Reliability and Profit Analysis of a Single-Unit System with Inspection under Warranty

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Abstract. The purpose of the present paper is to carry out reliability and profit analysis of a single-unit system considering the concept of inspection under warranty. Within warranty, failures are rectified by the manufacturer at no cost to the users provided warranty does not apply to product failure due to user-induced damage such as cracked screen, accident, misuse, physical damage, damage due to liquid and unauthorized modifications, etc. The cost to rectify failures beyond the warranty is borne by the users. After failure, unit goes under inspection within warranty. There is single repairman, which is always available with the system to do repair, inspection and replacement of the unit. Repairman inspects the failed unit to see the feasibility of its repair or replacement. If repair of the unit is not feasible, it is replaced by new one.The time to failure of the system follows negative exponential distribution while inspection and repair time distributions are taken as arbitrary. By using supplementary variable technique, various measures of system performance such as reliability, mean time to system failure (MTSF), availability of the system and profit function have been determined. The numerical results for reliability and profit function are also obtained in the form of tables for particular values of various parameters and repair cost.

1 Introduction: Warranty is a key promotional tool for the seller since it has become an essentially compititive stretegy employed by sellers to boost their market share, profitability and corporate image. Item sold under warranty often require post sale support in terms of repair or replacement. Several authors including Kadyan et al. [3], Kaur et al. [4], Kharoufeh et al. [5] and Xiaoning Jin et al. [6] studied single unit systems without considering any warranty of the systems. But, in the modern age, most products are sold with

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a warranty to gain some advantages in the highly competitive markets. Further, warranty plays an important role to assure reliability of a sold product and may increase sales. Also, repair of the failed unit is not always feasible due to its excessive use and increased cost of maintenance. In such cases, the failed unit may be replaced by new unit after getting necessary inspection in order to avoid unnecessary expenses on repair. Yeh et al. [7] have studied an inspection model with discount factor. But single-unit systems with warranty and inspection have not appeared in the literature so far.

Keeping in view of the above facts, here we studied a single unit reliability model with the concept of warranty and inspection. Within warranty, failures are rectified by the manufacturer at no cost to the users provided failures are not due to the negligence of users. After failure, unit goes under inspection within warranty. There is single repairman, which is always available with the system to do repair, inspection and replacement of the unit. Repairman inspects the failed unit to see the feasibility of repair. If repair of the unit is not feasible, it is replaced by new one.The time to failure of the system follows negative exponential distribution while inspection and repair time distributions are taken as arbitrary. The supplementary variable technique is adopted to derive the expressions for some economic measures such as reliability, MTSF, availability and profit function. The numerical results for reliability and profit function are also obtained in the form of tables for particular values of various parameters and repair cost.

2 Assumptions:

1. The system has a single unit.
2. There is single repairman, which is always available with the system to do repair, inspection and replacement of the unit.
3. The cost of repair of the failed unit during warranty is borne by the manufacturer provided failures are not due to the neglegence of users.
4. Under warranty, unit goes for inspection after failure.
5. Repairman inspects the failed unit to see the feasibility of repair or replacement.
6. The unit works as new after repair.
7. The distribution of failure time is taken as negative exponential while the inspection and repair time are considered as arbitrary.

3 State-Specification:

s_0/s_1 The unit is operative under warranty/ beyond warranty.

s_3/s_4 The unit is in failed state under warranty/ beyond warranty.

s_2 The failed unit is under inspection.

4 Notations:

λ/λ_1 Constant failure rate of the unit within warranty/beyond warranty.

α Constant rate of completion of warranty.

p/q Probability that repair is feasible/not feasible.

$\mu(x), s(x)$ Repair rate of the unit and probability density function, for the elapsed repair time 'x' in warranty.

$\mu_1(x), s_1(x)$ Repair rate of the unit and probability density function, for the elapsed repair time 'x' beyond warranty.

$h(y), s_2(y)$ Inspection rate of the failed unit and probability density function, for the elapsed inspection time 'y'.

$p_0(t)/p_1(t)$ The Probability that at time t the system is in good state in warranty/beyond warranty.

$p_3(x,t)\Delta$ The Probability that at time t the system is in failed state in warranty, the elapsed repair time lies in the interval $[x, x+\Delta)$.

$p_4(x,t)\Delta$ The Probability that at time t the system is in failed state beyond warranty, the elapsed repair time lies in the interval $[x, x+\Delta)$.

$p_2(y,t)\Delta$ The Probability that at time t the failed unit is under inspection, the elapsed inspection time lies in the interval $[y, y+\Delta)$.

$p(s)$ Laplace transform of function $p(t)$

$s(x)$ $= \mu(x)e^{\left[-\int_0^x \mu(x)dx\right]}$

$s_1(x)$ $= \mu_1(x)e^{\left[-\int_0^x \mu_1(x)dx\right]}$

$s_2(y)$ $= h(y)e^{\left[-\int_0^y h(y)dy\right]}$

5 Formulation of Mathematical Model: Using the probabilistic arguments and limiting transitions, we have the following difference-differential equations (Cox D.R. [2]):

(1) $$\left[\frac{d}{dt}+\lambda+\alpha\right]p_0(t)=\int_0^\infty \mu(x)p_3(x,t)dx+\int_0^\infty qh(y)p_2(y,t)dy$$

(2) $$\left[\frac{d}{dt}+\lambda_1\right]p_1(t)=\alpha p_0(t)+\int_0^\infty \mu_1(x)p_4(x,t)dx$$

(3) $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\ h(y)\right]p_2(y,t)=0$$

(4) $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu(x)\right]p_3(x,t)=0$$

(5) $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_1(x)\right]p_4(x,t)=0$$

Boundary conditions

(6) $$p_2(0,t)=\lambda p_0(t)$$

(7) $$p_3(0,t)=\int_0^\infty ph(y)p_2(y,t)dy$$

(8) $$p_4(0,t)=\lambda_1 p_1(t)$$

Initial conditions

$$p_i(0)=1;\quad \text{when}\quad i=0$$

(9) $$p_i(0)=0;\quad \text{when}\quad i\neq 0$$

6 Model analysis: The state transtion diagrame of the model is:

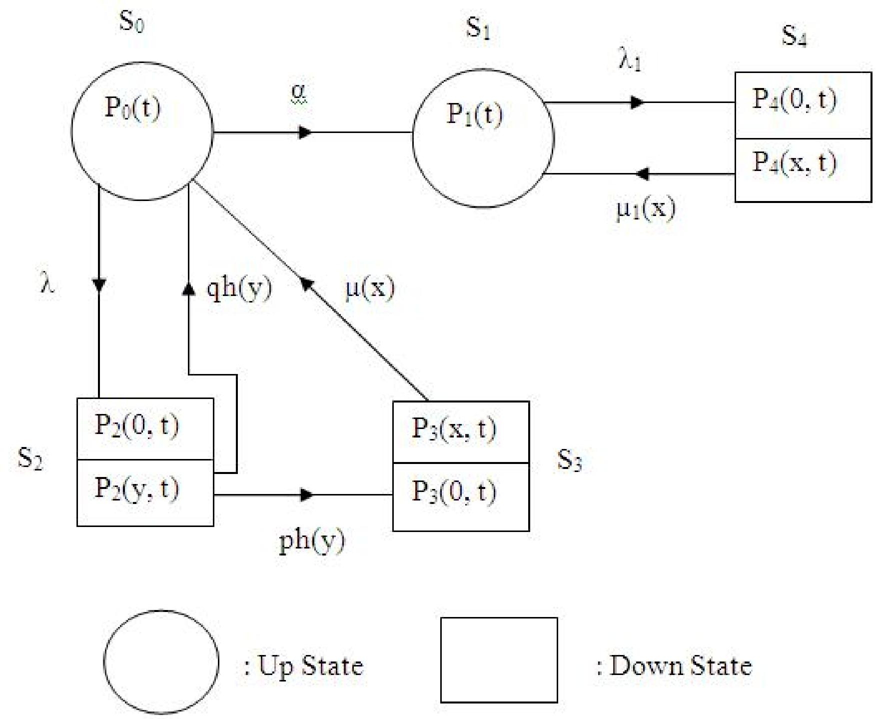

Figure 1

6.1 Solution of the equations: Taking Laplace transforms of equations (1)-(8) and using (9) we obtain

$$[s+\lambda+\alpha]\,p_0(s) = 1 + \int_0^\infty \mu(x)p_3(x,s)dx + \int_0^\infty qh(y)p_2(y,s)dy \tag{10}$$

$$[s+\lambda_1]\,p_1(s) = \alpha p_0(s) + \int_0^\infty \mu_1(x)p_4(x,s)dx \tag{11}$$

$$\left[\frac{\partial}{\partial y} + s + \ h(y)\right] p_2(y,s) = 0 \tag{12}$$

$$\left[\frac{\partial}{\partial x} + s + \mu(x)\right] p_3(x,s) = 0 \tag{13}$$

$$\left[\frac{\partial}{\partial x} + s + \mu_1(x)\right] p_4(x,s) = 0 \tag{14}$$

$$p_2(0,s) = \lambda p_0(s) \tag{15}$$

$$p_3(0,s) = \int_0^\infty ph(y)p_2(y,s)dy \tag{16}$$

$$p_4(0,s) = \lambda_1 p_1(s) \tag{17}$$

Integrating equations (12), (13) and (14), we get

$$p_2(y,s) = p_2(0,s)e^{\left[-sy-\int_0^y h(y)dy\right]} \tag{18}$$

$$p_3(x,s) = p_3(0,s)e^{\left[-sx-\int_0^x \mu(x)dx\right]} \tag{19}$$

and

$$p_4(x,s) = p_4(0,s)e^{\left[-sx-\int_0^x \mu_1(x)dx\right]} \tag{20}$$

Using equations (15) and (18), equation (16) yield

$$p_3(0,s) = \int_0^\infty ph(y)p_2(0,s)e^{\left[-sy-\int_0^y h(y)dy\right]}$$

$$p_3(0,s) = p\lambda p_0(s)S_2(s) \tag{21}$$

Using equation (21), equation (19) yields

$$p_3(x,s) = p\lambda p_0(s)S_2(s)e^{\left[-sx-\int_0^x \mu(x)dx\right]} \tag{22}$$

Using equations (15), (18) and (22), equation (10) yields

$$\begin{aligned}[s+\lambda+\alpha]p_0(s) &= 1 + p_3(0,s)\int_0^\infty \mu(x)e^{\left[-sx-\int_0^x \mu(x)dx\right]}dx \\ &\quad + p_2(0,s)q\int_0^\infty h(y)e^{\left[-sy-\int_0^y h(y)dy\right]}dy \\ &= 1 + p\lambda p_0(s)S(s)S_2(s) + q\lambda S_2(s)p_0(s)\end{aligned} \tag{23}$$

$$p_0(s) = \frac{1}{T(s)} \tag{24}$$

where

$$T(s) = s + \alpha + \lambda - \lambda p S(s) S_2(s) - q\lambda S_2(s) \tag{25}$$

Using equations (17) and (20), equation (11) yields

$$\begin{aligned}[s+\lambda]p_1(s) &= \alpha p_0(s) + p_4(0,s)\int_0^\infty \mu_1(x) e^{\left[-sx-\int_0^x \mu_1(x)dx\right]}dx \\ &= \alpha p_0(s) + \lambda p_1(s) S_1(s)\end{aligned} \tag{26}$$

$$p_1(s) = \frac{A(s)}{T(s)} \tag{27}$$

where

$$A(s) = \frac{\alpha}{(s + \lambda_1 - \lambda_1 S_1(s))} \tag{28}$$

Now, the Laplace transform of the probability that the system is in the failed state is given by

$$p_2(s) = \int_0^\infty p_2(s,y)dy = \lambda p_0(s)\frac{(1 - S_2(s))}{s} = \frac{\lambda B(s)}{T(s)} \tag{29}$$

where

$$B(s) = \frac{1 - S_2(s)}{s} \tag{30}$$

Similarly

$$p_3(s) = \int_0^\infty p_3(s,x)dx = \lambda p S_2(s) p_0(s)\frac{(1 - S(s))}{s} = \frac{\lambda p S_2(s) C(s)}{T(s)} \tag{31}$$

where

$$C(s) = \frac{1 - S(s)}{s} \tag{32}$$

similarly

$$p_4(s) = \int_0^\infty p_4(s,x)dx = \lambda_1 p_1(s)\frac{(1 - S_1(s))}{s} = \frac{\lambda_1 A(s) D(s)}{T(s)} \tag{33}$$

where

$$D(s) = \frac{1 - S_1(s)}{s} \tag{34}$$

It is worth noticing that

$$p_0(s) + p_1(s) + p_2(s) + p_3(s) + p_4(s) = \frac{1}{s} \tag{35}$$

6.2 Evaluation of Laplace transforms of up and down state probabilities: Let $Av(t)$ is the probability that the system is operating satisfactorily at time t. The Laplace transforms of $Av(t)$ or probabilities that the system is in up $P_{up}(t)$ (i.e. good) and down $P_{down}(t)$ (i.e. failed) state at time "t" are as follows

$$Av(s) \text{ or } P_{up}(s) = p_0(s) + p_1(s)$$

$$Av(s) = \frac{1 + A(s)}{T(s)} \tag{36}$$

$$P_{down}(s) = p_2(s) + p_3(s) + p_4(s)$$

$$P_{down}(s) = \frac{\lambda B(s) + \lambda p C(s) S_2(s) + \lambda_1 A(s) D(s)}{T(s)} \tag{37}$$

6.3 Steady-State Behaviour of the System: In the long run as t tends to infinity, the steady state behaviour of the system can be obtained by using Abel′s Lemma in Laplace transforms, viz.
$\lim_{s \to 0} s[Av(s)] = \lim_{n \to \infty}[Av(t)] = Av(\text{say})$, Provided the limit on the right hand side exists, the following time independent probabilities have been obtained.

$$Av = \frac{1}{1 - \lambda_1 S_1'(0)} \tag{38}$$

$$P_{down} = \frac{-\lambda_1 S_1'(0)}{1 - \lambda_1 S_1'(0)} \tag{39}$$

6.4 Reliability of the system: Let $R(t)$ is the probability that the system performs well in an interval $(0, t]$. Therefore in order to obtain $R(t)$, the differential-difference equations for reliability are:

$$\left[\frac{d}{dt} + \lambda + \alpha\right] p_0(t) = 0 \tag{40}$$

$$\left[\frac{d}{dt} + \lambda_1\right] p_1(t) = \alpha p_0(t) \tag{41}$$

Theorem 1. The reliability of the system is given by

$$R(t) = e^{-(\lambda+\alpha t)}\left[\frac{(\lambda - \lambda_1)}{(\lambda - \lambda_1 + \alpha)}\right] + e^{-(\lambda_1 t)}\left[\frac{\alpha}{(\lambda - \lambda_1 + \alpha)}\right]$$

Proof. Taking Laplace transforms of (40), (41) and using (9), we get

$$[s + \lambda + \alpha]p_0(s) = 1 \tag{42}$$

$$[s + \lambda_1]p_1(s) = \alpha p_0(s) \tag{43}$$

The solution can be written as

$$p_0(s) = \frac{1}{(s + \lambda + \alpha)} \tag{44}$$

$$p_1(s) = \frac{\alpha}{(s + \lambda + \alpha)(s + \lambda_1)} \tag{45}$$

$$R(s) = p_0(s) + p_1(s) = \frac{1}{(s + \lambda + \alpha)} + \frac{\alpha}{(s + \lambda + \alpha)(s + \lambda_1)}$$

Taking inverse Laplace transform, we get

$$R(t) = e^{-(\lambda + \alpha t)} \left[\frac{(\lambda - \lambda_1)}{(\lambda - \lambda_1 + \alpha)} \right] + e^{-(\lambda_1 t)} \left[\frac{\alpha}{(\lambda - \lambda_1 + \alpha)} \right] \tag{46}$$

Corollary 1. The mean time to system failure (MTSF) is:

$$MTSF = \left[\frac{(\lambda - \lambda_1)}{(\lambda - \lambda_1 + \alpha)(\lambda + \alpha)} \right] + \left[\frac{\alpha}{(\lambda - \lambda_1 + \alpha)\lambda_1} \right]$$

Proof. As MTSF is the expected time for which the system is in operation before it completely fails.

$$\therefore \quad MTSF = \int_0^\infty R(t)dt$$

$$MTSF = \int_0^\infty \left\{ e^{-(\lambda + \alpha)t} \left(\frac{(\lambda - \lambda_1)}{(\lambda - \lambda_1 + \alpha)} \right) + e^{-(\lambda_1)t} \left(\frac{\alpha}{(\lambda - \lambda_1 + \alpha)} \right) \right\} dt$$

$$MTSF = \left(\frac{(\lambda - \lambda_1)}{(\lambda - \lambda_1 + \alpha)(\lambda + \alpha)} \right) + \left(\frac{\alpha}{(\lambda - \lambda_1 + \alpha)\lambda_1} \right) \tag{47}$$

7 Particular cases:

7.1 Availability of the system:

When repair and inspection time follows exponential distribution i.e. setting

$$S(s) = \frac{\mu}{(s+\mu)}, S_1(s) = \frac{\mu_1}{(s+\mu_1)} \text{ and } S_2(s) = \frac{h}{(s+h)}$$

where μ and μ_1 are constant repair rates and h is constant inspection rate. Putting these values in equations (24)-(28), we get

$$p_0(s) = \frac{1}{I(s)} \tag{48}$$

where

$$I(s) = \frac{s^3 + s^2(\lambda+\alpha+\mu+h) + s(\mu h + \lambda h + \lambda\mu + \alpha h + \alpha\mu - q\lambda h) + \alpha\mu h}{(s+\mu)(s+h)} \tag{49}$$

$$p_1(s) = \frac{E(s)}{I(s)} \tag{50}$$

where

$$E(s) = \left[\frac{\alpha(s+\mu_1)}{s(s+\lambda_1+\mu_1)}\right] \tag{51}$$

$$Av(s) \text{ or } P_{up}(s) = p_0(s) + p_1(s)$$

$$Av(s) = \left[\frac{(s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0)}{s(s+\lambda_1+\mu_1)(s^3 + a_2 s^2 + a_1 s + a_0)}\right] \tag{52}$$

where $b_3 = (\lambda_1+\mu+\alpha+\mu_1+h), b_2 = (\lambda_1\mu+\mu\alpha+\alpha\mu_1+\mu_1 h+\mu h+\mu\mu_1+\lambda_1 h+h\alpha)$, $b_1 = (\mu\mu_1 h + \lambda_1\mu h + \alpha\mu\mu_1 + h\alpha\mu + h\alpha\mu_1)$ and $b_0 = (\alpha\mu\mu_1 h)$ and $a_2 = (\lambda+\mu+\alpha+h), a_1 = (\lambda\mu+\mu\alpha+\mu h+\lambda h+h\alpha-qh\lambda)$ and $a_0 = \mu\alpha h$

Taking inverse Laplace transforms of equation (52), we get

(53)

$$Av(t) = \frac{-b_0}{(\lambda_1+\mu_1)z_1z_2z_3} + \left\{\frac{(\lambda_1+\mu_1)^4 - b_3(\lambda_1+\mu_1)^3 + b_2(\lambda_1+\mu_1)^2 - b_1(\lambda_1+\mu_1) + b_0}{(\lambda_1+\mu_1)(\lambda_1+\mu_1+z_1)(\lambda_1+\mu_1+z_2)(\lambda_1+\mu_1+z_3)}\right\} e^{-(\lambda_1+\mu_1)t} + \left\{\frac{(z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0)}{z_1(\lambda_1+\mu_1+z_1)(z_1-z_2)(z_1-z_3)}\right\} e^{z_1t} + \left\{\frac{(z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0)}{z_2(\lambda_1+\mu_1+z_2)(z_2-z_2)(z_2-z_3)}\right\} e^{z_2t} + \left\{\frac{(z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0)}{z_3(\lambda_1+\mu_1+z_3)(z_3-z_1)(z_3-z_2)}\right\} e^{z_3t}$$

z_1, z_2 and z_3 are three roots of the equation $s^3 + s^2a_2 + sa_1 + a_0 = 0$ (E. Balagurusamy [1])

7.2 Profit analysis of the user: Suppose that the warranty period of the system is $(0, w]$. Since the repairman is always available with the system, therefore beyond warranty period, it remains busy during the interval $(w, t]$. Let K_1 be the revenue per unit time and K_2 be the repair cost per unit time, then the expected profit $H(t)$ during the interval $(0, t]$ is given by

$$H(t) = K_1 \int_0^t Av(t)dt - K_2(t-w) \tag{54}$$

$$= K_1 \left[\frac{-b_0t}{(\lambda_1+\mu_1)z_1z_2z_3} + \left\{\frac{(\lambda_1+\mu_1)^4 - b_3(\lambda_1+\mu_1)^3 + b_2(\lambda_1+\mu_1)^2 - b_1(\lambda_1+\mu_1) + b_0}{(\lambda_1+\mu_1)^2(\lambda_1+\mu_1+z_1)(\lambda_1+\mu_1+z_2)(\lambda_1+\mu_1+z_3)}\right\} (1 - e^{-(\lambda_1+\mu_1)t}) + \left\{\frac{(z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0)}{z_1^2(\lambda_1+\mu_1+z_1)(z_1-z_2)(z_1-z_3)}\right\} (e^{z_1t} - 1) + \left\{\frac{(z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0)}{z_2^2(\lambda_1+\mu_1+z_2)(z_2-z_2)(z_2-z_3)}\right\} (e^{z_2t} - 1) + \left\{\frac{(z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0)}{z_3^2(\lambda_1+\mu_1+z_3)(z_3-z_1)(z_3-z_2)}\right\} (e^{z_3t} - 1)\right] - K_2(t-w)$$

8 Numerical Computations: In order to study the behaviours of reliability $R(t)$ and expected profit $H(t)$ mentioned in equations (46) and (54) respectively, some numerical results are presented in the form of tables for $R(t)$ and $H(t)$ for particular values of various parameters w.r.t. time t as:

Table 1: Effect of failure rates (λ and λ_1) on Reliability ($R(t)$)

Time(t)	$\lambda_1 = 0.02$, $\alpha = 0.003$		$\lambda = 0.01$, $\alpha = 0.003$	
	$R(t)$ (for $\lambda = 0.01$)	$R(t)$ (for $\lambda = 0.02$)	$R(t)$ (for $\lambda_1 = 0.01$)	$R(t)$ (for $\lambda_1 = 0.03$)
10	0.90353744	0.8187308	0.9048374	0.9023208
11	0.89428347	0.8025188	0.8958341	0.8928418
12	0.88510119	0.7866279	0.8869204	0.8834209
13	0.87599061	0.7710516	0.8780954	0.8740593
14	0.86695174	0.7557837	0.8693582	0.864758
15	0.85798456	0.7408182	0.860708	0.8555182
16	0.84908904	0.726149	0.8521438	0.8463406
17	0.84026512	0.7117703	0.8436648	0.8372263

Table 2: Effect of rate of completion of warranty (α) on Reliability ($R(t)$)

Time(t)	λ	λ_1	$R(t)$ (for $\alpha = 0.005$)	$R(t)$ (for $\alpha = 0.004$)	$R(t)$ (for $\alpha = 0.003$)
10	0.01	0.02	0.9026852	0.90310989	0.9035374
11	0.01	0.02	0.89326861	0.89377417	0.8942835
12	0.01	0.02	0.88391256	0.884504484	0.8851012
13	0.01	0.02	0.87461773	0.875301177	0.8759906
14	0.01	0.02	0.86538475	0.866164563	0.8669517
15	0.01	0.02	0.85621422	0.857094929	0.8579846
16	0.01	0.02	0.84710669	0.848092533	0.849089
17	0.01	0.02	0.83806267	0.838063	0.840265

Table 3: Effect of repair cost (K_2) on Expected Profit ($H(t)$)

Time(t)	$\lambda = 0.01$,$\lambda_1 = 0.02$, $h = 0.5$, $\alpha = 0.003$,$\mu = 0.2$, $\mu_1 = 0.1$, $q = 0.3$				
	K_1	W	$H(t)$ (for $K_2 = 150$)	$H(t)$ (for $K_2 = 100$)	$H(t)$ (for $K_2 = 50$)
10	500	3	3799.487	4149.487	4499.487
11	500	3	4125.804	4525.804	4925.804
12	500	3	4451.44	4901.44	5351.44
13	500	3	4776.5	5276.5	5776.5
14	500	3	5101.066	5651.066	6201.066
15	500	3	5425.205	6025.205	6625.205
16	500	3	5748.973	6398.973	7048.973
17	500	3	6072.412	6772.412	7472.412

Table 4: Effect of repair cost (K_2) and constant inspection rate (h) on Expected Profit ($H(t)$)

Time(t)	$\lambda = 0.01$,$\lambda_1 = 0.02$, $\alpha = 0.003$, $\mu = 0.2$, $\mu_1 = 0.1$, $q = 0.3$, $h = 0.5$		$\lambda = 0.01$,$\lambda_1 = 0.02$, $\alpha = 0.003$, $\mu = 0.2$, $\mu_1 = 0.1$, $q = 0.3$, $h = 0.6$	
	$H(t)$ (for $K_2 = 150$)	$H(t)$ (for $K_2 = 100$)	$H(t)$ (for $K_2 = 150$)	$H(t)$ (for $K_2 = 100$)
10	3799.487	4149.487	3805.808	4155.808
11	4125.804	4525.804	4133.338	4533.338
12	4451.44	4901.44	4460.236	4910.236
13	4776.5	5276.5	4786.594	5286.594
14	5101.066	5651.066	5112.49	5662.49
15	5425.205	6025.205	5437.982	6037.982
16	5748.973	6398.973	5763.121	6413.121
17	6072.412	6772.412	6087.945	6787.945

9 Interpretation and Conclusion Tables 1 and 2 show the behavior of system reliability. Table 1 indicates that the reliability of the system decreases with the increase of failure rates (λ and λ_1) with respect to (w.r.t.) time 't' and for fixed values of other parameters. From table 2, it is analyzed that the reliability of the system increases with the decrease of rate of completion of warranty (α) w.r.t. time 't'. It reveals that the system becomes more reliable for users as we increase time duration of warranty because any failure during warranty is rectified free of cost to the users. Table 3, shows that expected profit $H(t)$ during the interval $(0, t]$ increases with the decrease of repair cost (K_2) w.r.t. time 't' . Also, table 4 represents that the expected profit increase with the increase of inspection rate (h) of the failed unit w.r.t time

't'. This shows that inspection during warranty is profitable to manufacturer because it protected manufacturer about unnecessary expenses on repair of a continuously usage system or unit.

Hence, on the basis of the above discussion and the results obtained for a particular case (as mentioned in section 7), it is concluded that the concept of reliability and profit analysis of a single-unit system with inspection and warranty can be made more reliable and profitable to user and manufacturer both by decreasing the rate of completion of warranty, repair cost and increasing inspection rate.

References

[1] Balagurusamy E. 2009. Reliability Engineering, McGraw-Hill, New York.

[2] Cox D.R. 1962. Renewal theory, John Wiley and Sons INC, New York.

[3] Kadyan M S, Promila, Jitender Kumar 2013. Reliability modeling of a single-unit system with arbitrary distribution subject to different weather condition. Int J Syst Assur Eng Manag, DOI10.1007/s13198-013-0168-3.

[4] Kaur M, Lal A K, Bhatia S S, Reddy A S (2013) Numerical solution of stochastic partial differential difference equation arising in reliability engineering. Applied Mathematics and Computation 219: 7645-7652.

[5] Kharoufeh J.P., Solo C.J. and Ulukus M.Y. 2010. Semi-Markov models for degradation-based reliability, IIE Transactions, 42, 599612.

[6] Xiaoning Jin, Lin Li and Jun Ni 2009. Option model for joint production and preventive maintenance system, Int. J. Production Economics, 119, 347-353.

[7] Yeh R.H., Chen H.D. and Wang C.H. 2005. An inspection model with discount factor for products having weibull lifetime, Int. Journal of Operation Research, 2(1), 77-81.