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Neuroscientific consideration of the educational effect achieved using illustrated course materials

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Abstract. It is our pedagogical challenge to introduce various mathematical concept in an educationally acceptable way and to prepare course materials that make students understand that deeply. As described in this paper, we present some of our attempts to verify the effects of using carefully prepared course materials with high-quality graphs in collegiate education of mathematics. Through our experiment, we detected the change of students' brain activity by conducting behavioral observation and neuroimaging simultaneously. In an experiment aimed at helping students understand the concept of an exponential growth comparing exponential and power functions, we prepared several graphs for that purpose. Seven students observed the graphs while we tracked their responses. Simultaneously, we monitored their brain activities using electroencephalography (EEG). Three students altered their judgments, we found, on viewing the triggering graph. Some changes in the trend of their EEG signal were recognized while they were viewing the graph. These results of our experiments show that the use of favorable graphs as course materials might promote learners' reasoning processes.

1 Introduction According to our questionnaire survey, a major opinion of teachers of collegiate mathematics in Japan denies the necessity of using high-quality graphs as course materials[4]. However our experiences indicate that the use of graphs plays a crucial role in some classroom environments. Educators usually use various graphs edited using a popular TEX tool as course materials. Additionally, we have verified the effectiveness of using graphs by comparing the responses of students to whom we showed high-quality graphs to those of students to whom we did not do that[3]. Generation of high-quality graphs is preferably done by computer algebra system (CAS) because of its computing and programming capabilities. However, it is not always easy to handle graphical images of outputs in documents edited using TEX. For instance, some elaborations are necessary to locate generated images to suitable positions in documents and to arrange the layout of other components flexibly and in a balanced manner. Although some TEX graphic systems exist such as PStricks[7] and TikZ[8], their computing capabilities remain restricted. As a handy tool for both generation of high-quality graphs with CAS and the easy arrangement of TEX document components, we have been using KETpic, a macro package designed to generate TEX-readable code for CAS-created graphical output. That package and related documentation can be freely downloaded from the website: **http://ketpic.com**.

The aim of this paper is to present some new attempts to verify the effectiveness of using carefully prepared course materials with high-quality graphs in collegiate mathematics education. Based on this experimental study, we claim that our methodology demonstrates great possibilities for providing an objective means to verify the effects of course materials of various types in collegiate education of mathematics.

2010 *Mathematics Subject Classification.* Mathematics Education .
Key words and phrases. exponential growth, graphics, TeX, brain activity, variance property .

2 How to teach the nature of exponential growth The exponential function is an important item in science and engineering education. Exponential growth is the most fundamental and the most discriminative natures in various characteristics of the exponential function. In Section 7.2 of his book titled Calculus[6], Stewart wrote, showing Figure 4,"Figure 4 shows how the exponential function $y = 2^x$ compares with the power function $y = x^2$. The graphs intersect three times, but ultimately the exponential curve $y = 2^x$ grows far more rapidly than the parabola $y = x^2$ (see also Figure 5)".

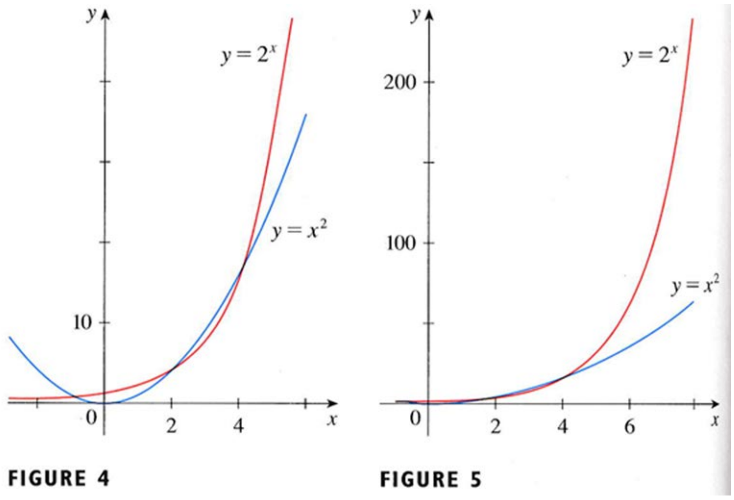

Figure 1. Graphs in Section 7.2 of Stewart's book

This nature is connected closely with the differential equation of $\frac{dy}{dx} = ky$ (k is any positive constant) that appears in phenomena treated in science and engineering. At Section 7.3* of the same book, he wrote for the case of $k = 1$. "The geometric interpretation of Formula 8 (i.e. the above Formula) is that the slope of a tangent line to the curve $y = e^x$ at any point is equal to the y-coordinate of the point. This property implies that the exponential curve $y = e^x$ grows very rapidly". Whereas the exponential growth is represented by $a^x > x^n \;:\; a > 1$, n is any natural number. Also x is large unboundedly. It is our pedagogical challenge to demonstrate this characteristic in an educationally acceptable way and to prepare course materials that make students understand that deeply.

Approaches to this matter in high school and college in our country are inappropriate. In high school mathematics textbooks (from major five textbook publishers), the description of

$$\lim_{x\to\infty} \frac{e^x}{x^n} = \infty, \qquad \lim_{x\to\infty} \frac{x^n}{e^x} = 0 \qquad \text{(for every natural number } n\text{)}$$

is done to supplement the inequality of $e^x > 1 + x$, $e^x > 1 + x + \frac{1}{2}x^2$ in the section of application of differential calculus to inequality, as a tip. Furthermore, the textbook from Suken Shuppan exceptionally describes the following. "Therefore, $y = e^x$ increases more rapidly than x^n when $x \to \infty$". Our experience has just taught us that the concept of "grows more rapidly" is tough even for college students, and high school students. In standard textbooks and reference books for college mathematics, it is designated as a problem for which l'Hopital's rule is applied.

An animated course material displayed by a projector has been developed to promote deep understanding and fixing of the concept of "exponential growth". In the first frame, two graphs of power function $y = x^4$ and exponential function $y = 2^x$ are drawn with x-y axes with the same scale reduction. With scale reduction of the y-axis increase, two graphs begin to intersect with each other. Then they exchange their magnitude relation. The animation comprises 37 frames. The final scale reduction of the y-axis is $\frac{1}{10{,}000}$.

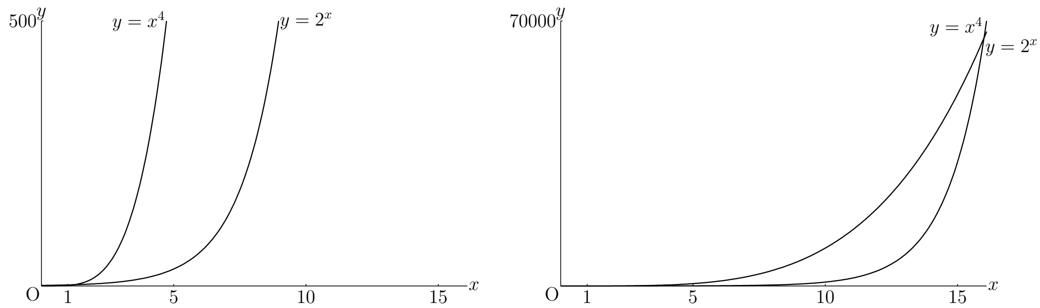

Figure 2. Examples of animation.

We used the program feature of the KETpic command "Texcom" for automatic generation of a TEX file that shows graphs for each scale reduction by the KETpic slide of TEX. The programming is extremely simple, as shown below.

```
Openfile('SisuPR.tex');
FC=0;
for K=[1,10,100,1000,10000],
  if K==10000 then JJ=[1];
  else JJ=[1,2,3,4,5,6,7,8,9]; end
  for J=JJ,
    I=J*K;FC=FC+1;
    Texcom('\newslide[0]{}%');
    Texcom('\begin{layer}{130}{0}');
    Texcom('\putnotese{35}{0}{{\Large Scale of $y=$ 1/'+string(I)+'}}');
    Texcom('\putnotese{5}{15}{\input{'+Fname+string(FC)+'Zu.tex}}');
    Texcom('\end{layer}');
    Texcom('%');
  end
end
Closefile();
```

Avoiding movement of the coordinate axis and comments in the graph is the most important tip to demonstrate a series of slides as an animation. It is the layer environment which realizes that allocating figures, mathematical symbols, and sentences to the desired place in a page[5].

Using this course material, we taught a class for about 50 students at Shibaura Institute of Technology. Then we obtained their favorable responses. The questionnaire, which surveyed their understanding of exponential growth before and after the class, revealed the following: 69.4% of students did not know that before the class but understood it correctly after the class, 14.3% students reported after the class, that their understanding of it had been inaccurate before the class. Summing the first two groups above, 83.7% of students reported gaining a correct understanding of the concept through the class. It is noteworthy that 12.2% knew the material even before the class. The remaining 4.1% students did not understand it well even after the class. Although collecting students' opinions and feelings through questionnaire surveys or interviews is important to improve course material, it might not work as an objective evaluation for a course materials. The KETpic research group has been performing an objective evaluation for the course materials using a statistical method. In academic year 2013, we taught an experimental lesson at Nagano National College of Technology and Gunma National College of Technology, making use of course materials for trigonometric functions and polar coordinates and dividing subjects into experimental and control groups for purposes of statistical analysis[2]. Although this analytical procedure is necessary to guarantee statistical objectivity, it requires too many subject students and might interfere with other ordinary lessons. Selecting a proper method to analyze statistical data from an objective standpoint is among several challenges and difficulties in this type of experiment.

Recently some researchers, mainly from Nakagawa Laboratory, Nagaoka University of Technology, have developed a method to capture a time-series of brain activities by analysis of emotional information collected through electroencephalography (EEG). This study was conducted for objective evaluation of course materials to teach the concept of "exponential growth" using evolution of time-series brain activity captured using EEG.

3 Developments until measuring experiments Recently, researchers mainly from Nakagawa Laboratory, Nagaoka University of Technology have strived to develop a method to trace activation in the brain using fractal analysis of time-series data of blood currents in the brain measured through Near Infrared Red Spectroscopy (NIRS). Employing this method, we conducted NIRS measurement for 20 first-year students in March, 2013 at Kisarazu National College of Technology. The target was to draw a graph of trigonometric functions including other fundamental items. Kurimoto Laboratory was also involved.

Experiments with and without EEG measurements are described below. The task for these experiments was exclusively to understand "exponential growth". Although the measuring task in August 2014 and the improvement to it are introduced below, the prior tasks were the same as those shown here. We passed out the first handout (sheet) describing the task. It reads as follows:

> Shall we find on the graph the difference of growth rates between power and exponential functions when x tends to infinity unboundedly?

Time measurements started when the second handout was passed out and subjects were asked to answer Sheets 1∼7 with one minute each. In sheet 1, $y = x^2$ and $y = x^4$ were compared. Two curves were drawn on the same coordinate plane in the range of $0 \leq y \leq 10$. The problem read:

1. Which one increases more rapidly when x increases unboundedly?
(1) $y = x^2$ (2) $y = x^4$

2. What is the magnitude of $y = \dfrac{x^2}{x^4}$ when x increases unboundedly?

On Sheet 2, $y = x^2$ and $y = 2^x$ were compared, two curves were drawn as in Sheet 1. The problem were similar to those in Sheet 1, as well. Sheets 3∼7 posed the main problems related to the task. There, $y = x^4$ and $y = 2^x$ were compared. Two curves were drawn on the same coordinate plain in the range of $0 \leq x \leq 15$. The scale reductions of y-axis on each sheet were $1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1{,}000}, \frac{1}{10{,}000}$, respectively. The problems were similar to those of other sheets.

In collaboration with Nakagawa Laboratory of the Nagaoka University of Technology, we monitored brain waves using two electrodes concurrently with NIRS measurements in March 2014. Subjects were three fifth-year Technical College students: two men and one woman. We later recognized the importance of recording the answering time in conjunction with the action observation. All three subjects made mistakes at the beginning, noticed the mistake, and answered correctly afterward. Comparison of records of answering time and brain waves revealed that the standard deviation SD of brain wave when they noticed their mistake was quite different from that of other instances. After they realized their mistake and answered confidently, they answered more quickly than ever. They answered slowly when wavering, we found. Table 1 presents the answering time (s) and Right-Wrong R/W ratios of each subject.

	Sheet1	Sheet2	1/1	1/10	1/100	1/1,000	1/10,000
Sub1	16, R	10, W	18, W	13, W	09, W	17, R	15, R
Sub2	28, R	32, W	33, W	36, W	45, R	15, R	16, R
Sub3	18, R	18, W	15, W	14, W	10, W	17, W	45, R

Table 1. Answering time and Right-Wrong

Motivated by this experiment, similar experiments for the same task, but without brain wave monitoring, were conducted at Pennsylvania State University and Toho University for some 20 first-year students in May, 2014. After these experiments, we improved the method of task explanation and question posing because the concept of "difference of two functions in increasing speed" was difficult and subjects did not understand that, we found. We improved the explanation of the task direction as presented below.

> Shall we find which of power function and exponential function grows larger when x is extremely large?

Problems were improved as presented below (these are questions for Sheet 4).

> Problem 1. Which of the following three ranges does include the value of x for which $y = 500$ in the graph of $y = 2^x$?
> (1-1) $0 < x < 5$ (1-2) $5 < x < 10$ (1-3) $10 < x < 15$
>
> Problem 2. Which of the following three is right when x is made very large?
> (2-1) $x^4 > 2^x$ (2-2) $x^4 < 2^x$ (2-3) None of them

Problem 1 is the reference problem for Problem 2. It is explained in the next section. In Problem 2, subjects were asked to select a correct inequality. It asked a simple magnitude relation.

At the end of this section, the major improvement on methods of selecting answers and measuring the answering time is to be reported. In the experiment in August, subjects simply pushed down switches of a Response Analyzer instead of putting a mark on a sheet by writing material. This eliminated unnecessary movement of the body and electric noise caused by it. The Response Analyzer is a test product of one of the authors (Usui). It has since been modified. The special features of this device are the following:

- Many devices (extensions) can be connected to a base unit by wireless communication, enabling simultaneous counting.
- Base unit is Raspberry Pi.
- Equipped with Linux-based OS

- High scalability
- Allow future network communication

The Response Analyzer improved the precision of the measurement of answering time and provided great merits of visualization of subjects' thinking processes. Many extensions might be used to monitor subjects with the same task. Analysis of data from such experiments will bring about a new method of evaluating course materials.

4 Method and result of measurement We conducted brain wave measurements to be discussed in this report on August 4 and 5 at Nagaoka University of Technology. To collect emotional information of five different kinds (Complete rest, Pleasant, Unpleasant, Joyful, Angry), measurements were done at 16 positions based on the International 10-20 system. Electrodes were set at positions 1∼16 as shown in Fig. 3, 18 at the top of the head were to remove noise caused by eye movement; A2 at the right ear was designed to remove that caused by heart beats (Fig. 3). Seven male subjects were tested, 6 graduate students (Sub 1 through Sub 6) and one-fourth year student of technical college (Sub 7). A TEAC Polymate V (16 channel, 8000 [Hz]) was used as the measuring instrument.

Two measurement tasks were set: one collected data used as reference data of emotion analysis and another data for mathematical task. Reference data of emotion analysis were:

Wait, Complete rest, Rest, Pleasant, Rest, Unpleasant, Rest, Joyful, Rest, Angry

All measurements were taken for one minute.

The mathematical task was done using seven handouts (sheets). Subjects were asked to be seated on their chairs along a long table. Handouts were put on the desk with the reverse side up. Therefore, the task description was hidden. Subjects flipped the handouts one by one and read the task. Each handout posed two multiple-choice questions. Sheets 1∼6 (Tasks A1 through A6) had to be answered in one minute each. Sheets 7 (Task A7) in three minutes. Although subjects were able to look at digital-clocks in front of them, the ending of answering time was declared orally by a time-keeper. Because that declaration was made to subjects, they were able to concentrate on the task with no concern about running out of time. Subjects answered by pushing down any one of four buttons of the Response Analyzer set up at the right-hand side of each subject. The Response Analyzer played an important role in this measurement, as explained in the previous section. Table 2 presents a description in this section.

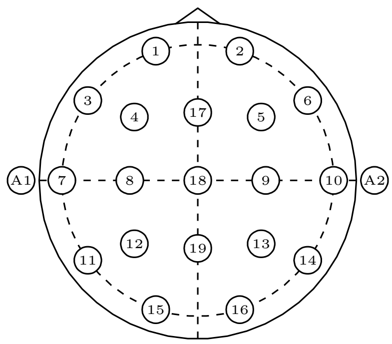

Figure 3. The positions where electrodes are attached

Term	Substance
Subject	Male
EEG Device	TEAC Polymate V(16ch, 8000[Hz])
Measuring Sites	16 positions(1∼16) in the international 10-20 system
Task	To understand the concept of "exponential growth"

Table 2. Measurement environment

The mathematical task proceeded as below. The first handout merely explained the contents of task and subjects' operations to answer questions. No measurement was done for this handout. Having written the improvement in the method of explaining task in the previous section (no description such as "difference of two functions in increasing speed") we merely introduced the tips in operation for answering questions here. These were added and modified on the introduction of Response Analyzer.

◇ You are asked to turn to the next page when you hear "All right!".
◇ Each option goes like (Number of problem−Number of answer).
◇ When you answer, you push the Number of the problem first and then the Number of the answer.
  Example: For (1-3), you push button 1, 3, For (2-2), you push button 2, 2.
◇ Unless otherwise noted, you can read one page for one minute each.
  When one minute has elapsed, you will hear "All right!".
◇ You can change your answer as many times as you wish during the answering time allowed.
  When you change your answer, push the number of the problem first and then the number of the answer.
  When you forgot how you pushed, you can push again.
  Example: To change to (1-4), you push button 1, 4
  Example: To select (1-3) again for confirmation, you push button 1, 3

Then timing measurement started and subjects entered into the first task (Task A1). Task A1 was a reference task for other mathematical tasks. It was similar to all other mathematical tasks. Any subject might have answered correctly. The reference task worked as the reference data for brain wave analysis later. Subjects answered, examining the graphs, two questions about magnitude relation between $y = x^2$ and $y = 2^x$**.** In addition, Problem 1 was the reference task for Task A1.

Problem 1. Which of the following two curves passes through origin?
(1-1) $y = x^2$ (1-2) $y = 2^x$

Problem 2. Which of the following three is right when x is very large?
(2-1) $x^2 > 2^x$ (2-2) $x^2 < 2^x$ (2-3) None of them

Tasks A1 through A6 were structured similarly to Task A1. Subjects answered, examining graphs, two questions about the magnitude relation between $y = x^4$ and $y = 2^x$**.** Problem 1 was the reference one; Problem 2 purposeful one. Graphs for these tasks are presented below. We monitored the change of subjects' brain activity when they examined graphs with scale reductions of y-axis $\frac{1}{1}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1{,}000}$, and $\frac{1}{10{,}000}$, serially. These graphs in Figure 4 respectively correspond to Tasks A2, A4, A5, and A6.

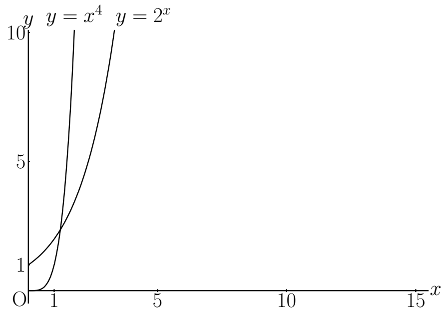

Fig.4-1. Sheet 2: 1/1

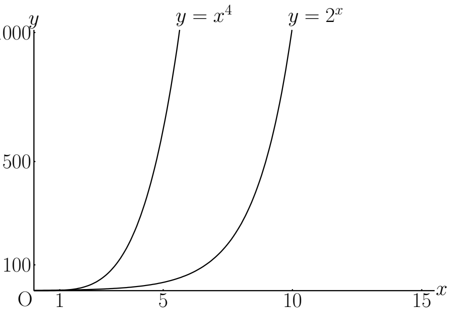

Fig.4-2. Sheet 4: 1/100

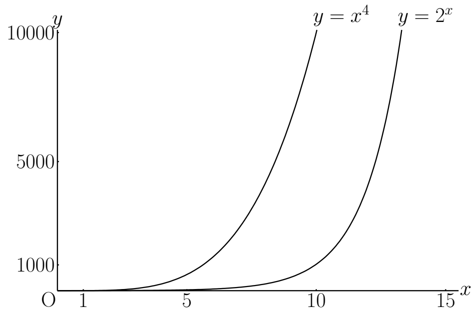

Fig.4-3. Sheet 5: 1/1,000

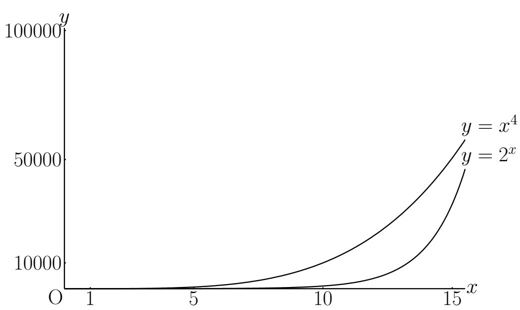

Fig.4-4. Sheet 6: 1/10,000

Figure 4. Graphs used for the EEG measurement

The last task of Task B asked subjects to understand the nature of exponential growth through Taylor's theorem. It used examples of exponential function $y = e^x$ and power function $y = x^2$ and asked subjects to read the explanation and answer questions in 3 min.

The next table presents a summary of records of answering time and right or wrong of the answer R/W measured using the Response Analyzer. Subjects were asked to push the button twice for each problem. The first push inputted the Number of the problem, which was k1 in the table. The second push inputted the Number of the answer and k2 in the table is R/W of that answer.

Task	Sub1				Sub2				Sub3				Sub4			
	Time	k1	k2	R/W	Time	k1	k2	R/W	Time	k1	k2	R/W	Time	k1	k2	R/W
A1	0:13.0	1	1	R	0:12.7	1	1	R	0:12.1	1	1	R	0:44.7	1	1	R
	0:41.7	2	2	R	0:23.4	2	2	R	0:23.3	2	2	R	1:01.9	2	2	R
A2	1:08.4	1	1	R	1:10.5	1	1	R	1:08.5	1	1	R	1:34.1	1	1	R
	1:17.3	2	1	W	1:21.2	2	2	R	1:18.4	2	2	R	1:51.0	1	1	R
													1:56.8	2	1	W
A3	2:29.6	1	2	R	2:59.5	1	2	R	2:20.9	1	2	R	2:26.3	1	2	R
	2:49.8	2	1	W					2:32.0	2	2	R	2:44.6	2	1	W
A4	3:31.6	1	3	W	3:15.3	1	2	R	3:15.3	1	2	R	3:14.3	1	2	R
	3:46.1	2	1	W	3:19.9	2	2	R	3:22.8	2	2	R	3:32.3	2	1	W
A5	4:14.5	1	3	R	4:16.7	1	3	R	4:12.5	1	3	R	4:13.1	1	3	R
	4:52.5	2	1	W	4:21.8	2	2	R	4:21.4	2	2	R	4:22.6	2	1	W
A6	5:20.5	1	3	R	5:11.3	1	3	R	5:22.6	1	3	R	5:31.4	1	3	R
	5:34.8	2	1	W	5:19.6	2	2	R	5:50.9	2	2	R	5:35.3	2	2	R
	5:52.5	2	2	R												
B	7:52.7	1	3	R	7:17.5	1	1	W	7:08.2	1	3	R	6:36.8	1	3	R
	8:22.6	2	2	R	7:39.8	2	1	W	7:19.0	2	1	W	7:06.6	2	2	R

Table 3-1. Answering time, buttons that had been pushed, and Right-Wrong

Task	Sub5				Sub6				Sub7			
	Time	k1	k2	R/W	Time	k1	k2	R/W	Time	k1	k2	R/W
A1	0:17.2	1	1	R	0:18.6	1	1	R	0:11.0	1	1	R
	0:58.9	2	2	R	0:30.5	2	2	R	0:24.5	2	2	R
A2	1:14.0	1	1	R	1:11.2	1	1	R	1:06.0	1	1	R
	1:44.2	2	1	W	1:54.2	2	2	R	1:52.0	2	2	R
A3	2:22.7	1	2	R	2:37.3	1	2	R	2:18.1	1	2	R
					2:46.0	2	2	R	2:59.2	2	1	W
A4	3:20.0	1	2	R	3:19.0	1	2	R	3:15.1	1	2	R
	3:37.5	2	2	R	3:23.8	2	2	R	3:58.0	2	1	W
A5	4:12.9	1	3	R	4:15.7	1	3	R	4:09.4	1	3	R
	4:32.3	2	2	R	4:22.7	2	2	R	4:32.0	2	2	R
A6	5:13.7	1	3	R	5:16.5	1	2	W	5:07.3	1	3	R
	5:30.5	2	2	R	5:18.0	1	3	R	5:08.2	2	2	R
					5:23.1	2	2	R				
B	6:54.9	1	3	R	7:15.0	1	3	R	6:53.1	1	3	R
	7:19.0	2	2	R	7:40.8	2	2	R	7:03.5	2	2	R

Table 3-2. Answering time, buttons that had been pushed, and Right-Wrong

Although data of brain waves at 16 positions for emotion analysis were collected, we have not conducted emotion analysis itself. We calculated the variance property $\alpha(t)$ from brain waves monitored at electrodes 4 and 5 using self-similarity analysis. Then we discussed the characteristics of brain activity. An outline of the relation between variance property and electrode voltage is shown here[1]. First we calculated the next quantity comparable to deviation of electrode voltage $x(t)$.

$$\text{a variogram at lag } \tau\text{:} \quad V(\tau) = \frac{1}{2} E\left[(x(t) - x(t+\tau))^2\right]$$

We set as $\tau = 0.25\,\text{s}$. If $x(t)$ has self-similarity, then proportionality of $V(\tau) \approx |\tau|^\alpha$ holds. On taking logarithm of both sides of this equation, we obtain the following.

$$\log V(\tau) = \log A + \alpha \log |\tau| \text{ , } A \text{ is a constant of proportionality}$$

Consequently, an almost linear relation is obtained when $\log V(\tau)$ is shown against $\log |\tau|$ on the logarithmic graph and α is the slope of the curve. Graphs of $\alpha(t)$ for Subject 7 are portrayed below. In these graphs red line represent the graph of $\alpha(t)$ for EEG channel 4 (left brain) and green line that for channel 5 (right brain).

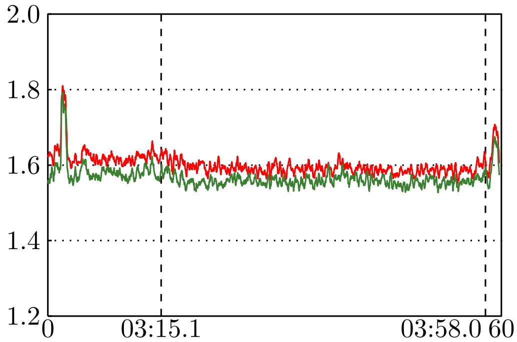

Figure 5-1. Task A4

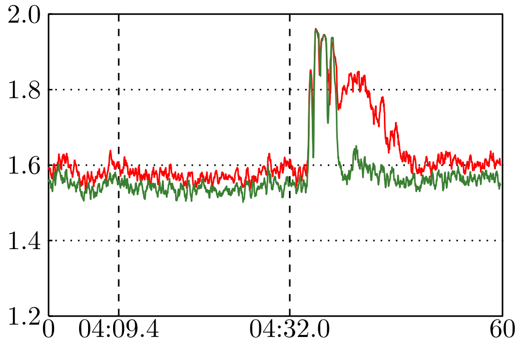

Figure 5-2. Task A5

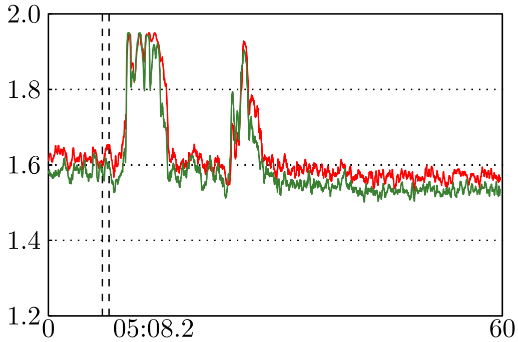

Figure 5-3. Task A6

Figure 5. Graph of $\alpha(t)$ for Subject 7

5 Summary and future challenge Making use of a few graphs of variance property $\alpha(t)$ in the previous section, we analyzed their characteristics. Herein, we provide conclusions to that analysis. The variance property is $1 \leq \alpha \leq 2$ if the input signal $x(t)$ does not include much noise and has little bias. If the brain activity is in the state of rest, then α oscillates at around 1.2. When α oscillates around a high value, the brain is estimated as activated with a number of synchronized brain signals. Herein, we examine data of Subject 7 when he engaged in Task A.

1. When he noticed the essence of the problem and changed his wrong answer to the correct one, it was reflected in the data (Fig. 5-2 Task A5).

2. Data show that he answered with certainty (Fig. 5-3 Task A6).

(1) In Fig. 5-2, we show that oscillation of α around a high value and long duration of it after answering correctly demonstrates his consent to his answer. In Figure 5-1, the rise of α value reflecting brain activity after selecting the wrong answer is apparent at left end of the figure for Task A3 and at the right end of the figure for Task A4. However, the rise of α value is not remarkable. Moreover, its duration is extremely short .
(2) Figure 5-3 shows oscillation of α around a high value and long duration of it when answered correctly to Problem 2 immediately after answering Problem 1 correctly.

Subject 7 answered our questionnaire after the tasks describing that "Although I did not know the magnitude relation between power function and exponential function, I noticed it at Task A5 and was convinced it at Task A6".

He answered all tasks of Problem 1 correctly. His brain wave was stable after pushing the button. No special activation was observed. Furthermore, because he answered Problem 2 of Task A6 just 0.9 s after answering Problem 1 of the same task, no change of α value reflecting activity related to Problem 1 was extracted.

The following three points remain as important challenges for future study.

1. Performing emotion analysis using brain wave data collected in this study to find new information.

2. Continuing similar experimental measurements using proper course materials to facilitate the development of course materials.

3. Continual improvement of the Response Analyzer to establish a course material evaluation method.

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