Partial Differential Equations Method to Analyze Nonlinear Resilience Force in a Long Clearance Seal and Rotor's Dynamics.

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Abstract. The authors have developed a method of calculating the non-linear elastic force that arises in the clearance seal of a finite length. The authors have analyzed the force impact on the dynamics of the rotor of a centrifugal machine. In this work authors studied impact of nonlinear hydrostatic force in an arbitrary long seal, opposite to the previous literature where this force assumed to be linear, and seals are short (e.g. the ratio of the length of the clearance seal to the seal's radius is less than 0.5).

1 Introduction

1.1 Metodological novelty. In this work authors present a model of the non-linear hydrostatic force in an arbitrary long seal. After establishing the model authors present numerical experiments in order to illustrate obtained results. Mainly, the novelty of presented work consists of relaxing the linearity assumption made in previous works [1] and [2]. Also, authors generalized the model in order to fit arbitrary seal length, and only so called "short seal" as previously (e.g. the ratio of the length of the clearance seal to the seal's radius is less than 0.5).

1.2 The partial differential equation role in the model. Basically, the usage of partial differential equation for dynamic processes was established by Newton by his famous relation $\overline{F} = m \cdot \overline{a}$. Rewriting this relation in terms of X and Y projections we will get the following system:

$$\begin{cases} m\ddot{x} = F_x \\ m\ddot{y} = F_y \end{cases}.$$

Solution of the system above will present rotor linear movements, or fractions. They should be kept as low as possible in order to keep the rotor well balanced.

1.3 Historical background. One of the pioneer investigators in this area was Prof. Dr. Lomakin. In 1953, he established a study of rotor seals frictions [1]. He has studied the problem connected with fast rebalancing of CBP-220-280. He resolved the problem connected with vibration by using the clearance seals of a different shape.

Similar problems encountered the NASA team headed by Dr. Childs, in [9] he mentioned that dynamic instability of main engine of the shuttle was explained by vibrations due to hydrodynamic forces in the rotor seal. Again, experimental change of the seals shape allowed resolving the issue.

1.4 Technical background. In the flow of the hydraulic machines, for the removal of significant flows of fluid from the high pressure zone to a zone of the lower pressure, clearance seals are used. Their sealing effect is determined by the large hydraulic resistance of the O-ring throttle with a small (0.1-0.35 mm) radial clearance. In literature there are numerous publications, which demonstrate that clearance seals of centrifugal machines significantly affect the rotor dynamic characteristics: arising hydrodynamic forces in the seal, depending on the design and operating conditions of the seal, may reduce vibroactivity of the rotor, or vice versa, lead to its dynamic imbalance. Most fully this problem is indicated in [1, 2, 3]. However, they consider a model of so-called "short" seals in which the circumferential component of fluid velocity, due to the pressure field, is neglected. When using the seals, wherein the circumferential component of the axial fluid is comparable to, or even exceeds it, the dynamic characteristics of the rotor vary significantly [4]. In [6, 7] they propose a model of a clearance seal of a finite length, for which there are obtained analytical expressions for the dimensionless coefficients of elastic and damping forces of the clearance seal.

It is shown that these coefficients depend only on two dimensionless parameters $\frac{l}{r}$, $\frac{r}{h_0}$, which are determined by geometry of the clearance (l- length, r- radius, h_0 - medium radial clearance). The ratio of the coefficients of elastic (K_c) and damping (K_b) forces, obtained by techniques of the short and the finite length clearance seals, are shown in Figure 1.

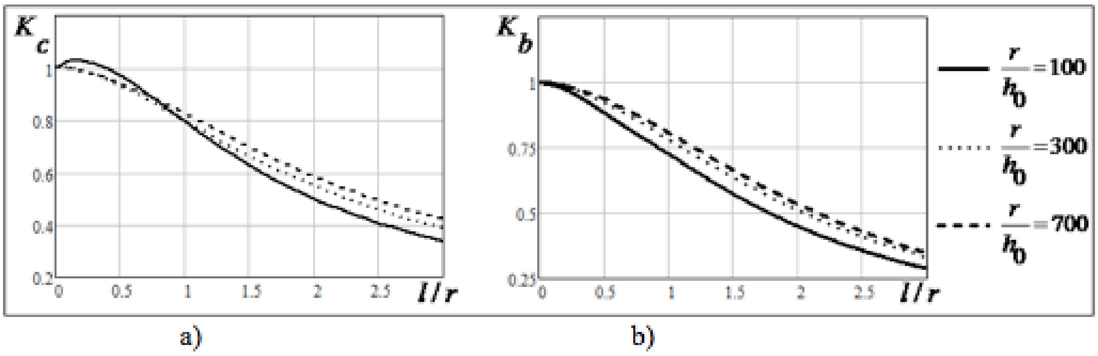

Figure 1: Dependence of the ratio of the coefficients:
a) for elastic force; b) for damping force

Apparently, with level growth of $\frac{l}{r}$ the rigidity and damping coefficients are significantly reduced (due to influence of circumferential overflows of fluid in the annular channel due to pressure field). That leads to significant deterioration of the vibratory state. Therefore, the problem of studying the impact of the circumferential overflows of fluid on hydrodynamic forces in the clearance seal of finite length is currently of great importance.

In this paper, we investigate the influence of fluid overflows in the circumferential gaps caused by pressure field on the nonlinearity of the elastic force, and the influence of the latter on the rotor dynamics.

The problem is solved with the following simplifying assumptions:

1) We considered the annular channels, for which the radial clearance is substantially less than the diameter.

2) The flow pattern across the gap is a self-similar region of the turbulent flow.

3) We consider the isothermal flow.

Assumptions 1-3) are quite natural for solving engineering problems connected to power machinery and rotor industry. Indeed, assumption 1) is applicable to general features of heavy industrial rotors. Assumption 2) reflects the best existing way to describe the fluid movements under high pressure and high rotation intensity. 3) Modern cooling systems give us abilities to assume that the fluid temperature will not change through the process. The rest of the paper will be presented as follows: in Part 2 authors will present and develop their model, Part 3 presents obtained numerical results and part 4 (Conclusions) finishes the paper.

2 The Model.

2.1 The study of the elastic force. Fluid motion in the clearance seal without inertial components is described by the system of equations introduced in [1]

$$\begin{cases} \dfrac{\partial p(z,\varphi)}{\partial \varphi} = -\dfrac{\lambda r}{2h_0}\dfrac{\rho w_0}{2}u(z,\varphi), \\ \dfrac{\partial p(z,\varphi)}{\partial z} = -\dfrac{\lambda l}{2h(\varphi)}\dfrac{\rho w^2(z,\varphi)}{2}, \\ \dfrac{\partial(w(z,\varphi)\cdot h(\varphi))}{\partial z} + \dfrac{l}{r}\dfrac{\partial(u(z,\varphi)\cdot h_0)}{\partial \varphi} = 0 \end{cases}$$

with boundary conditions

$$\begin{cases} p(0,\varphi) = p_{10} - \xi_1 \cdot \dfrac{\rho \cdot w^2(0,\varphi)}{2}, \\ p(1,\varphi) = p_{20} - \xi_2 \cdot \dfrac{\rho \cdot w^2(1,\varphi)}{2}, \end{cases}$$

where

$p(z,\varphi)$ - pressure of the fluid in the annular gap;
$w(z,\varphi)$ - axial velocity caused by pressure field;
$u(z,\varphi)$ - circumferential speed caused by pressure field;
$h(\varphi)$ - the value of the radial clearance;
ρ - density of the fluid;
λ - coefficient of the hydraulic friction;
p_{10} - pressure of the fluid in front of the clearance seal;
p_{20} - pressure of the fluid behind the clearance seal;
ξ_1 - coefficient of input losses;
ξ_2 - recovery ratio of the axial velocity downstream of the seal.

This system is transformed to quasilinear elliptic equation

$$\frac{(1-\varepsilon\cdot\cos\varphi)^2}{2\cdot l_r^2}\cdot\frac{\partial^2 p}{\partial z^2} + \sqrt{\frac{-(1-\varepsilon\cdot\cos\varphi)\cdot\xi_0}{\xi_1\cdot\Delta p}\cdot\frac{\partial p}{\partial z}}\cdot\frac{\partial^2 p}{\partial \varphi^2} = 0$$

with boundary conditions

$$\begin{cases} p(0,\varphi) = p_{10} + \xi_1 \cdot \dfrac{1-\varepsilon\cdot\cos\varphi}{\xi_1}\cdot\dfrac{\partial p(0,\varphi)}{\partial z}, \\ p(1,\varphi) = p_{20} + \xi_2 \cdot \dfrac{1-\varepsilon\cdot\cos\varphi}{\xi_1}\cdot\dfrac{\partial p(1,\varphi)}{\partial z}, \\ p(z,0) = p(z,2\pi), \end{cases}$$

where

$l_r = \dfrac{l}{r}$, $\varepsilon = \dfrac{e}{h_0}$ - dimensionless parameters;
ξ_1 - loss coefficient along the clearance seal;
ξ_0 - the total loss coefficient in the clearance seal;
Δ_p - pressure drop across the gap.

To solve this equation we used the grid method applying the method of successive approximations. The solution is the fluid pressure values at the mesh point at a predetermined relative eccentricity. After interpolation of the obtained values by two-dimensional cubic spline, we got the distribution of pressure in the gap. The elastic force is then determined by formula

$$F(\varepsilon) = -r\cdot l\cdot\int_0^{2\pi}\int_0^1 p(z,\varphi,\varepsilon)\cdot\cos\varphi dz d\varphi.$$

where

$P_i(\varepsilon)$ - the ith Legendre polynomial;
$a_i = \dfrac{2i+1}{2}\int_{-1}^1 f(\varepsilon)\cdot P_i(\varepsilon)d\varepsilon$ - are expansion coefficients;

$f(\varepsilon)$ - spline of the table data.

This result can be represented as

$$F(\varepsilon) = -k_c(0) \cdot h_0 \cdot \varepsilon \cdot \alpha(\varepsilon),$$

where
$k_c(0)$ - stiffness coefficient of linearized elastic force;
$\alpha(\varepsilon) = 1 + \alpha_1\varepsilon + \alpha_2\varepsilon^2 + \ldots + \alpha_n\varepsilon^n$ - dimensionless coefficient of nonlinear elastic force.

Figure 2 shows dependence of this ratio for some types of clearance seals. As one can see from the figure, the amount of elastic force decreases with increasing eccentricity. At the same time, the influence of parameter $\frac{r}{h_0}$ is irrelevant.

Analytical expression for stiffness of elastic force, depending on the displacement of the shaft is determined by formula

$$k_c(\varepsilon) = \frac{dF(\varepsilon)}{de} = k_c(0) \cdot \beta(\varepsilon),$$

where
$\beta(\varepsilon) = 1 + 2\alpha_1\varepsilon + 3\alpha_2\varepsilon^2 + \ldots + (n+1)\alpha_n\varepsilon^n$ - dimensionless coefficient of nonlinear stiffness.

Dependence of coefficient $\beta(\varepsilon)$ is shown in Figure 3. It's evident that stiffness of elastic force decreases with increasing displacement of the shaft, i.e. this system has a soft characteristic of stiffness. As far as we know, this fact deteriorates the vibration characteristics of the rotor.

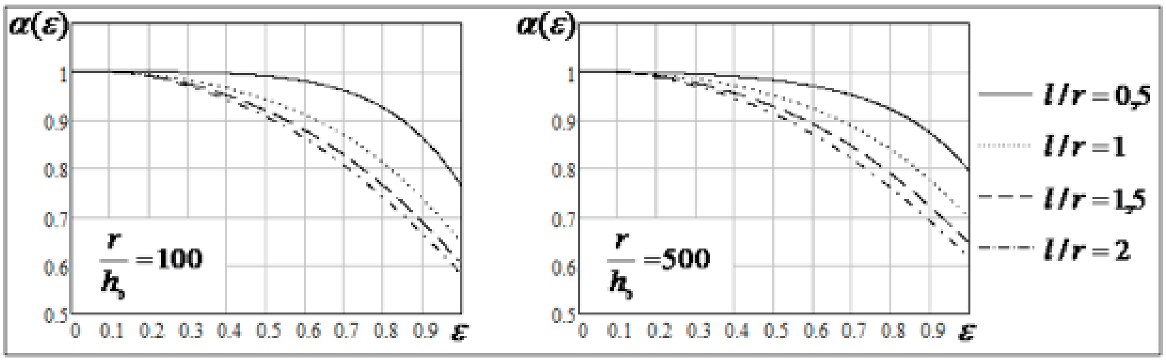

Figure 2: Dependence of nonlinearity coefficient of elastic force on relative eccentricity

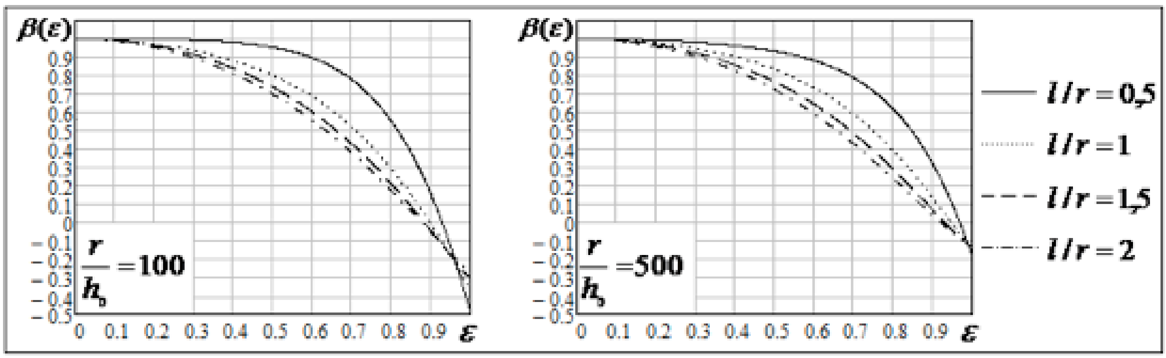

Figure 3: Dependence of nonlinearity coefficient of stiffness on relative eccentricity

2.2 Investigation of dynamic characteristics of the rotor To study the effect of nonlinear force on the dynamic characteristics of the rotor we consider a single-mass rotor model (Figure 4) with the parameters of the shaft: length $l = 520$ mm and diameter $d = 25$ mm; rotor mass $m = 18$ kg; clearance seal geometry: length $l = 48$ mm and radius $r = 25$ mm; average radial clearance $h_0 = 0.3$ mm, and pressure drop across the gap $\Delta_p = 1.25$ MPa.

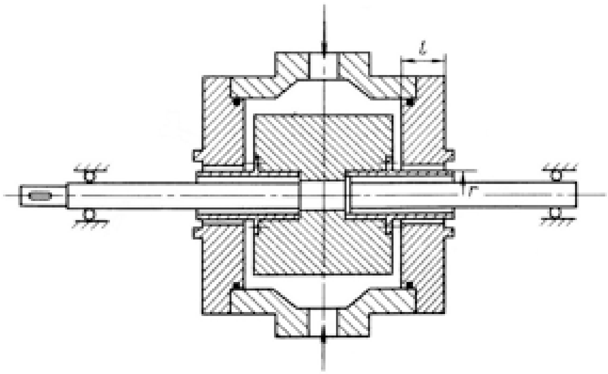

Figure 4: A single-mass rotor model

The structure of hydrodynamic forces, arising in the gap sealing, can be assumed as

$$\begin{cases} F_x = -b \cdot \dot{x} - k_c(0)\alpha(r) \cdot x - q \cdot y, \\ F_y = -b \cdot \dot{y} - k_c(0)\alpha(r) \cdot y - q \cdot x, \end{cases}$$

where
b - damping coefficient;
$q = 0.5b\omega$ - circulation ratio;
x, y - displacement coordinates of the shaft center in the fixed coordinate system;
$r = \sqrt{x^2 + y^2}$ - radius of the shaft movement orbit.

The damping coefficient was determined according to [6], nonlinear elastic force projection on fixed axes by the above method. For the considered clearance seals we have

$$b = 5079 \frac{H \cdot c}{\mathcal{M}}, \qquad k_c(0) = 1,035 \cdot 10^6 \frac{H}{\mathcal{M}},$$

$\alpha(r) = 1 + 0,057 \cdot r - 0,515 \cdot r^2 + 0,076 \cdot r^3 - 0,047 \cdot r^4 - 0,104 \cdot r^5 - 0,068 \cdot r^6 + 0,075 \cdot r^7 + 0,032 \cdot r^8 - 0,02 \cdot r^9$

Differential equations of motion of this model have the form

$$\begin{cases} m\ddot{x} + b\dot{x} + c_b x + k_c(0)\alpha(r) \cdot x + qy &= me_1\omega^2 \cos(\omega t); \\ m\ddot{y} + b\dot{y} + c_b y + k_c(0)\alpha(r) \cdot y - qx &= me_1\omega^2 \sin(\omega t); \end{cases} \tag{1}$$

where
m - rotor weight; c_b - stiffness of the shaft; me_1 - rotor imbalance; ω - speed of rotation.

Having entered the designations

$$\tau = t \cdot \omega_0,\ \frac{d}{d\tau} = \omega_0 \frac{d}{dt},\ \omega_0 = \sqrt{\frac{c_b + k_c(0)}{m}},\ \overline{x} = \frac{x}{h_0},\ \overline{y} = \frac{y}{h_0},\ \overline{b} = \frac{b\omega_0}{c_b + k_c(0)},\ \overline{\omega} = \frac{\omega}{\omega_0},\ \overline{e} = \frac{e_1}{h_0},$$

system (1) is written in the dimensionless form (hereinafter, for the convenience, $\overline{x}$ and $\overline{y}$ will be written in the form x and y).

$$
\begin{cases}
\ddot{x} + b\dot{x} + x\left(1 + \dfrac{k_c(0)}{c_b + k_c(0)}\left(\alpha_1 r + \alpha_2 r^2 + \ldots + \alpha_9 r^9\right)\right) + 0,5\bar{b}\overline{\omega} y = & \overline{e\omega}^2\cos(\overline{\omega}\tau) \\
\ddot{y} + b\dot{y} + y\left(1 + \dfrac{k_c(0)}{c_b + k_c(0)}\left(\alpha_1 r + \alpha_2 r^2 + \ldots + \alpha_9 r^9\right)\right) - 0,5\bar{b}\overline{\omega} x = & \overline{e\omega}^2\sin(\overline{\omega}\tau)
\end{cases}
\tag{2}
$$

3 Results of numerical experiments Numerical solution of (2) was performed using software package Mathcad. We obtained the area of sustainable movement of the shaft, as well as its rotational speed with boundary condition of stability. In this case the rotational speed with boundary condition of stability was determined by the appearance of a subharmonic self-oscillating imposition when the relative speed of the rotor $\overline{\omega}$ varied.

As an example, Figure 5 shows the oscillation of the shaft center in a horizontal plane (a), the orbit of the shaft center (b), and the relevant spectrum (c) at the speed of rotation $\overline{\omega} = 1,5$. As can be seen, the shaft makes a steady circular motion with the rotational frequency.

Figure 6 shows the oscillation of the shaft center in the horizontal plane (a), the orbit of the shaft center (b), and a corresponding spectrum (c) at the boundary of the unstable region of rotation $\overline{\omega} = 2,01$. Thus, along with a synchronous component, there is a subharmonic one with an amplitude which exceeds the amplitude of a synchronous component.

Figure 7 shows the oscillations of the shaft center in the horizontal plane (a) and the orbit of the shaft center (b) in the unstable region of rotation. In this case, we observe a rapid growth of the shaft displacement, leading to an emergency mode.

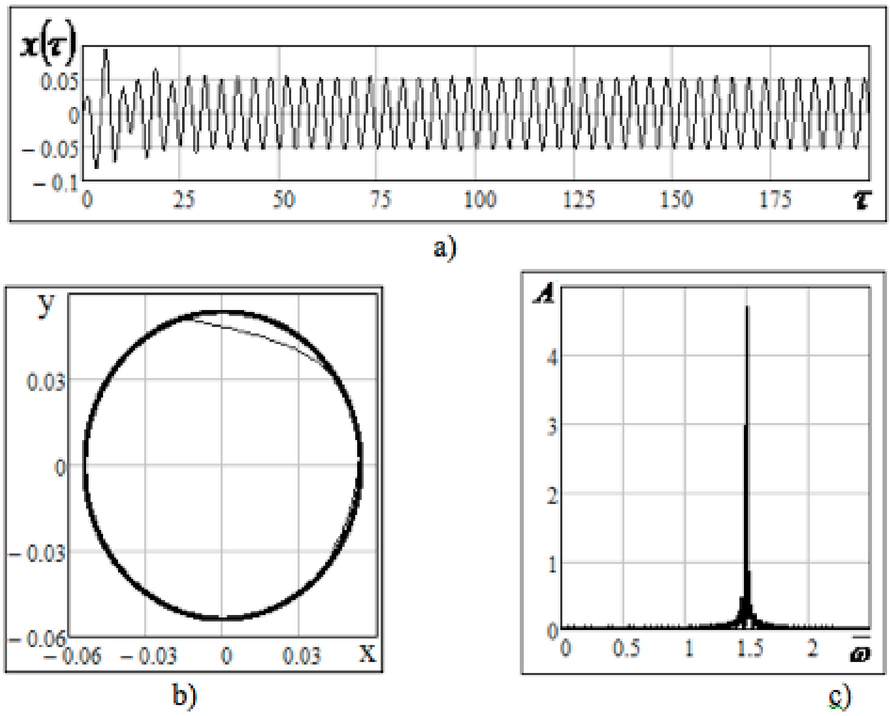

Figure 5: Oscillations of the shaft center in a horizontal plane (a), the orbit of the shaft movement (b), and the oscillation spectrum (c) in the stable region of rotation $\overline{\omega} = 1.5$

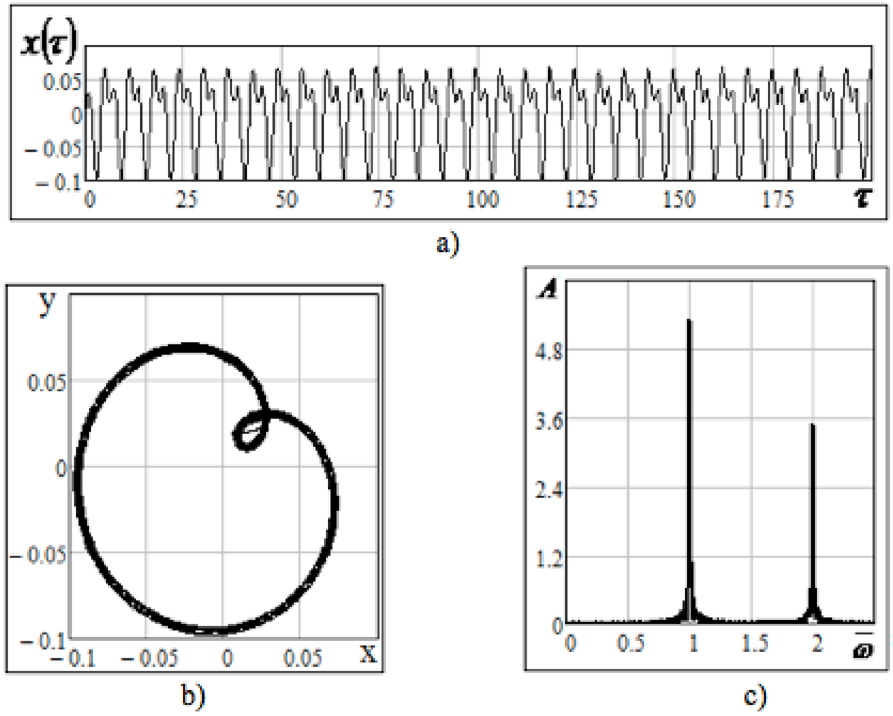

Figure 6: Oscillations of the shaft center in the horizontal plane (a), the orbit of the shaft movement (b) and the oscillation spectrum (c) at the stable region of rotation $\overline{\omega} = 2.01$

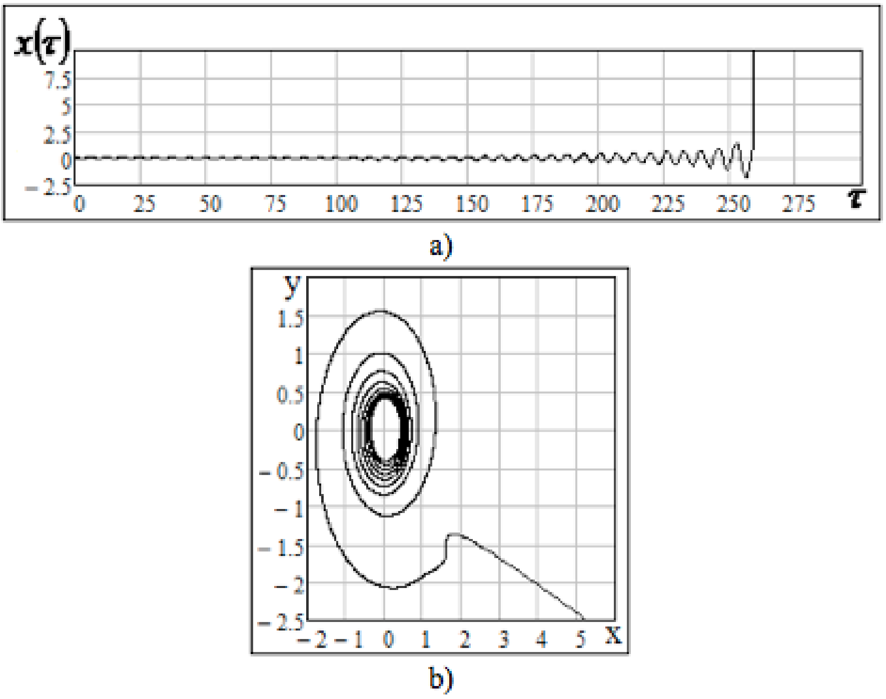

Figure 7: Oscillations of the shaft center in the horizontal plane (a), the orbit of the shaft movement (b) in the unstable region of rotation $\overline{\omega} = 2.05$

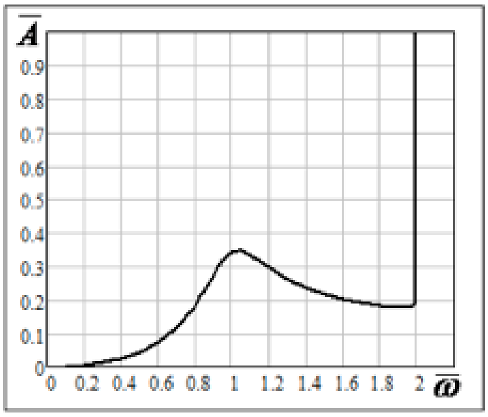

Figure 8: Dependence of the relative amplitude of the synchronous component of the rotor oscillations on the speed of rotation

Studies show that the differences between the calculations of the dynamic characteristics of the rotor according to the procedures of short clearance seals and clearance seals of finite length can be both quantitative and qualitative. For example, from Figure 8 we can see that in the unstable region of rotation the rotor performs oscillations with steady amplitude. Moreover, the total vibration level for speed of rotation above the boundary remains within acceptable limits. At the same time, Figures 6,7 and 8 show that for the investigated rotor model, which has a mild characteristic of stiffness, self-oscillating mode takes place only on the stability boundary; then shaft displacement increases rapidly, i.e. for the "long clearance" model the "emergency" effect occurs immediately after buckling.

4 Conclusions: There are two main points that make presented work different from previous ones, as [1] and [2]: First, a new method of calculating the *nonlinear* quasi-elastic force in a relatively long clearance seal is developed and some numerical results are presented. Assumption of linearity is too general and may lead to substantial problems why applied to the real world machinery.

Second, the rotor dynamics is investigated, taking into account the nonlinearity of the quasi-elastic force. It is shown that flows in the circumferential direction, due to the pressure field, reduce the elastic force in the gap sealing, which leads to deterioration of the dynamic characteristics of the rotor, which is quite interesting result and have some direct application in machinery industry involving rotor systems.

As for future plans: authors will do theoretical investigation of *nonlinear* damper and circular forces in short and long clearance seals, in order to evaluate rotor dynamics for these cases.

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