Soft BCI-positive implicative ideals of soft BCI-algebras

M. Aslam Malik *and M. Touqeer †

Received 13 October,2014

Abstract

The notion of soft BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras is introduced and their basic properties are discussed. Relations between soft ideals and soft BCI-positive implicative ideals of soft BCI-algebras are provided. Also idealistic soft BCI-algebras and BCI-positive implicative idealistic soft BCI-algebras are being related. The intersection, union, "AND" operation and "OR" operation of soft BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras are established. The characterizations of (fuzzy) BCI-positive implicative ideals in BCI-algebras are given by using the concept of soft sets. Relations between fuzzy BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras are discussed.

Keywords: Soft set; (BCI-positive implicative idealistic) soft BCI-algebra; Soft ideal; Soft BCI-positive implicative ideal.

1 Introduction

The real world is inherently uncertain, imprecise and vague. Because of various uncertainties, classical methods are not successful for solving complicated problems in economics, engineering and environment. The theories such as the probability theory, the (intuitionistic) fuzzy sets theory, the vague set theory, the theory of interval mathematics and the rough set theory, which are used for handling uncertainties have their own difficulties. One of the reasons for these difficulties is due to the inadequacy of the parametrization tool of the theory, which was pointed out by Molodtsov [16]. To overcome these difficulties, Molodtsov introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties. Soft set is a parameterized general mathematical tool which deals with a collection of approximate description of objects. In the soft set theory, the initial description of the object has an approximate nature and there is no need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Applications of soft set theory in different disciplines and real life problems are now catching momentum some of which are being discussed here.

Min [15] studied the concept of similarity between soft sets, which is an extension of the equality for soft set theory. He introduced the concepts of conjunction parameter and disjunction parameter of ordered pair parameter for soft set theory and investigated modified operations of soft set theory in terms of ordered parameters. Yang and Guo [19] introduced the notions of anti-reflexive kernel, symmetric kernel, reflexive closure and symmetric closure of a soft set relation. Soft set relation mappings and inverse soft set relation mappings were also discussed. Kalayathankal and Singh [9] introduced a fuzzy soft flood alarm model which was applied to five selected sites of Karal, India to predict potential flood. Shabir and Naz [18] introduced soft topological spaces defined over an initial universe with a fixed set of parameters. They introduced the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Zhan and Jun [20] discussed soft BL-algebras based on fuzzy sets. They proved that a soft set is an implicative filteristic soft BL-algebra if and only if it is both a positive implicative filteristic soft BL-algebra and a fantastic filteristic soft BL-algebra. Z. Zhang [21] presented a rough set approach to intuitionistic fuzzy soft set based decision making. Jiang et al. [4] discussed interval-valued intuitionistic fuzzy soft sets and their properties. Agarwal et al. [1] generalized the concept of intuitionistic fuzzy soft set by including a parameter reflecting a moderator's opinion about the validity of the information provided. We refer the readers to [2, 17] for further informa-

tion regarding development of soft set theory.
Jun [5] applied the concept of soft sets by Molodtsov to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft subalgebras. Jun and song [8] defined soft subalgebras and soft ideals in BCK/BCI-algebras related to fuzzy set theory. Jun et al. [6] introduced the notion of soft p-ideals and p-idealistic soft BCI-algebras and provided the relations between fuzzy p-ideals and p-idealistic soft BCI-algebras. In this paper, we introduce the notion of soft BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras. Using soft sets, we give characterizations of (fuzzy) BCI-positive implicative ideals in BCI-algebras. We provide relations between fuzzy BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras.

2. Basic results on BCI-algebras

BCK/BCI-algebras are important classes of logical algebras introduced by Y. Imai and K. Iséki [3] and were extensively investigated by several researchers.

An algebra $(X, *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

(I) $((x * y) * (x * z)) * (z * y) = 0$
(II) $(x * (x * y)) * y = 0$
(III) $x * x = 0$
(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$

for all $x, y, z \in X$. In a BCI-algebra X, we can define a partial ordering " $\leq$ " by putting $x \leq y$ if and only if $x * y = 0$.

If a BCI-algebra X satisfies the identity:
(V) $0 * x = 0$,
for all $x \in X$, then X is called a BCK-algebra.

In any BCI-algebra the following hold:

(VI) $(x * y) * z = (x * z) * y$
(VII) $x * 0 = x$
(VIII) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$

(IX) $0 * (x * y) = (0 * x) * (0 * y)$
(X) $x * (x * (x * y)) = (x * y)$
(XI) $(x * z) * (y * z) \leq x * y$

for all $x, y, z \in X$.

A non-empty subset S of a BCI-algebras X is called a subalgebra of X if $x * y \in S$ for all $x, y \in S$. A non-empty subset $\mathcal{I}$ of a BCI-algebra X is called an ideal of X if for any $x \in X$

($\mathcal{I}$1) $0 \in \mathcal{I}$
($\mathcal{I}$2) $x * y \in \mathcal{I}$ and $y \in \mathcal{I}$ implies $x \in \mathcal{I}$

Any ideal $\mathcal{I}$ of a BCI-algebra X satisfies the following implication:

$$x \leq y \text{ and } y \in \mathcal{I} \Rightarrow x \in \mathcal{I}, \forall\, x \in X$$

A non-empty subset $\mathcal{I}$ of a BCI-algebra X is called an BCI-positive implicative ideal (see Liu and Zhang [12]) of X if it satisfies ($\mathcal{I}$1) and

($\mathcal{I}$3) $((x * z) * z) * (y * z) \in \mathcal{I}$ and $y \in \mathcal{I} \Rightarrow x * z \in \mathcal{I}$ for all $x, z \in X$.

We know that every BCI-positive implicative ideal of a BCI-algebra X is also an ideal of X.

We refer the readers to [11, 14] for further study about ideals in BCK/BCI-algebras.

3. Basic results on soft sets

In [16] the soft set is defined in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathfrak{P}(U)$ denotes the power set of U and $A \subset E$.

Definition 3.1 (Molodtsov [16]). A pair $(\mathcal{F},\ A)$ is called a soft set over U, where $\mathcal{F}$ is a mapping given by

$\mathcal{F} : A \rightarrow \mathfrak{P}(U)$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $\mathcal{F}(a)$ may be considered as the set of a-approximate elements of the soft set $(\mathcal{F},\ A)$.

Definition 3.2 (Maji et al. [13]). Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. The intersection of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cap B$
(ii) $\mathcal{H}(x) = \mathcal{F}(x)$ or $\mathcal{G}(x)$ for all $x \in C$, (as both are same sets)

In this case, we write $(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$.

Definition 3.3 (Maji et al. [13]). Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. The union of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cup B$
(ii) for all $x \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(x) & if\ x \in A \setminus B \\ \mathcal{G}(x) & if\ x \in B \setminus A \\ \mathcal{F}(x) \cup \mathcal{G}(x) & if\ x \in A \cap B \end{cases}$$

In this case, we write $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$.

Definition 3.4 (Maji et al. [13]). Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. Then "$(\mathcal{F},\ A)\ AND\ (\mathcal{G},\ B)$" denoted by $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B)$ is defined as $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$, where $\mathcal{H}(x, y) = \mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ for all $(x, y) \in A \times B$.

Definition 3.5 (Maji et al. [13]). Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. Then "$(\mathcal{F},\ A)\ OR\ (\mathcal{G},\ B)$" denoted by $(\mathcal{F},\ A)\ \tilde{\vee}\ (\mathcal{G},\ B)$ is defined as $(\mathcal{F},\ A)\ \tilde{\vee}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$, where $\mathcal{H}(x, y) = \mathcal{F}(x)\ \cup\ \mathcal{G}(y)$ for all $(x, y) \in A \times B$.

Definition 3.6 (Maji et al. [13]). For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U, we say that $(\mathcal{F},\ A)$ is a soft subset of $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A)\ \tilde{\subset}\ (\mathcal{G},\ B)$, if it satisfies:

(i) $A \subset B$
(ii) For every $a \in A$, $\mathcal{F}(a)$ and $\mathcal{G}(a)$ are identical approximations.

4. Soft BCI-positive implicative ideals

In what follows let X and A be a BCI-algebra and a nonempty set, respectively and R will refer to an arbitrary binary relation between an element of A and an element of X, that is, R is a subset of $A \times X$ without otherwise specified. A set valued function $\mathcal{F} : A \to \mathfrak{P}(X)$ can be defined as $\mathcal{F}(x) = \{y \in X \mid xRy\}$ for all $x \in A$. The pair $(\mathcal{F},\ A)$ is then a soft set over X.

Definition 4.1 (Jun and Park [7]). Let S be a subalgebra of X. A subset $\mathcal{I}$ of X is called an ideal of X related to S (briefly, S-ideal of X), denoted by $\mathcal{I} \lhd S$, if it satisfies:

(i) $0 \in \mathcal{I}$
(ii) $x * y \in \mathcal{I}$ and $y \in \mathcal{I} \Rightarrow x \in \mathcal{I}$ for all $x \in S$

Definition 4.2. Let S be a subalgebra of X. A subset $\mathcal{I}$ of X is called a BCI-positive implicative ideal of X related to S (briefly, $S-(BCI-PI)-$ideal of X), denoted by $\mathcal{I} \lhd_{bci-pi} S$, if it satisfies:

(i) $0 \in \mathcal{I}$

(ii) $((x * z) * z) * (y * z) \in \mathcal{I}$ and $y \in \mathcal{I} \Rightarrow x * z \in \mathcal{I}$ for all $x, z \in S$

Example 4.3. Let $X = \{0, a, b, c\}$ be a BCI-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	0	0	c
a	a	0	0	c
b	b	b	0	c
c	c	c	c	0

Then $S = \{0, b\}$ is a subalgebra of X and $\mathcal{I} = \{0, a, b\}$ is an $S - (BCI - PI)-$ideal of X.

Note that every $S - (BCI - PI)-$ideal of X is an S-ideal of X.

Definition 4.4 (Jun [5]). Let $(\mathcal{F},\ A)$ be a soft set over X. Then $(\mathcal{F},\ A)$ is called a soft BCI-algebra over X if $\mathcal{F}(x)$ is a subalgebra of X for all $x \in A$.

Definition 4.5 (Jun and Park [7]). Let $(\mathcal{F},\ A)$ be a soft BCI-algebra over X. A soft set $(\mathcal{G},\ \mathcal{I})$ over X is called a soft ideal of $(\mathcal{F},\ A)$, denoted $(\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}\ (\mathcal{F},\ A)$, if it satisfies:

(i) $\mathcal{I} \subset A$
(ii) $\mathcal{G}(x) \lhd \mathcal{F}(x)$ for all $x \in \mathcal{I}$

Definition 4.6. Let $(\mathcal{F},\ A)$ be a soft BCI-algebra over X. A soft set $(\mathcal{G},\ \mathcal{I})$ over X is called a soft BCI-positive implicative ideal of $(\mathcal{F},\ A)$, denoted $(\mathcal{G},\ \mathcal{I})\ \tilde{\lhd}_{bci-pi}\ (\mathcal{F},\ A)$, if it satisfies:

(i) $\mathcal{I} \subset A$
(ii) $\mathcal{G}(x) \lhd_{bci-pi} \mathcal{F}(x)$ for all $x \in \mathcal{I}$

Let us illustrate this definition using the following example.

Example 4.7. Consider a BCI-algebra $X = \{0, a, b, c\}$ which is given in Example 4.3. Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{0\} \cup \{y \in X \mid y * (y * x) \in \{0, a\}\}$$

for all $x \in A$. Then $\mathcal{F}(0) = \mathcal{F}(a) = X$, $\mathcal{F}(b) = \{0, a, c\}$, $\mathcal{F}(c) = \{0\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X. Let $\mathcal{I}=\{0,a,b\}\subset A$ and $\mathcal{G}:\mathcal{I}\rightarrow\mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x)=\begin{cases} Z(\{0,a\}) & if\ x=b \\ \{0\} & if\ x\in\{0,a\}\end{cases}$$

where $Z(\{0,a\})=\{x\in X\mid 0*(0*x)\in\{0,a\}\}$. Then $\mathcal{G}(0)=\{0\}\lhd_{bci-pi} X=\mathcal{F}(0)$, $\mathcal{G}(a)=\{0\}\lhd_{bci-pi} X=\mathcal{F}(a)$, $\mathcal{G}(b)=\{0,a,b\}\lhd_{bci-pi}\{0,a,c\}=\mathcal{F}(b)$. Hence $(\mathcal{G},\ \mathcal{I})$ is a soft BCI-positive implicative ideal of $(\mathcal{F},\ A)$.

Note that every soft BCI-positive implicative ideal is a soft ideal but the converse is not true as seen in the following example.

Example 4.8. Let $X=\{0,a,b,c,d\}$ be a BCK-algebra and hence a BCI-algebra, with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	d	d	c	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A=X$ and $\mathcal{F}:A\rightarrow\mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x)=\{y\in X\mid y*(y*x)\in\{0,a\}\}$$

for all $x\in A$. Then $\mathcal{F}(0)=\mathcal{F}(a)=X$, $\mathcal{F}(b)=\{0,a,c,d\}$ and $\mathcal{F}(c)=\mathcal{F}(d)=\{0,a\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X.
Let $(\mathcal{G},\ \mathcal{I})$ be a soft set over X, where $\mathcal{I}=\{a,b\}\subset A$ and $\mathcal{G}:\mathcal{I}\rightarrow\mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x)=\{y\in X\mid y*x=0\}$$

for all $x\in\mathcal{I}$. Then $\mathcal{G}(a)=\{0,a\}\lhd X=\mathcal{F}(a)$, $\mathcal{G}(b)=\{0,a,b\}\lhd$

$\{0, a, c, d\} = \mathcal{F}(b)$. Hence $(\mathcal{G}, \mathcal{I})$ is a soft ideal of $(\mathcal{F}, A)$ but it is not a soft BCI-positive implicative ideal of $(\mathcal{F}, A)$ because $\mathcal{G}(a)$ is not a BCI-positive implicative ideal of X related to $\mathcal{F}(a)$ since $((d * c) * c) * (0 * c) = 0 \in \mathcal{G}(a)$ and $0 \in \mathcal{G}(a)$ but $d * c = c \notin \mathcal{G}(a)$.

Theorem 4.9. Let $(\mathcal{F}, A)$ be a soft BCI-algebra over X. For any soft sets $(\mathcal{G}_1, \mathcal{I}_1)$ and $(\mathcal{G}_2, \mathcal{I}_2)$ over X where $\mathcal{I}_1 \cap \mathcal{I}_2 \neq \emptyset$, we have

$$(\mathcal{G}_1, \mathcal{I}_1) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A), (\mathcal{G}_2, \mathcal{I}_2) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A) \Rightarrow (\mathcal{G}_1, \mathcal{I}_1) \tilde{\cap} (\mathcal{G}_2, \mathcal{I}_2) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$$

Proof. Using Definition 3.2, we can write

$$(\mathcal{G}_1, \mathcal{I}_1) \tilde{\cap} (\mathcal{G}_2, \mathcal{I}_2) = (\mathcal{G}, \mathcal{I})$$

where $\mathcal{I} = \mathcal{I}_1 \cap \mathcal{I}_2$ and $\mathcal{G}(e) = \mathcal{G}_1(e)$ or $\mathcal{G}_2(e)$ for all $e \in \mathcal{I}$. Obviously, $\mathcal{I} \subset A$ and $\mathcal{G} : \mathcal{I} \to \mathfrak{P}(X)$ is a mapping. Hence $(\mathcal{G}, \mathcal{I})$ is a soft set over X. Since $(\mathcal{G}_1, \mathcal{I}_1) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$ and $(\mathcal{G}_2, \mathcal{I}_2) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$, it follows that $\mathcal{G}(e) = \mathcal{G}_1(e) \lhd_{bci-pi} \mathcal{F}(e)$ or $\mathcal{G}(e) = \mathcal{G}_2(e) \lhd_{bci-pi} \mathcal{F}(e)$ for all $e \in \mathcal{I}$. Hence

$$(\mathcal{G}_1, \mathcal{I}_1) \tilde{\cap} (\mathcal{G}_2, \mathcal{I}_2) = (\mathcal{G}, \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$$

This completes the proof. □

Corollary 4.10. Let $(\mathcal{F}, A)$ be a soft BCI-algebra over X. For any soft sets $(\mathcal{G}, \mathcal{I})$ and $(\mathcal{H}, \mathcal{I})$ over X, we have

$$(\mathcal{G}, \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A), (\mathcal{H}, \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A) \Rightarrow (\mathcal{G}, \mathcal{I}) \tilde{\cap} (\mathcal{H}, \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$$

Proof. Straightforward. □

Theorem 4.11. Let $(\mathcal{F}, A)$ be a soft BCI-algebra over X. For any soft sets $(\mathcal{G}, \mathcal{I})$ and $(\mathcal{H}, \mathcal{J})$ over X in which $\mathcal{I}$ and $\mathcal{J}$ are disjoint, we have

$$(\mathcal{G}, \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A), (\mathcal{H}, \mathcal{J}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A) \Rightarrow (\mathcal{G}, \mathcal{I}) \tilde{\cup} (\mathcal{H}, \mathcal{J}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$$

Proof. Assume that $(\mathcal{G}, \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$ and $(\mathcal{H}, \mathcal{J}) \tilde{\lhd}_{bci-pi} (\mathcal{F}, A)$. By means of Definition 3.3, we can write $(\mathcal{G}, \mathcal{I}) \tilde{\cup} (\mathcal{H}, \mathcal{J}) = (\mathcal{R}, \mathcal{U})$, where $\mathcal{U} = \mathcal{I} \cup \mathcal{J}$ and for every $e \in \mathcal{U}$,

$$\mathcal{R}(x) = \begin{cases} \mathcal{G}(e) & if\ e \in \mathcal{I} \setminus \mathcal{J} \\ \mathcal{H}(e) & if\ e \in \mathcal{J} \setminus \mathcal{I} \\ \mathcal{G}(e) \cup \mathcal{H}(e) & if\ e \in \mathcal{I} \cap \mathcal{J} \end{cases}$$

Since $\mathcal{I} \cap \mathcal{J} = \emptyset$, either $e \in \mathcal{I} \setminus \mathcal{J}$ or $e \in \mathcal{J} \setminus \mathcal{I}$ for all $e \in \mathcal{U}$. If $e \in \mathcal{I} \setminus \mathcal{J}$, then $\mathcal{R}(e) = \mathcal{G}(e) \lhd_{bci-pi} \mathcal{F}(e)$ since $(\mathcal{G},\ \mathcal{I}) \tilde{\lhd}_{bci-pi} (\mathcal{F},\ A)$. If $e \in \mathcal{J} \setminus \mathcal{I}$, then $\mathcal{R}(e) = \mathcal{H}(e) \lhd_{bci-pi} \mathcal{F}(e)$ since $(\mathcal{H},\ \mathcal{J}) \tilde{\lhd}_{bci-pi} (\mathcal{F},\ A)$. Thus $\mathcal{R}(e) \lhd_{bci-pi} \mathcal{F}(e)$ for all $e \in \mathcal{U}$ and so

$$(\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J}) = (\mathcal{R},\ \mathcal{U})\ \tilde{\lhd}_{bci-pi}\ (\mathcal{F},\ A) \qquad \square$$

It $\mathcal{I}$ and $\mathcal{J}$ are not disjoint in Theorem 4.11, then Theorem 4.11 is not true in general as seen in the following example.

Example 4.12. Let $X = \{0, a, b, c, d\}$ be a BCK-algebra and hence a BCI-algebra, with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	0
c	c	c	c	0	0
d	d	d	c	b	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid y * (y * x) \in \{0, b\}\}$$

for all $x \in A$. Then $\mathcal{F}(0) = X$, $\mathcal{F}(a) = \mathcal{F}(b) = \{0, b, c, d\}$ and $\mathcal{F}(c) = \mathcal{F}(d) = \{0, b\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X.

Let $(\mathcal{G},\ \mathcal{I})$ be a soft set over X, where $\mathcal{I} = \{b, c, d\} \subset A$ and $\mathcal{G} : \mathcal{I} \to \mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x) = \{y \in X \mid y * x = 0\}$$

for all $x \in \mathcal{I}$. Then $\mathcal{G}(b) = \{0, a, b\} \lhd_{bci-pi} \{0, b, c, d\} = \mathcal{F}(b)$, $\mathcal{G}(c) = \{0, a, c\} \lhd_{bci-pi} \{0, b\} = \mathcal{F}(c)$, $\mathcal{G}(d) = X \lhd_{bci-pi} \{0, b\} = \mathcal{F}(d)$. Hence $(\mathcal{G},\ \mathcal{I})$ is a soft BCI-positive implicative ideal of $(\mathcal{F},\ A)$.
Now consider $\mathcal{J} = \{b\}$ which is not disjoint with $\mathcal{I}$ and let $\mathcal{H} : \mathcal{J} \to \mathfrak{P}(X)$ be a set valued function by:

$$\mathcal{H}(x) = \{y \in X \mid y * (y * x) = 0\}\}$$

for all $x \in \mathcal{J}$. Then $\mathcal{H}(b) = \{0, c\} \lhd_{bci-pi} \{0, b, c, d\} = \mathcal{F}(b)$. Hence $(\mathcal{H},\ \mathcal{J})$ is a soft BCI-positive implicative ideal of $(\mathcal{F},\ A)$. But if $(\mathcal{R},\ \mathcal{U}) = (\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J})$, then $\mathcal{R}(b) = \mathcal{G}(b) \cup \mathcal{H}(b) = \{0, a, b, c\}$, which is not a BCI-positive implicative ideal of X related to $\mathcal{F}(b)$ since
$((d * 0) * 0) * (b * 0) = d * b = c \in \mathcal{R}(b)$ and $b \in \mathcal{R}(b)$ but $d * 0 = d \notin \mathcal{R}(b)$. Hence $(\mathcal{R},\ \mathcal{U}) = (\mathcal{G},\ \mathcal{I})\ \tilde{\cup}\ (\mathcal{H},\ \mathcal{J})$ is not a soft BCI-positive implicative ideal of $(\mathcal{F},\ A)$.

5. BCI-positive implicative idealistic soft BCI-algebras

Definition 5.1 (Jun and Park [7]). Let $(\mathcal{F},\ A)$ be soft set over X. Then $(\mathcal{F},\ A)$ is called an idealistic soft BCI-algebra over X if $\mathcal{F}(x)$ is an ideal of X for all $x \in A$.

Definition 5.2. Let $(\mathcal{F},\ A)$ be soft set over X. Then $(\mathcal{F},\ A)$ is called a BCI-positive implicative idealistic soft BCI-algebra over X if $\mathcal{F}(x)$ is a BCI-positive implicative ideal of X for all $x \in A$.

Example 5.3. Consider a BCI-algebra $X = \{0, a, b, c\}$ which is given in Example 4.3. Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \begin{cases} Z(\{0, a\}) & if\ x \in \{b, c\} \\ X & if\ x \in \{0, a\} \end{cases}$$

where $Z(\{0, a\}) = \{x \in X \mid 0 * (0 * x) \in \{0, a\}\}$. Then $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

For any element x of a BCI-algebra X, we define the order of x, denoted by $o(x)$, as

$$o(x) = min\{n \in N \mid 0 * x^n = 0\}$$

where $0 * x^n = (...((0 * x) * x)...) * x$, in which x appears n-times.

Example 5.4. Let $X = \{0, a, b, c, d, e, f, g\}$ be a BCI-algebra defined by the following Cayley table:

$*$	0	a	b	c	d	e	f	g
0	0	0	0	0	d	d	d	d
a	a	0	0	0	e	d	d	d
b	b	b	0	0	f	f	d	d
c	c	b	a	0	g	f	e	d
d	d	d	d	d	0	0	0	0
e	e	d	d	d	a	0	0	0
f	f	f	d	d	b	b	0	0
g	g	f	e	d	c	b	a	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = \{a, b, c\} \subset X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid o(x) = o(y)\}$$

for all $x \in A$. Then $\mathcal{F}(a) = \mathcal{F}(b) = \mathcal{F}(c) = \{0, a, b, c\}$ is a BCI-positive implicative ideal of X. Hence $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. But if we take $B = \{a, b, f, g\} \subset X$ and defined a set-valued function $\mathcal{G} : B \to \mathfrak{P}(X)$ by:

$$\mathcal{G}(x) = \{0\} \cup \{y \in X \mid o(x) = o(y)\}$$

for all $x \in B$, then $(\mathcal{G},\ B)$ is not a BCI-positive implicative idealistic soft BCI-algebra over X, since $\mathcal{G}(f) = \{0, d, e, f, g\}$ is not a BCI-positive implicative ideal of X because $((g * d) * d) * (f * d) = g * b = e \in \mathcal{G}(f)$ and $f \in \mathcal{G}(f)$ but $g * d = c \notin \mathcal{G}(f)$.

Example 5.5 Consider a BCI-algebra X $= \{0, a, b, c\}$ with the following cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{y \in X \mid y = x^n,\ n \in N\}$$

for all $x \in A$. Then $\mathcal{F}(0) = \{0\}$, $\mathcal{F}(a) = \{0, a\}$, $\mathcal{F}(b) = \{0, b\}$, $\mathcal{F}(c) = \{0, c\}$, which are BCI-positive implicative ideals of X. Hence $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

Obviously, every BCI-positive implicative idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X, but the converse is not true in general as seen in the following example.

Example 5.6. Consider a BCI-algebra $X := Y \times Z$, where $(Y, *, 0)$ is a BCI-algebra and $(Z, -, 0)$ is the adjoint BCI-algebra of the additive group $(Z, +, 0)$ of integers. Let $\mathcal{F} : X \to \mathfrak{P}(X)$ be a set-valued function defined as follows:

$$\mathcal{F}(y, n) = \begin{cases} Y \times N_\circ & if\ n \in N_\circ \\ \{(0, 0)\} & otherwise \end{cases}$$

for all $(y, n) \in X$, where $N_\circ$ is the set of all non-negative integers. Then $(\mathcal{F},\ X)$ is an idealistic soft BCI-algebra over X but it is not a BCI-positive implicative idealistic soft BCI-algebra over X since $\{(0, 0)\}$ may not be a BCI-positive implicative ideal of X.

Proposition 5.7. Let $(\mathcal{F},\ A)$ and $(\mathcal{F},\ B)$ be soft sets over X where $B \subseteq A \subseteq X$. If $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X, then so is $(\mathcal{F},\ B)$.

Proof. Straightforward. $\square$

The converse of Proposition 5.7 is not true in general as seen in the following example.

Example 5.8. Consider a BCI-positive implicative idealistic soft BCI-algebra over X which is described in Example 5.4. If we take $B = \{a, b, c, d\} \supseteq A$, then $(\mathcal{F},\ B)$ is not a BCI-positive implicative idealistic soft BCI-algebra over X since $\mathcal{F}(d) = \{d, e, f, g\}$ is not a BCI-positive implicative ideal of X.

Theorem 5.9. Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two BCI-positive implicative idealistic soft BCI-algebras over X. If $A \cap B \neq \emptyset$, then the intersection $(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

Proof. Using Definition 3.2, we can write

$$(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$$

where $C = A \cap B$ and $\mathcal{H}(e) = \mathcal{F}(e)$ or $\mathcal{G}(e)$ for all $e \in C$. Note that $\mathcal{H} : C \rightarrow \mathfrak{P}(X)$ is a mapping, therefore $(\mathcal{H},\ C)$ is a soft set over X. Since $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ are BCI-positive implicative idealistic soft BCI-algebras over X, it follows that $\mathcal{H}(e) = \mathcal{F}(e)$ is a BCI-positive implicative ideal of X or $\mathcal{H}(e) = \mathcal{G}(e)$ is a BCI-positive implicative ideal of X for all $e \in C$. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. □

Corollary 5.10. Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ A)$ be two BCI-positive implicative idealistic soft BCI-algebras over X. Then their intersection $(\mathcal{F},\ A)\ \tilde{\cap}\ (\mathcal{G},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

Proof. Straightforward. □

Theorem 5.11. Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two BCI-positive implicative idealistic soft BCI-algebras over X. If A and B are disjoint, then the union $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

Proof. By means of Definition 3.3, we can write $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \cup B$ and for every $e \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(e) & if\ e \in A \setminus B \\ \mathcal{G}(e) & if\ e \in A \setminus B \\ \mathcal{F}(e) \cup \mathcal{G}(e) & if\ e \in A \cap B \end{cases}$$

Since $A \cap B = \emptyset$, either $e \in A \setminus B$ or $e \in B \setminus A$ for all $e \in C$. If $e \in A \setminus B$, then $\mathcal{H}(e) = \mathcal{F}(e)$ is a BCI-positive implicative ideal of X since $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. If $e \in B \setminus A$, then $\mathcal{H}(e) = \mathcal{G}(e)$ is a BCI-positive implicative ideal of X since $(\mathcal{G},\ B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. □

Theorem 5.12. Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two BCI-positive implicative idealistic soft BCI-algebras over X, then $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

Proof. By means of Definition 3.4, we know that

$$(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B),$$

where $H(x,y) = \mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ for all $(x,y) \in A \times B$. Since $\mathcal{F}(x)$ and $\mathcal{G}(y)$ are BCI-positive implicative ideals of X, the intersection $\mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ is also a BCI-positive implicative ideal of X. Hence $H(x,y)$ is a BCI-positive implicative ideal of X for all $(x,y) \in A \times B$.
Hence $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. □

Definition 5.13. A BCI-positive implicative idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X is said to be trivial (resp., whole) if $\mathcal{F}(x) = 0$ (resp., $\mathcal{F}(x) = X$) for all $x \in A$.

Example 5.14. Let X be a BCI-algebra which is given in Example 5.5 and let $\mathcal{F} : X \to \mathfrak{P}(X)$ be a set-valued function defined by

$$\mathcal{F}(x) = \{0\} \cup \{y \in X \mid o(x) = o(y)\}$$

for all $x \in X$. Then $\mathcal{F}(0) = \{0\}$ and $\mathcal{F}(a) = \mathcal{F}(b) = \mathcal{F}(c) = X$, which are BCI-positive implicative ideals of X. Hence $(\mathcal{F},\ \{0\})$ is a trivial BCI-positive implicative idealistic soft BCI-algebra over X and $(\mathcal{F},\ X \setminus \{0\})$ is a whole BCI-positive implicative idealistic soft BCI-algebra over X.

The proofs of the following three lemmas are straightforward, so they are omitted.

Lemma 5.15. Let $f : X \to Y$ be an onto homomorphism of BCI-algebras. If I is an ideal of X, then $f(I)$ is an ideal of Y.

Lemma 5.16. Let $f : X \to Y$ be an isomorphism of BCI-algebras. If I is a BCI-positive implicative ideal of X, then $f(I)$ is a BCI-positive implicative ideal of Y.

Let $f : X \to Y$ be a mapping of BCI-algebras. For a soft set $(\mathcal{F},\ A)$ over X, $(f(\mathcal{F}),\ A)$ is soft set over Y, where $f(\mathcal{F}) : A \to \mathfrak{P}(Y)$ is defined by $f(\mathcal{F})(x) = f(\mathcal{F}(x))$ for all $x \in A$.

Lemma 5.17 Let $f : X \to Y$ be an isomorphism of BCI-algebras. If $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X, then $(f(\mathcal{F}),\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over Y.

Theorem 5.18. Let $f : X \to Y$ be an isomorphism of BCI-algebras and let $(\mathcal{F},\ A)$ be a BCI-positive implicative idealistic soft BCI-algebra over X.

(1) If $\mathcal{F}(x) = ker(f)$ for all $x \in A$, then $(f(\mathcal{F}),\ A)$ is a trivial BCI-positive implicative idealistic soft BCI-algebra over Y.

(2) If $(\mathcal{F},\ A)$ is whole, then $(f(\mathcal{F}),\ A)$ is a whole BCI-positive implicative idealistic soft BCI-algebra over Y.

Proof. (1) Assume that $\mathcal{F}(x) = ker(f)$ for all $x \in A$. Then $f(\mathcal{F})(x) = f(\mathcal{F}(x)) = \{0_Y\}$ for all $x \in A$. Hence $(\mathcal{F},\ A)$ is a trivial BCI-positive implicative idealistic soft BCI-algebra over Y by Lemma 5.17 and Definition 5.13.

(2) Suppose that $(\mathcal{F},\ A)$ is whole. Then $\mathcal{F}(x) = X$ for all $x \in A$ and so $f(\mathcal{F})(x) = f(\mathcal{F}(x)) = f(X) = Y$ for all $x \in A$. It follows from Lemma 5.17 and Definition 5.13 that $(f(\mathcal{F}),\ A)$ is a whole BCI-positive implicative idealistic soft BCI-algebra over Y. □

Definition 5.19 (Liu and Meng [11]). A fuzzy set μ in X is called a fuzzy BCI-positive implicative ideal of X, if for all $x, y, z \in X$,

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x * z) \geq min\{\mu(((x * z) * z) * (y * z)),\ \mu(y)\}$

The transfer principle for fuzzy sets described in [10] suggest the following theorem.

Lemma 5.20 (Liu and Meng [11]). A fuzzy set μ in X is a fuzzy BCI-positive implicative ideal of X if and only if for any $t \in [0, 1]$, the level subset $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is either empty or a BCI-positive implicative ideal of X.

Theorem 5.21. For every fuzzy BCI-positive implicative ideal μ of X, there exists a BCI-positive implicative idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X.

Proof. Let μ be a fuzzy BCI-positive implicative ideal of X. Then $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is an BCI-positive implicative ideal of X for all $t \in Im(\mu)$. If we take $A = Im(\mu)$ and consider a set valued function $\mathcal{F} : A \to \mathfrak{P}(X)$ given by $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$, then $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X. □

Conversely, the following theorem is straightforward.

Theorem 5.22. For any fuzzy set μ in X, if a BCI-positive implicative idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X is given by $A = Im(\mu)$ and $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$, then μ is a fuzzy BCI-positive implicative ideal of X.

Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = Im(\mu)$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by

$$\mathcal{F}(t) = \{x \in X \mid \mu(x) + t > 1\} \tag{5.2}$$

for all $t \in A$. Then there exists $t \in A$ such that $\mathcal{F}(t)$ is not a BCI-positive implicative ideal of X as seen in the following example.

Example 5.23. For any BCI-algebra X, define a fuzzy set μ in X by $\mu(0) = t_\circ < 0.5$ and $\mu(x) = 1 - t_\circ$ for all $x \neq 0$. Let $A = Im(\mu)$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ be a set-valued function defined by (5.2). Then $\mathcal{F}(1 - t_\circ) = X \setminus \{0\}$, which is not a BCI-positive implicative ideal of X.

Theorem 5.24. Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = [0, 1]$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is given by (5.2). Then the following assertions are equivalent:

(1) μ is a fuzzy BCI-positive implicative ideal of X.

(2) for every $t \in A$ with $\mathcal{F}(t) \neq \emptyset$, $\mathcal{F}(t)$ is an BCI-positive implicative ideal of X.

Proof. Assume that μ is a fuzzy BCI-positive implicative ideal of X. Let $t \in A$ be such that $\mathcal{F}(t) \neq \emptyset$. Then for any $x \in \mathcal{F}(t)$, we have $\mu(0)+t \geq \mu(x)+t > 1$, that is, $0 \in \mathcal{F}(t)$. Let $((x*z)*z)*(y*z) \in \mathcal{F}(t)$ and $y \in \mathcal{F}(t)$ for any $t \in A$ and $x, y, z \in X$. Then $\mu(((x*z)*z)*(y*z))+t > 1$ and $\mu(y) + t > 1$. Since μ is a fuzzy BCI-positive implicative ideal of X, it follows that

$$\mu(x * z) + t \ \geq min\{\mu(((x * z) * z) * (y * z)),\ \mu(y)\} + t$$

$$= min\{\mu(((x * z) * z) * (y * z)) + t,\ \mu(y) + t\} > 1$$

so that $x * z \in \mathcal{F}(t)$. Hence $\mathcal{F}(t)$ is a BCI-positive implicative ideal of X for all $t \in A$ such that $\mathcal{F}(t) \neq \emptyset$.

Conversely, suppose that (2) is valid. If there exists $x_\circ \in X$ such that $\mu(0) < \mu(x_\circ)$, then there exists $t_\circ \in A$ such that $\mu(0) + t_\circ \leq 1 < \mu(x_\circ) + t_\circ$. It follows that $x_\circ \in \mathcal{F}(t_\circ)$ and $0 \notin \mathcal{F}(t_\circ)$, which is a contradiction. Hence $\mu(0) \geq \mu(x)$ for all $x \in X$. Now assume that

$$\mu(x_\circ * z_\circ) < min\{\mu(((x_\circ * z_\circ) * z_\circ) * (y_\circ * z_\circ)),\ \mu(y_\circ)\}$$

for some $x_\circ, y_\circ, z_\circ \in X$. Then there exists some $s_\circ \in A$ such that

$$\mu(x_\circ * z_\circ) + s_\circ \leq 1 < min\{\mu(((x_\circ * z_\circ) * z_\circ) * (y_\circ * z_\circ)),\ \mu(y_\circ)\} + s_\circ$$

$$\Rightarrow \mu(x_\circ * z_\circ) + s_\circ \leq 1 < min\{\mu(((x_\circ * z_\circ) * z_\circ) * (y_\circ * z_\circ)) + s_\circ,\ \mu(y_\circ) + s_\circ\}$$

which implies that $((x_\circ * z_\circ) * z_\circ) * (y_\circ * z_\circ) \in \mathcal{F}(s_\circ)$ and $y_\circ \in \mathcal{F}(s_\circ)$ but $x_\circ * z_\circ \notin \mathcal{F}(s_\circ)$. This is a contradiction. Therefore

$\mu(x * z) \geq min\{\mu(((x * z) * z) * (y * z)),\ \mu(y)\}$ for all $x, y, z \in X$

Thus μ is fuzzy BCI-positive implicative ideal of X. □

Corollary 5.25. Let μ be a fuzzy set in X such that $\mu(x) > 0.5$ for all $x \in X$ and let $(\mathcal{F},\ A)$ be a soft set over X in which

$$A := \{t \in Im(\mu) \mid t > 0.5\}$$

and $\mathcal{F} : A \to \mathfrak{P}(X)$ is given by (5.2). If μ is a fuzzy BCI-positive implicative ideal of X, then $(\mathcal{F},\ A)$ is a BCI-positive implicative idealistic soft BCI-algebra over X.

Proof. Straightforward. □

Theorem 5.26. Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = (0.5, 1]$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is defined by

$$\mathcal{F}(t) = U(\mu; t) \text{ for all } t \in A$$

Then $\mathcal{F}(t)$ is a BCI-positive implicative ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$ if and only if the following assertions are valid:

(1) $max\{\mu(0),\ 0.5\} \geq \mu(x)$ for all $x \in X$.

(2) $max\{\mu(x*z),\ 0.5\} \geq min\{\mu(((x*z)*z)*(y*z)),\ \mu(y)\}$ for all $x, y, z \in X$.

Proof. Assume that $\mathcal{F}(t)$ is a BCI-positive implicative ideal of X for all $t \in A$

with $\mathcal{F}(t) \neq \emptyset$. If there exists $x_\circ \in X$ such that $max\{\mu(0),\ 0.5\} < \mu(x_\circ)$, then there exists $t_\circ \in A$ such that $max\{\mu(0),\ 0.5\} < t_\circ \leq \mu(x_\circ)$. It follows that $\mu(0) < t_\circ$, so that $x_\circ \in \mathcal{F}(t_\circ)$ and $0 \notin \mathcal{F}(t_\circ)$. This is a contradiction. Therefore (1) is valid. Suppose that there exist $a, b, c \in X$ such that

$$max\{\mu(a * c),\ 0.5\} < min\{\mu(((a * c) * c) * (b * c)),\ \mu(b)\}$$

Then there exists $s_\circ \in A$ such that

$$max\{\mu(a * c),\ 0.5\} < s_\circ \leq min\{\mu(((a * c) * c) * (b * c)),\ \mu(b)\}$$

which implies that $((a*c)*c)*(b*c) \in \mathcal{F}(s_\circ)$ and $b \in \mathcal{F}(s_\circ)$, but $a*c \notin \mathcal{F}(s_\circ)$. This is a contradiction. Hence (2) is valid.
Conversely, suppose that (1) and (2) are valid. Let $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. Then for any $x \in \mathcal{F}(t)$, we have

$$max\{\mu(0),\ 0.5\} \geq \mu(x) \geq t > 0.5$$

which implies $\mu(0) \geq t$ and thus $0 \in \mathcal{F}(t)$. Let $((x * z) * z) * (y * z) \in \mathcal{F}(t)$ and $y \in \mathcal{F}(t)$, for any $x, y, z \in X$. Then $\mu(((x * z) * z) * (y * z)) \geq t$ and $\mu(y) \geq t$. It follows from the second condition that

$$max\{\mu(x * z),\ 0.5\} \geq min\{\mu(((x * z) * z) * (y * z)),\ \mu(y)\} \geq t > 0.5$$

$$\Rightarrow \mu(x * z) \geq t$$

so that $x * z \in \mathcal{F}(t)$. Therefore $\mathcal{F}(t)$ is a BCI-positive implicative ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. $\square$

CONCLUSION

The concept of soft set, which is introduced by Molodtsov [16], is a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Soft sets are deeply related to fuzzy sets and rough sets. We introduced the notion of soft BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras and discussed related properties. We established the intersection, union, "AND" operation and "OR" operation of soft BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras. From the above discussion it can be observed that fuzzy BCI-positive implicative ideals can be characterized using the concept of soft sets. For a soft set $(\mathcal{F},\ A)$ over X, a fuzzy set μ in X is a fuzzy BCI-positive implicative ideal of X if and only if for every $t \in A$ with $\mathcal{F}(t) = \{x \in X \mid \mu(x) + t > 1\} \neq \emptyset$, $\mathcal{F}(t)$ is a BCI-positive implicative ideal of X. Finally we have discussed the relations between fuzzy BCI-positive implicative ideals and BCI-positive implicative idealistic soft BCI-algebras.

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Communicated by *Arto Salomaa*

M.AslamMalik∗ and M.Touqeer†

∗malikpu@math.pu.edu.pk †touqeer-fareed@yahoo.com

Department of Mathematics, University of the Punjab, Quaid-e-AzamCampus, Lahore-54590,Pakistan.