Optimal Service Hours with Special Offers

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Received January 7, 2014 ; revised April 4, 2014

Abstract. In managing service provider businesses, it is important to decentralize consumers at peak time and increase sales not at peak time as well. Shy and Stanbacka[5] have dealt with this problem to explore optimal service hours under a specific ideal time distribution, and discussed the existence of optimal opening and closing times. In the actual environments, however, service providers strategically introduce a wide variety of special offers such as discounted price to collect more consumers.

In this study, we deal with optimal service hours with a special offer of price discount immediately after the opening time and just before the closing time with the view to attracting extra consumers whose ideal and convenient service times are before the opening time and after the closing time. Under the ideal service time distribution by Shy and Stanbacka[5], the provider's profit is first formulated as an objective function to be maximized and then clarified is the condition under which the service provider can earn more profit by special offers than without special offers. An optimal business hours is also explored to clarify the conditions where there exist optimal opening and closing times. Numerical examples are also presented to illustrate the proposed model formulation.

1 Introduction

Business hours have been traditionally regulated particularly in many European countries although the liberalization of business hours generated debates in these three decades(see, e.g., De Meza[4], Ferris[2], Clemenz [1], Inderst and Irmen[3]).

On the other hand, service providers as well as retailers are eager to make more profits by strategic managements. It is especially important for service industries to decentralize consumers in peak time and increase sales not in peak time. Shy and Stanbacka[5] have dealt with this problem to explore optimal service hours. Under a specific ideal service time distribution of consumers, they discussed the existence of optimal service hours.

A special offer such as early birds specials and/or closing time discount/sale is one of the effective strategies for service industries as well as retailers since they can possibly increase the sales not in peak time. We can observe, in the real circumstances, special offers in a wide variety of service and retailing industries, e.g., morning perm at a beauty salon, happy hour at a hotel, midnight discount of a telecommunications industry, special time discount in business logistics and so forth.

This study confines itself to a service provider having a special offer of price discount immediately after the opening time and just before the closing time. This type of special offer is effective since they can attract extra consumers whose ideal or convenient service times are before the opening time or after the closing time.

First, we formulate the provider's profit as an objective function under the ideal service time distribution introduced by Shy and Stanbacka[5]. Clarified are the conditions under which the service provider can increase his profit by the special offer.

2000 *Mathematics Subject Classification.* 90B05, 90B50 .

Key words and phrases. expected profit, special offer, service hours.

Second, we explore optimal business hours maximizing the provider's profit to show the conditions where there exist unique optimal opening and closing times. Numerical examples are also presented to illustrate the characteristics of the proposed model.

2 Model Formulation

2.1 Assumptions and notations The assumptions and the relevant notations of this study are as follows;

(a) Each individual consumer has her own ideal service time to visit the provider or to receive his service.

(b) The utility of each consumer is given by U_t when she purchase a service product at time t.

(c) Each consumer obtains utility, u_0, by purchasing a service product.

(d) The regular selling price of service is p.

(e) The provider sells his service product at price αp as his special offer, where $0 < \alpha < 1$.

(f) The time during which a special offer is provided is denoted by $\tau (> 0)$.

(g) A consumer owes ω per unit time in visiting the provider or receiving his service earlier or later than her ideal time.

(h) The opening and closing times are, respectively, t_o and t_c, where we have $0 \leq t_o \leq t_c \leq 1$.

(i) The raw price per service product is given by c_1, while the operation cost of the service provider per unit of time is c_2.

2.2 Ideal time distribution In this study, we assume that a demand quantity, q_t, at ideal time t is given by

$$q_t = \begin{cases} n[\mu + 4(1-\mu)t], & 0 \leq t < \frac{1}{2} \\ n[4 - 3\mu - 4(1-\mu)t], & \frac{1}{2} \leq t \leq 1 \end{cases}, \tag{21}$$

where n repsresents the population size and μ $(0 \leq \mu \leq 1)$ measures the degree of uniformity. Figure 1 shows the ideal time distribution given by Eq. (21) for $n = 1$ against various values of μ.

Shy and Stanbacka[5] have assumed the above ideal time distribution on the unit circle with the view to formalizing the idea that there are spillovers between time periods. In this study, however, we assume the same structure of the ideal time distribution on the unit time interval $[0, 1]$. This is because spillovers are an important factor only when the service provider sells his products for almost whole unit time period, and in such a situation the strategic determination of service hours might not be necessary.

We here introduce an additional assumption as follows:

(j) When the selling price is reduced to αp at t, demand quantity q_t increases to $[1+\beta(\alpha)]q_t$ with $\beta(\alpha) > 0$ for $0 < \alpha < 1$.

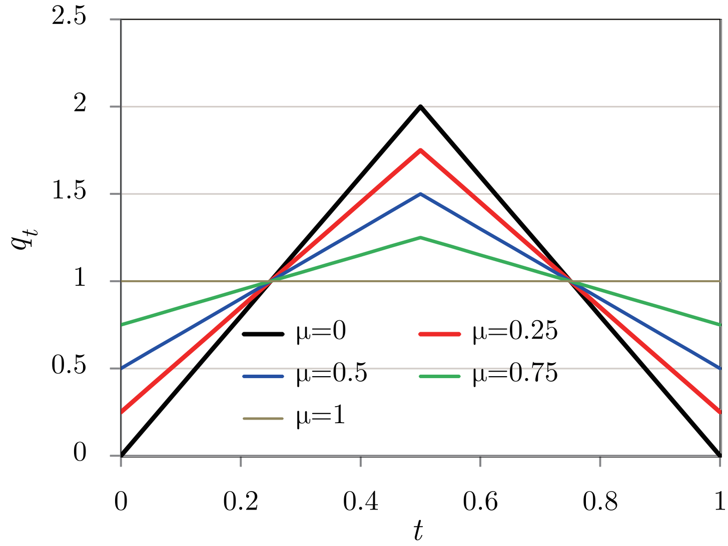

Figure 1: Ideal time distribution($n = 1$).

Assumption (j) signifies the price elasticity η of demand is given by

$$\eta = -\frac{\frac{\beta(\alpha)q_t}{q_t}}{\frac{(\alpha-1)p}{p}} = \frac{\beta(\alpha)}{1-\alpha}, \ 0 < \alpha < 1,$$

where $\lim_{\alpha\to 1-0}\beta(\alpha) = 0$.

In the following, consumers represented by the demand quantity q_t are called type $\mathscr{A}$, while those expressed by $\beta(\alpha)q_t$ are referred to type $\mathscr{B}$. Moreover, we concentrate upon the case where values of α and $\beta(\alpha)$ are both specified to specific values, and $\beta(\alpha)$ is written as β for simplicity.

3 Consumers' Behavior

3.1 Best response Since the ideal time distribution by Eq. (21) reveals a symmetrical shape, the opening time, t_o, and the closing time, t_c, are also symmetrical with respect to $t = \frac{1}{2}$, accordingly we have

$$t_c = 1 - t_o.$$

Hence, we focus on the former half of period $[0, \frac{1}{2}]$ to discuss the opening time, t_o, hereafter.
(1) Type $\mathscr{A}$ consumers' response When the provider introduces early birds specials and/or closing time discount/sale, the best response of type $\mathscr{A}$ consumers with ideal time t becomes as follows:

i) If $t \in [0, t_o^{(1a)}]$, type $\mathscr{A}$ consumers are reluctant to wait until t_o, and purchase no service product, where

$$t_o^{(1a)} = t_o - \frac{u_0 - \alpha p}{\omega}. \tag{31}$$

Consequently, their net utility is given by

$$U_t = 0.$$

ii) If $t \in (t_o^{(1a)}, t_o]$, type $\mathscr{A}$ consumers purchase a service product at the discounted price, αp, by waiting until t_o. In this case, their net utility becomes

$$U_t = u_0 - \alpha p - \omega(t_o - t).$$

iii) Type $\mathscr{A}$ consumers with $t \in (t_o, t_o+\tau]$ purchase a service product at their ideal service time at αp, and hence we have

$$U_t = u_0 - \alpha p.$$

iv) In the case of $t \in (t_o + \tau, t_o^{(2)}]$, the consumers purchase a service product earlier than their ideal time at αp, and thereby

$$U_t = u_0 - \alpha p - \omega[t - (t_o + \tau)],$$

where

$$t_o^{(2)} = t_o + \tau + \frac{(1-\alpha)p}{\omega}. \tag{32}$$

It should be noted in Eq. (32) that $t_o^{(2)} \neq t_o + \tau + \frac{u_0 - \alpha p}{\omega}$ since consumers with ideal time t can obtain positive utility $u_0 - p$ even at t, and therefore $t_o^{(2)}$ should be derived from the condition in reference to t; $u_0 - \alpha p - \omega[t - (t_o + \tau)] \geq u_0 - p$.

v) When $t \in (t_o^{(2)}, \frac{1}{2}]$, type $\mathscr{A}$ consumers will purchase a service product at p, at their ideal time t, and hence

$$U_t = u_0 - p.$$

(2) Type $\mathscr{B}$ consumers' response The best response of type $\mathscr{B}$ consumers with ideal time t is described as follows:

i) If $t \in [0, t_o^{(1b)}]$, type $\mathscr{B}$ consumers would not wait until t_o and purchase no service product at αp, where

$$t_o^{(1b)} = t_o - \frac{(1-\alpha)p}{\omega}. \tag{33}$$

Consequently, their net utility becomes

$$U_t = 0.$$

ii) If $t \in (t_o^{(1b)}, t_o]$, type $\mathscr{B}$ consumers purchases a service product at αp, by waiting until t_o. In this case, the maximum value of their net utility can be represented by

$$U_t = (1-\alpha)p - \omega(t_o - t).$$

iii) Consumers with $t \in (t_o, t_o + \tau]$ purchase a service product at discounted price, αp, at their ideal service time, and hence their maximum net utility can be expressed as

$$U_t = (1-\alpha)p.$$

iv) In the case of $t \in (t_o+\tau, t_o^{(2)}]$, type $\mathscr{B}$ consumers purchase a product earlier than their ideal time at αp, and their maximum net utility becomes

$$U_t = (1-\alpha)p - \omega[t - (t_o + \tau)].$$

v) When $t \in (t_o^{(2)}, \frac{1}{2}]$, type $\mathscr{B}$ consumers would purchase no service product yielding

$$U_t = 0.$$

3.2 Domain of opening time It is neither reasonable nor proper for a consumer with ideal time $t < 0$ to wait until t_o, and we assume

$$\min\left(t_o^{(1a)}, t_o^{(1b)}\right) = t_o^{(1a)} = t_o - \frac{u_0 - \alpha p}{\omega} \geq 0,$$

which constrains the opening time to satisfy

$$t_o \geq \frac{u_0 - \alpha p}{\omega}. \tag{34}$$

The right-hand-side of this equation is denoted by t_L in the following.

Likewise, it is reasonable to assume

$$t_o^{(2)} \leq \frac{1}{2},$$

which is equivalent to

$$t_o \leq \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega}. \tag{35}$$

The right-hand-side of the above equation is denoted by t_U.

It should be noted here that Eqs.(34) and (35) yield,

$$\frac{u_0 - \alpha p}{\omega} \leq \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega},$$

which singifies, at the same time, that ω should satisfy

$$\omega \geq \frac{2[u_0 + (1-2\alpha)p]}{1 - 2\tau}. \tag{36}$$

From Eqs. (34) and (35), the domain of t_o is, as a result, given by

$$t_L \equiv \frac{u_0 - \alpha p}{\omega} \leq t_o \leq \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega} \equiv t_U. \tag{37}$$

4 Provider's Profit

Let $Q_{1A}(t_o)$ express the number of type $\mathscr{A}$ consumers who purchase a service product at αp, then

$$\begin{aligned} Q_{1A}(t_o) &= \int_{t_o^{(1a)}}^{t_o^{(2)}} q_t dt \\ &= 2n\left[\tau + \frac{u_0 + (1-2\alpha)p}{\omega}\right]\left[\mu + 2(1-\mu)\left(2t_o + \tau - \frac{u_0 - p}{\omega}\right)\right]. \end{aligned} \tag{41}$$

By letting $Q_{1B}(t_o)$ signify the number of type $\mathscr{B}$ consumers who purchase a service product at αp, we have

$$\begin{aligned} Q_{1B}(t_o) &= 2\int_{to^{(1b)}}^{t_o^{(2)}} \beta q_t dt \\ &= 2n\beta\mu\left[\tau + \frac{2(1-\alpha)p}{\omega}\right] + 4n\beta(1-\mu)\left[\tau + \frac{2(1-\alpha)p}{\omega}\right](2t_o + \tau). \end{aligned} \tag{42}$$

On the other hand, let us denote, by $Q_2(t_o)$, the number of consumers who purchase a service product at the regular price, p, then we have

$$\begin{aligned} Q_2(t_o) &= 2\int_{t_o^{(2)}}^{\frac{1}{2}} q_t dt \\ &= 2n\left[\frac{1}{2} - t_o - \tau - \frac{(1-\alpha)p}{\omega}\right]\left\{\mu + 2(1-\mu)\left[\frac{1}{2} + t_o + \tau + \frac{(1-\alpha)p}{\omega}\right]\right\}. \end{aligned} \tag{43}$$

Hence, the provider's profit becomes

$$\Pi(t_o) = (\alpha p - c_1)\left[Q_{1A}(t_o) + Q_{1B}(t_o)\right] + (p - c_1)Q_2(t_o) - c_2(1 - 2t_o). \tag{44}$$

We here introduce the following additional constraints so that the provider's profit can take on a positive value at its demand peak and a negative value at its demand off-peak;

$$n(2-\mu)(p - c_1) > c_2, \tag{45}$$

$$n\mu(p - c_1) < c_2. \tag{46}$$

Further, we also assume

$$c_1 < \alpha p,$$

not to lose profit by the special offer. This provides a lower bound for α and consequently the domain of α is given by

$$\frac{c_1}{p} < \alpha < 1. \tag{47}$$

The above observations yield the following proposition:

Proposition 1 *If $u_0 - p \le (1-\alpha)p$ and $\beta(\alpha p - c_1) > (1-\alpha)p$, the service provider can increase his profit by introducing a special offer.*

Proof. When the service provider should not offer the discounted price, αp, type $\mathscr{A}$ consumers with ideal service time t satisfying $t_o - \frac{u_0 - p}{\omega} \le t \le t_o + \tau + \frac{(1-\alpha)p}{\omega}$ would purchase a service product at its regular price, p ($> \alpha p$). This indicates that the service provider prepares himself for decrease in profit due to the special offer given by

$$\Pi_1 = (1-\alpha)p \int_{t_o - \frac{u_0 - p}{\omega}}^{t_o + \tau + \frac{(1-\alpha)p}{\omega}} q_t dt. \tag{48}$$

At the same time, however, the special offer will induce type $\mathscr{A}$ consumers with ideal time t satisfying $t_o - \frac{u_0 - \alpha p}{\omega} \le t < t_o - \frac{u_0 - p}{\omega}$ to enjoy the special offer by shifting their ideal time, and thereby the provider can increase his profit by

$$\Pi_2 = (\alpha p - c_1) \int_{t_o - \frac{u_0 - \alpha p}{\omega}}^{t_o - \frac{u_0 - p}{\omega}} q_t dt. \tag{49}$$

In addition, type $\mathscr{B}$ consumers with ideal time t satisfying $t_o - \frac{(1-\alpha)p}{\omega} \le t \le t_o + \tau + \frac{(1-\alpha)p}{\omega}$ would purchase a service product at αp, the provider can further increase his profit by

$$\Pi_3 = (\alpha p - c_1) \int_{t_o - \frac{(1-\alpha)p}{\omega}}^{t_o + \tau + \frac{(1-\alpha)p}{\omega}} \beta q_t dt. \tag{410}$$

Let Π_0 be defined by $\Pi_0 \equiv \Pi_2 + \Pi_3 - \Pi_1$, then we have

$$\begin{aligned} \Pi_0 &= [\beta(\alpha p - c_1) - (1-\alpha)p] \int_{t_o - \frac{u_0 - p}{\omega}}^{t_o + \tau + \frac{(1-\alpha)p}{\omega}} q_t dt \\ &\quad + (\alpha p - c_1)\left[\beta \int_{t_o - \frac{(1-\alpha)p}{\omega}}^{t_o - \frac{u_0 - p}{\omega}} q_t dt + \int_{t_o - \frac{u_0 - \alpha p}{\omega}}^{t_o - \frac{u_0 - p}{\omega}} q_t dt\right]. \end{aligned} \tag{411}$$

If $\beta(\alpha p - c_1) > (1-\alpha)p$, the first term in the right-hand-side of Eq. (411) takes on a positive value. In addition, if $u_0 - p \leq (1-\alpha)p$, the second term in the right-hand-side of Eq. (411) is also positive, and consequently, $\Pi_0 > 0$. ■

5 Optimal Strategy

This section seeks for an optimal opening time, t_o^*, and thereby an optimal closing time, t_c^*, can also be obtained by the symmetric structure of the ideal time distribution. Numerical examples are also presented to illustrate the proposed model formulation.

5.1 Analysis From Eq. (44), we have

$$\frac{d\Pi(t_o)}{dt_o} = (\alpha p - c_1)\left[\frac{dQ_{1A}(t_o)}{dt_o} + \frac{Q_{1B}(t_o)}{dt_o}\right] + (p - c_1)\frac{dQ_2(t_o)}{dt_o} + 2c_2. \tag{51}$$

By letting us denote, by $\pi(t_o)$, the right-hand-side of Eq. (51), we have

$$\begin{aligned} \pi(t_o) &= 8n(1-\mu)(\alpha p - c_1)\left\{\tau + \frac{u_0 + (1-2\alpha)p}{\omega} + \beta\left[\tau + \frac{2(1-\alpha)p}{\omega}\right]\right\} \\ &\quad -8n(1-\mu)(p - c_1)\left[t_o + \tau + \frac{(1-\alpha)p}{\omega}\right] - 2n\mu(p - c_1) + 2c_2, \end{aligned} \tag{52}$$

which indicates $\pi(t_o)$ is strictly decreasing in t_o.

In addition, we have

$$\begin{aligned} \pi\left(\frac{u_0 - \alpha p}{w}\right) &= -8n(1-\mu)(1-\alpha)p\left[\tau + \frac{u_0 + (1-2\alpha)p}{\omega}\right] \\ &\quad +8n\beta(1-\mu)(\alpha p - c_1)\left[\tau + \frac{2(1-\alpha)p}{\omega}\right] \\ &\quad -2n\mu(p - c_1) + 2c_2, \end{aligned} \tag{53}$$

$$\begin{aligned} \pi\left(\frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega}\right) &= 8n(1-\mu)(\alpha p - c_1)\left\{(\beta+1)\left[\tau + \frac{2(1-\alpha)p}{\omega}\right] + \frac{u_0 - p}{\omega}\right\} \\ &\quad -4n(1-\mu)(p - c_1) - 2n\mu(p - c_1) + 2c_2. \end{aligned} \tag{54}$$

Now, let A and B be defined by

$$\begin{aligned} A &\equiv -4n(1-\mu)(1-\alpha)p\left[\tau + \frac{u_0 + (1-2\alpha)p}{\omega}\right] \\ &\quad +4n\beta(1-\mu)(\alpha p - c_1)\left[\tau + \frac{2(1-\alpha)p}{\omega}\right] - n\mu(p - c_1) + c_2, \end{aligned} \tag{55}$$

$$\begin{aligned} B &\equiv 4n(1-\mu)(\alpha p - c_1)\left\{(\beta+1)\left[\tau + \frac{2(1-\alpha)p}{\omega}\right] + \frac{u_0 - p}{\omega}\right\} - 2n(1-\mu)(p - c_1) \\ &\quad -n\mu(p - c_1) + c_2, \end{aligned} \tag{56}$$

and then the optimal opening time, t_o^*, can be discussed under the following classification:

(a) If we have $A > 0$, further classification is necessary.

i) In the case of $B \geq 0$, t_o^* is given by

$$t_o^* = \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega} = t_U.$$

ii) On the contrary, in case we have $B < 0$, t_o^* is given by

$$\begin{aligned} t_o^* \quad = \quad & \frac{\alpha p - c_1}{p - c_1} \left\{ \tau + \frac{u_0 + (1-2\alpha)p}{\omega} + \beta \left[\tau + \frac{2(1-\alpha)p}{\omega} \right] \right\} \\ & - \frac{\mu}{4(1-\mu)} + \frac{c_2}{4n(1-\mu)(p-c_1)} - \tau - \frac{(1-\alpha)p}{\omega}. \end{aligned}$$

(b) If we have $A \leq 0$, then $\pi(t_o) \leq 0$ and hence

$$t_o^* = \frac{u_0 - \alpha p}{\omega} = t_L.$$

As for the optimal opening time, t_o^*, we have the following proposition:

Proposition 2 *For the ideal time distribution with $\mu = 1$, if $p - c_1 \geq \frac{c_2}{n}$, the optimal opening time becomes $t_o^* = \frac{u_0 - \alpha p}{\omega} = t_L$, otherwise we have $t_o^* = \frac{1}{2} - \tau - \frac{(1-\alpha)p}{\omega} = t_U$.*

Proof. In the case of $\mu = 1$, the relationship, $p - c_1 \geq \frac{c_2}{n}$, reveals $A \leq 0$ from Eq. (55) along with $\pi(t_o) \leq 0$. On the contrary, $p - c_1 < \frac{c_2}{n}$ signifies $B > 0$ and hence $\pi(t_o) > 0$. ■

5.2 Numerical examples This subsection presents numerical examples to illustrate the proposed model. Table 1 shows the optimal opening time, t_o^*, and its corresponding profit, $\Pi(t_o^*)$, together with t_L and t_U against various values of μ and α when $(n, \tau, u_0, p, \omega, c_1, c_2, \beta) = (1, 0.05, 10, 9, 40, 4, 3, 0.35)$. It is observed in Table 1 that the optimal opening time, t_o^*, satisfies $t_L < t_o^* < t_U$ in the case of $\mu = 0.25$. In the case of $\mu = 0.5$ as well, $\alpha = 0.80$ and 0.85 indicate $t_L < t_o^* < t_U$. In the other cases, we have $t_o^* = t_L$. This is because the ideal time distribution shows a faltter shape with a smaller value of q_t at its demand peak when μ increases.

Table 1: Optimal strategies

μ	0.25			0.5			0.75		
α	0.75	0.80	0.85	0.75	0.80	0.85	0.75	0.80	0.85
t_L	0.081	0.07	0.059	0.081	0.07	0.059	0.081	0.07	0.059
t_U	0.398	0.405	0.416	0.398	0.405	0.416	0.398	0.405	0.416
t_o^*	0.145	0.159	0.167	0.081	0.092	0.100	0.081	0.07	0.059
$\Pi(t_o^*)$	2.278	2.410	2.500	2.130	2.246	2.323	2.043	2.191	2.287

Figure 2 shows the shape of the profit function, $\Pi(t_o)$, for $\mu = 0.25$, 0.50 and 0.75 against $\alpha = 0.8$ with the other parameter values set to the same for Table 1. It is observed in Fig. 1 that $\Pi(t_o)$ apparently has its maximum when $\mu = 0.25$. In the case of $\mu = 0.5$, $\Pi(t_o)$ has its maximum at $t_o = 0.092$ as shown in Table 1, while $\Pi(t_o)$ decreases with increasing t_o for $\mu = 0.75$.

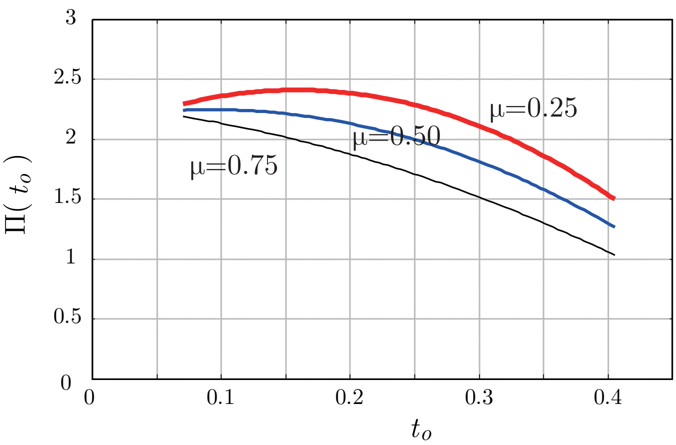

Figure 2: Behavior of profit function

6 Concluding Remarks

This study proposed a mathematical model of an optimal number of service hours for service providers that offer early birds specials and closing time discounts for a service product. By introducing the ideal service time distribution considered by Shy and Stancbaka, clarified were the conditions under which service proviers can earn more profit by special offers. The conditions were also shown that there exist optimal opening and closing times. Numerical examples were presented to illustrate the theoretical underpinnings of the proposed model formulation, and to show the effectiveness of introducing special offers for service providers.

Under the proposed model, however, optimality of the discounted price has not been discussed. One of useful extensions of our work is to explicitly introduce the price elasticity of demand to explore an optimal strategy with regard to the opening (and closing) time as well as the discounted price.

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