BERTRAND VERSUS COURNOT COMPETITION IN A VERTICAL DUOPOLY

DongJoon Lee * SangHeon Han † Joonghwa Oh ‡

Received 6 October 2014; Revised 15 December 2014, 28 December 2014, 25 January 2015

Abstract. This paper examines whether firms prefer to choose prices or quantities with a manufacturing duopoly in which each upstream firm sells its product to its own downstream firm. The degree of product differentiation plays an important role in whether firms set prices or quantities. We show that price competition performs better than quantity competition, from the upstream and downstream firms' point of view, regardless of the product differentiation. We also show that pay-offs are larger in Bertrand (price) competition than in Cournot (quantitiy) competition if both products are differentiated to a certain extent.

1 Introduction As we well know, two classical models in oligopoly theory are Cournot and Bertrand. In a non-cooperative profit maximization environment, one may wonder whether firms prefer to choose prices (Bertland) or quantities (Cournot). Singh and Vives (1984) first analyzed the issue of whether firms prefer to set prices or quantities. They show that consumer and total surplus in Bertrand competition are larger than those in Cournot competition regardless of the nature of goods.[1] They also show that Cournot equilibrium profits are higher than Bertrand equilibrium profits when the goods are substitutes, and vice versa when the goods are complements.[2]

During the past 30 years, many literatures have produced an array of extensions and generalizations of the analysis in Singh and Vives (1984). Previous literature on the issue has followed two separate streams. One stream focuses on extensions and generalizations of Singh and Vives (1984). For example, Dastidar (1997), Qiu (1997), Lambertini (1997), Häckner (2000), and Amir and Jin (2001), among others, have analyzed counter-examples based on the framework of Singh and Vives (1984) by allowing for a wider range of cost and demand asymmetries.[3] The other stream of the literature focuses on expanding the Bertrand-Cournot competition with vertically related duopoly. Correa-Lopez (2007) examines the Bertrand-Cournot profits ranking in a vertically related duopoly model focusing on substitutes and vertical product differentiation. They show that Bertrand profits may exceed Cournot profits when decentralized bargaining over the labor cost is introduced.[4]

2010 *Mathematics Subject Classification.* 91A80, 47N10.

Key words and phrases. Price and Quantity Competition, Imperfect Downstream Competition.

*Faculty of Management Administration Nagoya University of Commerce & Business 4-4 Sagamine, Komenoki-cho, Nisshin-shi, Aichi-ken, Japan 470-0193, Phone: +81-(0)561-73-2111, E-mail: dongjoon@nucba.ac.jp

†Faculty of Management Administration Nagoya University of Commerce & Business 4-4 Sagamine, Komenoki-cho, Nisshin-shi, Aichi-ken, Japan 470-0193, Phone: +81-(0)561-73-2111, E-mail: hansh@nucba.ac.jp

‡Faculty of Commerce Administration Nagoya University of Commerce & Business 4-4 Sagamine, Komenoki-cho, Nisshin-shi, Aichi-ken, Japan 470-0193, Phone: +81-(0)561-73-2111, E-mail: jhoh@nucba.ac.jp, This research was supported by JSPS KAKENHI Grant Number 26780262.

[1] When the goods are independent, they are equal.

[2] See Cheng (1985) for a graphical description of Singh and Vives' analysis.

[3] In particular, Zanchettin (2006) found that Singh and Vives's (1984) result that firms always make larger profits under Cournot competition than under Bertrand competition fails to hold.

[4] Symeonidis (2003, 2008) also analyzes the effects of downstream competition when there is bargaining between downstream firms and upstream agents (firms or unions).

Arya et al. (2008) explore the standard conclusions about duopoly competition when the production of key input is outsourced to a vertically integrated retail competitor with upstream market power. They show that prices and industry profits can be larger in Bertrand competition than in Cournot, while consumer and total surplus can be smaller in Bertrand than in Cournot. Mukherjee et al. (2012) compare Cournot with Bertrand competition in a vertical structure in which a monopoly upstream firm sells its product to two downstream firms, assuming there are asymmetric costs between downstream firms and homogeneous final goods. They demonstrate that the technology differences among the downstream firms and the pricing strategy (i.e., uniform pricing or price discrimination) of the upstream firm play an important role in the ranking of profit and social welfare. We revisit the profit ranking under Bertrand and Cournot competition in a vertically related duopoly in which each upstream firm sells its product to its own downstream firm. Our paper differs from the existing literature in at least two important aspects. First, previous studies consider Bertrand and Cournot competitions under wage bargaining and input prices negotiation. Our study examines them without negotiation. Second, previous ones produced the counter-results of Signs and Vives (1984) under costs and demand asymmetry. However, this paper analyzes the issue under symmetric conditions. This paper is organized as follows; in Section 2, we set up the model. Section 3 examines the Cournot competition, and then, Section 4 analyzes the Bertrand competition. Section 5 deals with comparative analysis. Finally, Section 6 contains concluding remarks.

Consider a manufacturing duopoly in which each upstream firm sells its product to its own downstream firm. There is a continuum of consumers of the same type with a utility function separable and linear in numeraire goods. Therefore, there are no income effects. The representative consumer maximizes $U(q_i, q_j) - \Sigma p_i q_i; i = 1, 2; i \neq j$, where q_i is the quantity of good i and p_i its price. U is assumed to be quadratic and strictly concave $U(q_i, q_j) = q_i + q_j - (q_i^2 + 2bq_iq_j + q_j^2)/2; i = 1, 2; i \neq j$. This utility function gives rise to a linear demand structure. Inverse demands are given by

$$p_i = 1 - q_i - bq_j, \ 0 \leq b \leq 1, \ i, j = 1, 2, i \neq j. \tag{1}$$

where p_i is the retail price for product i, and q_i and q_j are the amount of goods produced by channel i and j, respectively. Each unit of retail output requires exactly one unit of the input. The products are differentiated $(0 \leq b \leq 1)$. Upstream firms and downstream firms are risk-neutral and there are no production or retailing costs.

We posit a two-stage game. At stage one, each upstream firm sets an wholesale price. At stage two, each downstream firm also sets the retail price or quantity.

2 Cournot Competition We first consider Cournot competition in which each downstream firm sets a quantity. In this case the equilibrium concept is the sub-game perfect Nash equilibrium.

Stage Two (Quantity): At stage two, downstream firm i sets a quantity, q_i, so as to maximize its profit for a given input price, w_i. Downstream firm i's maximization problem is as follows:

$$\max \pi_i = (p_i - w_i)q_i, \ w.r.t. \ q_i.$$

where w_i is the input price. Therefore, downstream firm i sets the quantity, q_i, as the function of input prices as follows:

$$q_i(w_i, w_j) = \frac{2(1 - w_i) - b(1 - w_j)}{4 - b^2}. \tag{2}$$

Stage one (Wholesale Price): At stage one, upstream firm i sets wholesale, w_i, to maximize its profit for a given w_j. Upstream firm i's maximization problem is as follows:

$$\max \Pi_i = w_i q_i(w_i, w_j) = \frac{w_i[(2-w_i) - b(1-w_j)]}{4-b^2}, \ w.r.t. \ w_i.$$

The equilibrium wholesale price for upstream firm i is derived as follows:

$$w_i = \frac{2-b}{4-b}. \tag{3.1}$$

Substituting the wholesale price into Eq. (1) and Eq. (2), we obtain the retail price, p_i, the quantity, q_i, the upstream firm i's payoff, Π_i, and downstream firm i's payoff, π_i,

$$p_i^C = \frac{6-b^2}{(2+b)(4-b)}, \tag{3.2}$$

$$q_i^C = \frac{2}{(2+b)(4-b)}, \tag{3.3}$$

$$\Pi_i^C = \frac{2(2-b)}{(2+b)(4-b)^2}, \text{ and} \tag{3.4}$$

$$\pi_i^C = \frac{4}{(2+b)^2(4-b)^2}. \tag{3.5}$$

where superscripts C denote Cournot equilibrium.

3 Bertrand Competition We now turn to Bertrand competition in which each downstream firm sets a retail price. From Eq. (1), the following direct demand function can be derived as follows:

$$q_i = \frac{1-b-p_i+bp_j}{1-b^2}, \ 0 \le b \le 1, \ i,j = 1,2, i \neq j. \tag{4}$$

Stage Two (Retail Price): At stage two, downstream firm i sets retail price, p_i, so as to maximize its profit for a given wholesale price, w_i. Downstream firm i's maximization problem is as follows:

$$\max \pi_i = (p_i - w_i)q_i = \frac{(p_i - w_i)(1-b-p_i+bp_j)}{1-b^2}, \ w.r.t. \ p_i.$$

Therefore, downstream firm i sets the retail price, p_i, as the function of wholesale prices as follows:

$$p_i(w_i, w_j) = \frac{2(1-w_i) - b(1-w_j) - b^2}{4-b^2}. \tag{5}$$

Stage One (Wholesale Price): At stage one, upstream firm i sets a wholesale price, w_i, to maximize its profit for a given wholesale price, w_j. Upstream firm i's maximization problem is as follows:

$$\max \Pi_i = w_i q_i(w_i, w_j) = \frac{w_i[(2-b^2)(1-w_i) - b(1-w_j)]}{(4-b^2)(1-b^2)}, \ w.r.t. \ w_i.$$

The equilibrium wholesale price for upstream firm i is derived as follows:

$$w_i = \frac{2-b-b^2}{4-b-2b^2}. \tag{6.1}$$

Substituting the wholesale price into Eq. (4) and Eq. (5), we obtain the retail price, p_i, the quantity, q_i, the upstream firm i's payoff, Π_i, and downstream firm i's payoff, π_i,

$$p_i^B = \frac{2(1-b)(3-b^2)}{(2-b)(4-b-2b^2)}, \tag{6.2}$$

$$q_i^B = \frac{(2-b^2)}{(2-b)(4-b-2b^2)}, \tag{6.3}$$

$$\Pi_i^B = \frac{(1-b)(2+b)(2-b^2)}{(1+b)(2-b)(4-b-2b^2)^2}, \text{ and} \tag{6.4}$$

$$\pi_i^B = \frac{(1-b)(2-b^2)^2}{(1+b)(2-b)2(4-b-2b^2)^2}. \tag{6.5}$$

4 Comparative Analysis We turn now to compare the equilibrium under Bertrand and Cournot competition. Firstly, we compare wholesale prices between two types of contracts. From Eq. (3.1) and Eq. (6.1), we obtain the following results:

$$w_i^C - w_i^B = \frac{b^3}{(4-b)(4-b-2b^2)} \geq 0.$$

where superscripts B and C denote Bertrand and Cournot, respectively.

Lemma 1. Under Eq. (1) and Eq. (4), if $0 < b \leq 1$, the equilibrium wholesale prices are higher in Cournot than in Bertrand competition. If $b = 0$, both have the same wholesale prices.

Secondly, the equilibrium levels of retail prices and quantities are shown in Table 1.

Table 1: Equilibrium Levels of Retail Price and Quantity

	Retail Price	Quantity
Bertrand	$\frac{2(1-b)(3-b^2)}{(2-b)(4-b-2b^2)}$	$\frac{2-b^2}{(2-b)(4-b-2b^2)}$
Cournot	$\frac{(6-b^2)}{(2+b)(4-b)}$	$\frac{2}{(2+b)(4-b)}$

Lemma 2. Under Eq. (1) and Eq. (4), if $0 < b \leq 1$, the equilibrium prices for both downstream firms are higher in Cournot than in Bertrand competition. If $b = 0$, both have the same prices.

Lemma 3. Under Eq. (1) and Eq. (4), if $0 < b \leq 1$, the equilibrium outputs for both downstream firms are larger in Bertrand than in Cournot competition. If $b = 0$, both have the same input prices.

Quantities are larger and prices lower in Bertrand than in Cournot competition regardless of the nature of goods.[5] Lower prices and higher quantities are always better in welfare terms. Consumer and total surplus are decreasing as a function of prices. Therefore, in terms of consumer surplus and total surplus, the Bertrand equilibrium dominates the Cournot one. Proposition 1 summarizes the results thus far.

[5] When $b = 0$, they are equal.

Proposition 1. Under Eq. (1) and Eq. (4), if $0 < b \leq 1$, consumer surplus and total surplus are larger in Bertrand than in Cournot competition. If $b = 0$, they are equal.
For proof, see Appendix.

Thirdly, we turn to the equilibrium profits for Bertrand and Cournot competition. From Eq. (3.4) and Eq. (6.4), when $0 \leq b \leq 1$, notice that the following results are satisfied:

$$\Pi_i^B - \Pi_i^C = \frac{b^2(4+b-b^2)(16-b(2-b)(10+7b))}{(1+b)(2-b)(2+b)(4-b)^2(4-b-2b^2)^2},$$
$$\Pi_i^B > \Pi_i^C \Leftrightarrow 0 < b < 0.8868 \equiv \bar{b}.$$

Proposition 2. Under Eq. (1) and Eq. (4), if $0 < b \leq \bar{b}$, the Bertrand strategy is dominant for upstream firms. If $\bar{b} < b \leq 1$, the Cournot strategy is dominant for upstream firms. If $b = 0$, payoffs for both upstream firm are equal.

Proposition 2 can be explained as follows. If $0 < b < \bar{b}$, pay-offs in Bertrand competition are higher than those in Cournot, and vise versa. The degree of product differentiation plays an important role in equilibrium. As the degree of product differentiation decreases, the product market competition is more intense under Bertrand compared with Cournot competition. Therefore, pay-offs of Cournot competition are higher than those of Bertrand competition because of monopolistic effect. On the other hand, as the degree of product differentiation decreases, even if the wholesale price is lower in Bertrand competition than in Cournot competition, a more intense competition in the former helps to create a larger wholesale demand than in the latter. As a result, the upstream firm obtains higher pay-offs in Bertrand competition than in Cournot competition.

5 Concluding Remarks We may summarize the results derived from the model as follows:
(1) With linear demand function, if $0 < b \leq 1$, consumer and total surplus are larger in Bertrand than in Cournot competition.
(2) Pay-offs of both upstream firms are larger, equal, or smaller in Bertrand competition than in Cournot competition, according to whether $0 < b < \bar{b}$, or $\bar{b} < b \leq 1$.

We can also extend our analysis for each upstream firm and each downstream firm to make a precommitment to quantity or price contract in a vertically related market. In such a situation, we are wondering the results are the same as Singh and Vives (1984).

Appendix

Proof of Proposition 1. Consumer Surplus ranking of Bertrand and Cournot equilibria. In view of Lemma 2, consumer surplus is clearly higher under Bertrand than under Cournot competition. From the utility function, we get

$$CS = U(q_i, q_j) - (p_i q_i + p_j q_j) = q_i + q_j - \frac{(q_i^2 + 2bq_iq_j + q_j^2)}{2} - (p_i q_i + p_j q_j)$$
$$= q_i + q_j - \frac{(q_i + q_j)^2}{2} + (1-b)q_iq_j - (p_iq_i + p_jq_j) = (1-p_i)\frac{q_i}{2} + (1-p_j)\frac{q_j}{2}.$$

For $0 \leq b \leq 1$, inequality $CS^B > CS^C$ reduces to

$$CS^B - CS^C = \frac{b^2(8-3b^2)(32+8b-28b^2-4b^3+5b^4)}{(1+b)(2-b)^2(2+b)^2(4-b)^2(4-b-2b^2)^2} > 0.$$

This inequality holds for any $0 < b \leq 1$. For $b = 0$, consumer surplus is equal. From the utility function, we get

$$\begin{aligned} TS &= CS + \Pi_i + \Pi_j + \pi_i + \pi_j \\ &= U(q_i, q_j) - (p_i q_i + p_j q_j) + (w_i q_i + w_j q_j) + (p_i - w_i) q_i + (p_j - w_j) q_j \\ &= q_i + q_j - \frac{(q_i^2 + 2b q_i q_j + q_j^2)}{2} \\ &= q_i + q_j - \frac{(q_i + q_j)^2}{2} + (1-b) q_i q_j \\ &= \frac{(1-p_i) q_i}{2} + p_i q_i + \frac{(1-p_j) q_j}{2} + p_j q_j \\ &= \frac{(1+p_i) q_i}{2} + \frac{(1+p_j) q_j}{2}. \end{aligned}$$

For $0 \leq b \leq 1$, inequality $TS^B > TS^C$ reduces to

$$TS^B - TS^C = \frac{b^2(8-3b^2)(96-72b-60b^2+36b^3+9b^4-4b^5)}{(1+b)(2-b)^2(2+b)^2(4-b)^2(4-b-2b^2)^2} > 0$$

This inequality holds for any $0 < b \leq 1$. For $b = 0$, total surplus is equal.

References

[1] Amir, R. and J.Y. Jin. 2001. Cournot and Bertrand Equilibria Compared: Substitutability, Complementarity and Concavity, *International Journal of Industrial Organization*, 19; 303-317.

[2] Arya, A., B. Mittendorf, and D. E. M. Sappington. 2006. Outsourcing, Vertical Integration, and Price vs. Quantity Competition, *International Journal of Industrial Organization*, 26; 1-16.

[3] Belleflamme, P. and Peitz, M. 2010. *Industrial Organization Markets and Strategies*, Cambridge University Press.

[4] Dastidar, K.G. 1997. Comparing Cournot and Bertrand in a Homogenous Product Market, *Journal of Economic Theory*, 75; 205-212.

[5] Dixit, A. K. 1979. A Model of Duopoly Suggesting a Theory of Entry Barriers, *Bell Journal of Economics*, 10; 20-32.

[6] Häckner, J. 2000. A Note on Price and Quantity Competition in Differentiated Oligopolies, *Journal of Economic Theory*, 93; 233-239.

[7] Horn, H. and A. Wolinsky. 1988 Bilateral Monopolies and Incentives for Merger, *RAND Journal of Economics*, 19; 408-419.

[8] Lambertini, L. 1997. Prisoners' Dilemma in Duopoly (Super) Games, *Journal of Economic Theory*, 77; 181-191.

[9] Lopez, M. 2007. Price and Quantity Competition in a Differentiated Duopoly with Upstream Suppliers, *Journal of Economics Management Strategy*, 16; 469-505.

[10] Qiu, L. D. 1997. On the Dynamic Efficiency of Bertrand and Cournot Equilibria, *Journal of Economic Theory*, 75; 213-229.

[11] Singh, N. and X, Vives. 1984. Price and Quantity Competition in a Differentiated Duopoly, *Rand Journal of Economics*, 15; 546-554.

[12] Vives, X. 1984. Duopoly Information Equilibrium: Cournot and Bertrand, *Journal of Economic Theory*, 34; 71-94.

[13] Vives, X. 1985. On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation, *Journal of Economic Theory*, 36; 166-175.

Communicated by *Yosio Tabata*