{\pi_0}(X)$ is Δ-admissible under the LINEX loss function.

Example 3 Suppose that $Y_1, \ldots, Y_n$ are i.i.d. random variables according to the p.d.f.

$$p(y,\theta) = \begin{cases} C_0 e^{y-\theta} & (\theta < y < \theta+1), \\ 0 & \text{(otherwise)}, \end{cases}$$

where $\theta(\in \mathbb{R})$ is unknown and $C_0 := 1/(e-1)$ is the normalizing factor. Then the p.d.f. of the sufficient statistic $(X_1, X_2) = (\min_{1\le i\le n} Y_i, \max_{1\le i\le n} Y_i)$ is given by (2), where $q(\theta) = C_0^n e^{-n\theta}$, $r(x) = n(n-1)e^{x_1+x_2}(e^{x_2}-e^{x_1})^{n-2}$ and $\mathcal{X}_\theta = \{x \in \mathbb{R}^2 | \theta < x_1 < x_2 < \theta+1\}$. The GBE of θ w.r.t. $\pi_\eta(\theta) = e^{\eta\theta}$ $(\eta \in \mathbb{R})$ is given by

$$\delta_{\pi_\eta}(x) = \begin{cases} -\frac{1}{a}\log\left\{\frac{n-\eta}{n-\eta+a}\frac{e^{-(n-\eta+a)x_1}-e^{-(n-\eta+a)(x_2-1)}}{e^{-(n-\eta)x_1}-e^{-(n-\eta)(x_2-1)}}\right\} & (\eta \neq n, n+a), \\ -\frac{1}{a}\log\left\{\frac{a(x_1-x_2+1)}{e^{ax_1}-e^{a(x_2-1)}}\right\} & (\eta = n+a), \\ -\frac{1}{a}\log\left\{-\frac{1}{a}\frac{e^{-ax_1}-e^{-a(x_2-1)}}{x_1-x_2+1}\right\} & (\eta = n). \end{cases}$$

Of course, these estimators are location invariant. Further, by using the facts

$$r(x_1+\theta, x_2+\theta) = e^{n\theta} r(x_1,x_2), \quad F(x_1+\theta, x_2+\theta, \theta) = e^{-(n-\eta+a)\theta}F(x_1,x_2,0),$$

we get

$$\gamma(\theta) = \frac{e^{\eta\theta}}{C_0^n}\int_{\mathbb{R}^2} F^2(x,0)e^{a\delta_{\pi_\eta}(x)}r(x)I_{\mathcal{X}_0}(x)dx < \infty.$$

Therefore, Theorem 1 shows that $\delta_{\pi_\eta}(X)$ is Δ-admissible when $\eta = 0$. By the same reason as Example 1, $\delta_{\pi_0}(X)$ dominates $\delta_{\pi_\eta}(X)$ for $\eta \neq 0$.

Example 4 Suppose that $Y_1, \ldots, Y_n$ are i.i.d. random variables according to the p.d.f.

$$p(y,\theta) = \begin{cases} \frac{1}{\theta} & (\xi\theta < y < (\xi+1)\theta), \\ 0 & \text{(otherwise)}, \end{cases}$$

where $\xi \in (0,\infty)$ is known and $\theta \in (0,\infty)$ is unknown. In this case, the p.d.f. of the sufficient statistic $(X_1, X_2) = (\min_{1\le i\le n} Y_i, \max_{1\le i\le n} Y_i)$ is given by (2), where $q(\theta) = \theta^{-n}$, $r(x) = n(n-1)(x_2-x_1)^{n-2}$ and $\mathcal{X}_\theta = \{x \in \mathbb{R}^2 | \xi\theta < x_1 < x_2 < (\xi+1)\theta\}$. Consider the estimation problem of $g(\theta) = \log\theta$. Let the prior density of θ be $\pi_h(\theta)$ in (6) for $h(z) = e^{\varepsilon z}$ $(\varepsilon \in \mathbb{R})$. Then, the GBE of $\log\theta$ is given by

$$\delta_{\pi_h}(x) = \begin{cases} -\frac{1}{a}\log\left[\frac{\{x_2/(\xi+1)\}^{-a}-(x_1/\xi)^{-a}}{a\log\{(\xi+1)x_1/(\xi x_2)\}}\right] & (\varepsilon = 0), \\ -\frac{1}{a}\log\left[\frac{a\log\{(\xi+1)x_1/(\xi x_2)\}}{(x_1/\xi)^a-\{x_2/(\xi+1)\}^a}\right] & (\varepsilon = a), \\ -\frac{1}{a}\log\left[\frac{\varepsilon}{\varepsilon-a}\frac{(x_1/\xi)^{\varepsilon-a}-\{x_2/(\xi+1)\}^{\varepsilon-a}}{(x_1/\xi)^{\varepsilon}-\{x_2/(\xi+1)\}^{\varepsilon}}\right] & (\varepsilon \neq 0, a). \end{cases}$$

By a direct calculation, we get $\delta_{\pi_h}(\theta x_1, \theta x_2) = \delta_{\pi_h}(x_1,x_2) + \log\theta$. Further, we can easily obtain

$$r(\theta x_1, \theta x_2) = \theta^{n-2}r(x_1,x_2), \quad F_h(\theta x_1, \theta x_2, \theta) = \theta^{\varepsilon-a}F_h(x_1,x_2,1).$$

These imply that

$$\gamma_h(\theta) = \frac{\theta^{n+\varepsilon+1}}{n(n-1)}\int_{\mathbb{R}^2} F_h^2(x,1)e^{a\delta_{\pi_h}(x)}r(x)I_{\mathcal{X}_1}(x)dx < \infty.$$

Therefore, we see that $\delta_{\pi_h}(X)$ is Δ-admissible for $\varepsilon = -n$ from Corollary 2. By considering the logarithmic transformations of $y_1, \ldots, y_n$ and θ, the problem is essentially same as Example 3. So, we see that $\delta_{\pi_h}(X)$ is not Δ-admissible for $\varepsilon \neq -n$.

5 Appendix In this section, we give the proof of Theorem 1. The proof is resemble to Tanaka [16].

Proof of Theorem 1. Suppose that there exists an estimator $\delta(X) \in \Delta$ such that

$$E_\theta[L(\theta, \delta(X))] \leq E_\theta[L(\theta, \delta_\pi(X))]$$

for all $\theta \in \Theta$. ¿From the condition (A1), we see that this is equivalent to

$$\begin{aligned} &e^{-ag(\theta)} E_\theta\left[(e^{a\delta(X)/2} - e^{a\delta_\pi(X)/2})^2\right] \\ &\quad \leq E_\theta\left[a\left(\delta(X) - \delta_\pi(X)\right) - 2e^{-ag(\theta)} e^{a\delta_\pi(X)/2}(e^{a\delta(X)/2} - e^{a\delta_\pi(X)/2})\right]. \end{aligned} \tag{7}$$

Multiplying both sides of (7) by $\pi(\theta)$, and integrating w.r.t. θ over the finite interval $[u, v] \subset \Theta$, we obtain

$$\begin{aligned} &\int_u^v E_\theta\left[(e^{a\delta(X)/2} - e^{a\delta_\pi(X)/2})^2\right] e^{-ag(\theta)}\pi(\theta)d\theta \\ &\quad \leq \int_u^v E_\theta\left[a\left(\delta(X) - \delta_\pi(X)\right) - 2e^{-ag(\theta)} e^{a\delta_\pi(X)/2}(e^{a\delta(X)/2} - e^{a\delta_\pi(X)/2})\right]\pi(\theta)d\theta. \end{aligned}$$

An application of the Fubini theorem gives

$$\begin{aligned} &\int_u^v \int_{\mathbb{R}^2} (e^{a\delta(x)/2} - e^{a\delta_\pi(x)/2})^2 r(x) I_{\mathcal{X}_\theta}(x) dx e^{-ag(\theta)}\pi(\theta)q(\theta)d\theta \\ &\quad \leq \int_{\mathbb{R}^2} r(x) I_{\mathcal{X}}(x) \int_u^v \big\{a(\delta(x) - \delta_\pi(x)) \\ &\qquad -2e^{-ag(\theta)} e^{a\delta_\pi(x)/2}(e^{a\delta(x)/2} - e^{a\delta_\pi(x)/2})\big\} q(\theta)\pi(\theta) I_{\Theta_x}(\theta) d\theta dx, \end{aligned} \tag{8}$$

which is guaranteed by (A2). Using the inequality

$$x - y \leq e^{-y}(e^x - e^y)$$

for all $x, y \in \mathbb{R}$, the right hand side in (8) is dominated by

$$\begin{aligned} &2\int_{\mathbb{R}^2} r(x) I_{\mathcal{X}}(x)(e^{a\delta(x)/2} - e^{a\delta_\pi(x)/2}) e^{a\delta_\pi(x)/2} \\ &\quad \times \int_u^v (e^{-a\delta_\pi(x)} - e^{-ag(\theta)}) q(\theta)\pi(\theta) I_{\Theta_x}(\theta) d\theta dx. \end{aligned} \tag{9}$$

Clearly, we see that $F(x, v) = 0$ for $\beta^{-1}(x_2) > v$. Also, it follows from (4) that

$$F(x, v) = \int_{\underline{\theta}}^{\overline{\theta}} (e^{-a\delta_\pi(x)} - e^{-ag(t)}) q(t)\pi(t) I_{\Theta_x}(t) dt = 0$$

for $\alpha^{-1}(x_1) < v$. Thus, we get $F(x, v) = F(x, v) I_{\mathcal{X}_v}(x)$, consequently, (9) is rewritten by

$$\begin{aligned} &2\int_{\mathbb{R}^2} r(x)(e^{a\delta(x)/2} - e^{a\delta_\pi(x)/2}) e^{a\delta_\pi(x)/2} F(x, v) I_{\mathcal{X}_v}(x) dx \\ &\quad -2\int_{\mathbb{R}^2} r(x)(e^{a\delta(x)/2} - e^{a\delta_\pi(x)/2}) e^{a\delta_\pi(x)/2} F(x, u) I_{\mathcal{X}_u}(x) dx. \end{aligned} \tag{10}$$

Put

$$T(\theta) := \int_{\mathbb{R}^2} (e^{a\delta(x)/2} - e^{a\delta_\pi(x)/2})^2 r(x) I_{\mathcal{X}_\theta}(x) dx.$$

Then, by applying the Schwarz inequality to (10) and combining (8), we have

$$\begin{aligned}&\int_u^v e^{-ag(\theta)}\pi(\theta)q(\theta)T(\theta)d\theta \\ &\quad \le 2\left\{T(v)e^{-ag(v)}q(v)\pi(v)\right\}^{1/2}\gamma^{1/2}(v) + 2\left\{T(u)e^{-ag(u)}q(u)\pi(u)\right\}^{1/2}\gamma^{1/2}(u).\end{aligned}$$

By the same argument in Tanaka [16], the proof is completed. □

ACKNOWLEDGMENTS

The author would express his appreciation to the referee. This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Young Scientists (B), 21700309, 2009.

References

[1] Ghosh, M. and Meeden, G. Admissibility of linear estimators in the one parameter exponential family. *Ann. Statist.*, **5** (1977), 772–778.

[2] Hoffmann, K. Admissibility and inadmissibility of estimators in the one-parameter exponential family. *Statistics*, **16** (1985), 327–349.

[3] Karlin, S. Admissibility for estimation with quadratic loss. *Ann. Math. Statist.*, **29** (1958), 406–436.

[4] Kim, B. H. and Meeden, G. Admissible estimation in an one parameter nonregular family of absolutely continuous distributions. *Comm. Statist. Theory Methods*, **23** (1994), 2993–3001.

[5] Kuo, L. and Dey, D. K. On the admissibility of the linear estimators of the Poisson mean using linex loss functions. *Statist. Decisions*, **8** (1990), 201–210.

[6] Pandey, B. N. Testimator of the scale parameter of the exponential distribution using linex loss function. *Comm. Statist. Theory Methods*, **26** (1997), 2191–2202.

[7] Parsian, A. and Farsipour, N. S. Estimation of parameters of exponential distribution in the truncated space using asymmetric loss function. *Statist. Papers*, **38** (1997), 423–443.

[8] Parsian, A., Farsipour, N. S. and Nematollahi, N. On the minimaxity of Pitman type estimator under a LINEX loss function. *Comm. Statist. Theory Methods*, **22** (1993), 97–113.

[9] Pulskamp, R. J. and Ralescu, D. A. A general class of nonlinear admissible estimators in the one-parameter exponential case. *J. Statist. Plann. Inference*, **28** (1991), 383–390.

[10] Ralescu, D. and Ralescu, S. A class of nonlinear admissible estimators in the one-parameter exponential family. *Ann. Statist.*, **9** (1981), 177–183.

[11] Rojo, J. On the admissibility of $c\overline{X} + d$ with respect to the LINEX loss function. *Comm. Statist. Theory Methods*, **16** (1987), 3745–3748.

[12] Sacks, J. Generalized Bayes solutions in estimation problems. *Ann. Math. Statist.*, **34** (1963), 751–768.

[13] Sadooghi-Alvandi, S. M. Estimation of the parameter of a Poisson distribution using a LINEX loss function. *Austral. J. Statist.*, **32** (1990), 393–398.

[14] Sadooghi-Alvandi, S. M. and Nematollahi, N. A note on the admissibility of $c\overline{X} + d$ relative to the linex loss function. *Comm. Statist. Theory Methods*, **18** (1989), 1871–1873.

[15] Sinha, B. K. and Gupta, A. D. Admissibility of generalized Bayes and Pitman estimates in the nonregular family. *Comm. Statist. A–Theory Methods*, **13** (1984), 1709–1721.

[16] Tanaka, H. Sufficient conditions for the admissibility under LINEX loss function in regular case. *Comm. Statist. Theory Methods*, **39** (2010), 1477–1489.

[17] Tanaka, H. Sufficient conditions for the admissibility under the LINEX loss function in non-regular case. Accepted for publication in *Statistics*.

[18] Varian, H. R. A Bayesian approach to real estate assessment. *Studies in Bayesian econometrics and statistics*, In honor of Leonard J. Savage. Eds. Fienberg, S. E. and Zellner, A. North-Holland Amsterdam (1975), 195–208.

[19] Zellner, A. Bayesian estimation and prediction using asymmetric loss functions. *J. Amer. Statist. Assoc.*, **81** (1986), 446–451.

Communicated by *Asis SenGupta*