SEMI-G-STABLE IN DITOPOLOGICAL TEXTURE SPACES

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Received September 18, 2014 ; revised December 14, 2015

Abstract. In this paper, the author introduce and study new notions of continuity, compactness and stability in ditopological texture spaces based on the notions of semi-g-open and semi-g-closed sets and some of their characterizations are obtained.

1 Introduction Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study fuzzy topology. The study of compactness and stability in ditopological texture spaces was started to begin in [6]. In this paper, we introduce and study the concepts of semi-g-bicontinuity, semi-g-bi-irresolute, semi-g-compactness and semi-g-stability in ditopological texture spaces.

2 Preliminaries The following are some basic definitions of textures we will need later on.

Texture space: [6] Let S be a set. Then $\varphi \subseteq P(S)$ is called a texturing of S, and S is said to be textured by φ if

1. $(\varphi, \subseteq)$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \varphi$, $i \in I$, the meet $\bigwedge_{i \in I} A_i$ and the join $\bigvee_{i \in I} A_i$ in φ are related with the intersection and union in $P(S)$ by the equalities
   $\bigwedge_{i \in I} A_i = \bigcap_{i \in I} A_i$
   for all I, while
   $\bigvee_{i \in I} A_i = \bigcup_{i \in I} A_i$
   for all finite I.
2. φ is completely distributive.
3. φ separates the points of S. That is, given $s_1 \neq s_2$ in S we have $L \in \varphi$ with $s_1 \in L$, $s_2 \notin L$, or $L \in \varphi$ with $s_2 \in L$, $s_1 \notin L$.

If S is textured by φ then (S, φ) is called a texture space, or simply a texture.

Complementation: [6] A mapping $\sigma : \varphi \to \varphi$ satisfying $\sigma(\sigma(A)) = A$, $\forall A \in \varphi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$, $\forall A, B \in \varphi$ is called a complementation on (S, φ) and (S, φ, σ) is then said to be a complemented texture.

For a texture (S, φ), most properties are conveniently dened in terms of the p-sets

$$P_s = \bigcap\{A \in \varphi : s \in A\}$$

and the q-sets,

$$Q_s = \bigvee\{A \in \varphi : s \notin A\}.$$

Ditopology: [6] A dichotomous topology on a texture (S, φ), or ditopology for short, is a pair (τ, k) of subsets of φ, where the set of open sets τ satisfies

2010 *Mathematics Subject Classification.* Primary: 54A05, 54A10; Secondary: 54C05.
Key words and phrases. Texture, difunction, semi-g-bi-irresolute, semi-g-stability.

1. $S, \phi \in \tau$,
2. $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$, and
3. $G_i \in \tau, i \in I \Rightarrow \bigvee_i G_i \in \tau$,

and the set of closed sets k satisfies

1. $S, \phi \in k$,
2. $K_1, K_2 \in k \Rightarrow K_1 \cup K_2 \in k$, and
3. $K_i \in k, i \in I \Rightarrow \bigcap K_i \in k$.

Hence a ditopology is essentially a "topology" for which there is no a priori relation between the open and closed sets.
For $A \in \varphi$ we define the closure $[A]$ and the interior $]A[$ of A under (τ, k) by the equalities
$[A] = \bigcap\{K \in k : A \subseteq K\}$ and $]A[= \bigvee\{G \in \tau : G \subseteq A\}$
We refer to τ as the topology and k as the cotopology of (τ, k).

If (τ, k) is a ditopology on a complemented texture (S, φ, σ), then we say that (τ, k) is complemented if the equality $k = \sigma(\tau)$ is satisfied. In this study, a complemented ditopological texture space is denoted by $(S, \varphi, \tau, k, \sigma)$.
In this case we have $\sigma([A]) =]\sigma(A)[$ and $\sigma(]A[) = [\sigma(A)]$.

We denote by $O(S, \varphi, \tau, k)$, or when there can be no confusion by $O(S)$, the set of open sets in φ. Likewise, $C(S, \varphi, \tau, k)$, $C(S)$ will denote the set of closed sets.

Let (S_1, φ_1) and (S_2, φ_2) be textures. In the following definition we consider the product texture [3] $P(S_1) \otimes \varphi_2$, and denote by $\overline{P}_{(s,t)}$, $\overline{Q}_{(s,t)}$, respectively the p-sets and q-sets for the product texture $(S_1 \times S_2, P(S_1) \otimes \varphi_2)$.
Direlation: [5] Let (S_1, φ_1) and (S_2, φ_2) be textures. Then

1. $r \in P(S_1) \otimes \varphi_2$ is called a relation from (S_1, φ_1) to (S_2, φ_2) if it satisfies
   R1 $r \not\subseteq \overline{Q}_{(s,t)}$, $P_{s'} \not\subseteq Q_s \Rightarrow r \not\subseteq \overline{Q}_{(s',t)}$.
   R2 $r \not\subseteq \overline{Q}_{(s,t)} \Rightarrow \exists s' \in S_1$ such that $P_s \not\subseteq Q_{s'}$ and $r \not\subseteq \overline{Q}_{(s',t)}$.
2. $R \in P(S_1) \otimes \varphi_2$ is called a corelation from (S_1, φ_1) to (S_2, φ_2) if it satisfies
   CR1 $\overline{P}_{(s,t)} \not\subseteq R$, $P_s \not\subseteq Q_{s'} \Rightarrow \overline{P}_{(s',t)} \not\subseteq R$.
   CR2 $\overline{P}_{(s,t)} \not\subseteq R \Rightarrow \exists s' \in S_1$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{(s',t)} \not\subseteq R$.
3. A pair (r, R), where r is a relation and R a corelation from (S_1, φ_1) to (S_2, φ_2) is called a direlation from (S_1, φ_1) to (S_2, φ_2).

One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.
Difunctions: [5] Let (f, F) be a direlation from (S_1, φ_1) to (S_2, φ_2). Then (f, F) is called a difunction from (S_1, φ_1) to (S_2, φ_2) if it satisfies the following two conditions.
DF1 For $s, s' \in S_1$, $P_s \not\subseteq Q_{s'} \Rightarrow \exists t \in S_2$ such that $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s',t)} \not\subseteq F$.
DF2 For $t, t' \in S_2$ and $s \in S_1$, $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F \Rightarrow P_{t'} \not\subseteq Q_t$.
Image and Inverse Image: [5] Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction.

1. For $A \in \varphi_1$, the image $f^{\to}A$ and the co-image $F^{\to}A$ are defined by

$$f^{\to}A = \bigcap\{Q_t : \forall s, f \not\subseteq \overline{Q}_{(s,t)} \Rightarrow A \subseteq Q_s\},$$
$$F^{\to}A = \bigvee\{P_t : \forall s, \overline{P}_{(s,t)} \not\subseteq F \Rightarrow P_s \subseteq A\}.$$

2. For $B \in \varphi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by

$$f^{\leftarrow}B = \bigvee\{P_s : \forall t, f \not\subseteq \overline{Q}_{(s,t)} \Rightarrow P_t \subseteq B\},$$
$$F^{\leftarrow}B = \bigcap\{Q_s : \forall t, \overline{P}_{(s,t)} \not\subseteq F \Rightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.
Bicontinuity: [4] The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1) \to (S_2, \varphi_2, \tau_2, k_2)$ is called continuous if $B \in \tau_2 \Rightarrow F^{\leftarrow}B \in \tau_1$, cocontinuous if $B \in k_2 \Rightarrow f^{\leftarrow}B \in k_1$, and bicontinuous if it is both continuous and cocontinuous.
Surjective difunction: [5] Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condition
SUR. For $t, t^{'} \in S_2$, $P_t \not\subseteq Q_{t'} \Rightarrow \exists s \in S_1$ with $f \not\subseteq \overline{Q}_{(s,t')}$ and $\overline{P}_{(s,t)} \not\subseteq F$.

If (f, F) is surjective then $F^{\rightarrow}(f^{\leftarrow}B) = B = f^{\rightarrow}(F^{\leftarrow}B)$ for all $B \in \varphi_2$ [[5], Corollary 2.33]

[5] Let (f, F) be a difunction between the complemented textures $(S_1, \varphi_1, \sigma_1)$ and $(S_2, \varphi_2, \sigma_2)$. The complement $(f, F)^{'} = (F^{'}, f^{'})$ of the difunction (f, F) is a difunction, where $f^{'} = \bigcap\{\overline{Q}_{(s,t)} | \exists u, v$ with $f \not\subseteq \overline{Q}_{u,v}, \sigma_1(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \sigma_2(P_t)\}$ and $F^{'} = \bigvee\{\overline{P}_{(s,t)} | \exists u, v$ with $\overline{P}_{u,v} \not\subseteq F, P_u \not\subseteq \sigma_1(P_s)$ and $\sigma_2(Q_t) \not\subseteq Q_v\}$. If $(f, F) = (f, F)^{'}$ then the difunction (f, F) is called complemented.

[7] Let (S, φ, τ, k) be a ditopological texture space. A set $A \in \varphi$ is called semi-open (semi-closed) if $A \subseteq [\,]A[\,]$ $(][A][\subseteq A)$. We denote by $SO(S, \varphi, \tau, k)$, or when there can be no confusion by $SO(S)$, the set of semi-open sets in φ. Likewise, $SC(S, \varphi, \tau, k)$, or $SC(S)$ will denote the set of semi-closed sets. [2] Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be generalized closed (g-closed for short) if $A \subseteq G \in \tau$ then $[A] \subseteq G$. [2] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is said to be generalized open (g-open for short) if $\sigma(A)$ is g-closed. We denote by $gc(S, \varphi, \tau, k)$, or when there can be no confusion by $gc(S)$, the set of g-closed sets in φ. Likewise, $go(S, \varphi, \tau, k, \sigma)$, or $go(S)$ will denote the set of g-open sets.

[1] Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be semi-g-closed if $A \subseteq G \in SO(S)$ then $[A] \subseteq G$.

We denote by $semigc(S, \varphi, \tau, k)$, or when there can be no confusion by $semigc(S)$, the set of semi-g-closed sets in φ. [1] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is called semi-g-open if $\sigma(A)$ is semi-g-closed.

We denote by $semigo(S, \varphi, \tau, k, \sigma)$, or when there can be no confusion by $semigo(S)$, the set of semi-g-open sets in φ. [1] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. For $A \in \varphi$, we define the semi-g-closure $[A]_{semi-g}$ and the semi-g-interior $]A[_{semi-g}$ of A under (τ, k) by the equalities

$$[A]_{semi-g} = \bigcap\{K \in semigc(S) : A \subseteq K\} \text{ and }]A[_{semi-g} = \bigcup\{G \in semigo(S) : G \subseteq A\}.$$

3 semi-g-bicontinuous, semi-g-bi-irresolute, semi-g-compact and semi-g-stable

The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is called:

1. semi-g-continuous (semi-g-irresolute), if $F^{\leftarrow}(G) \in semigo(S_1)$, for every $G \in O(S_2)$ $(G \in semigo(S_2))$.

2. semi-g-cocontinuous (semi-g-co-irresolute), if $f^{\leftarrow}(G) \in semigc(S_1)$, for every $G \in k_2$ $(G \in semigc(S_2))$.

3. semi-g-bicontinuous, if it is semi-g-continuous and semi-g-cocontinuous.

4. semi-g-bi-irresolute, if it is semi-g-irresolute and semi-g-co-irresolute.

Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then:

1. Every continuous is semi-g-continuous.
2. Every cocontinuous is semi-g-cocontinuous.
3. Every semi-g-irresolute is semi-g-continuous.
4. Every semi-g-co-irresolute is semi-g-cocontinuous.

Clear. Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then:

1. The following are equivalent:
   (a) (f, F) is semi-g-continuous.
   (b) $]F^{\to}A[^{S_2} \subseteq F^{\to}]A[^{S_1}_{semi-g}$, $\forall A \in \varphi_1$.
   (c) $f^{\leftarrow}]B[^{S_2} \subseteq]f^{\leftarrow}B[^{S_1}_{semi-g}$, $\forall B \in \varphi_2$.
2. The following are equivalent:
   (a) (f, F) is semi-g-cocontinuous.
   (b) $f^{\to}[A]^{S_1}_{semi-g} \subseteq [f^{\to}A]^{S_2}$, $\forall A \in \varphi_1$.
   (c) $[F^{\leftarrow}B]^{S_1}_{semi-g} \subseteq F^{\leftarrow}[B]^{S_2}$, $\forall B \in \varphi_2$.

We prove (1), leaving the dual proof of (2) to the interested reader.
$(a) \Rightarrow (b)$. Let $A \in \varphi_1$. From [[5], Theorem 2.24 (2 a)] and the definition of interior,

$$f^{\leftarrow}]F^{\to}(A)[^{S_2} \subseteq f^{\leftarrow}(F^{\to}(A)) \subseteq A.$$

Since inverse image and co-image under a difunction is equal, $f^{\leftarrow}]F^{\to}(A)[^{S_2} = F^{\leftarrow}]F^{\to}(A)[^{S_2}$. Thus, $f^{\leftarrow}]F^{\to}(A)[^{S_2} \in semigo(S_1)$, by semi-g-continuity. Hence

$$f^{\leftarrow}]F^{\to}(A)[^{S_2} \subseteq]A[^{S_1}_{semi-g}$$

and applying [[5], Theorem 2.24 (2 b)] gives

$$]F^{\to}(A)[^{S_2} \subseteq F^{\to}(f^{\leftarrow}(]F^{\to}(A)[^{S_2}) \subseteq F^{\to}]A[^{S_1}_{semi-g},$$

which is the required inclusion.
$(b) \Rightarrow (c)$. Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^{\leftarrow}(B)$ and using [[5], Theorem 2.24 (2 b)] gives

$$]B[^{S_2} \subseteq]F^{\to}f^{\leftarrow}(B)[^{S_2} \subseteq F^{\to}]f^{\leftarrow}(B)[^{S_1}_{semi-g}.$$

Hence, we have $f^{\leftarrow}]B[^{S_2} \subseteq f^{\leftarrow}F^{\to}]f^{\leftarrow}(B)[^{S_1}_{semi-g} \subseteq]f^{\leftarrow}(B)[^{S_1}_{semi-g}$ by [[5], Theorem 2.24 (2 a)].
$(c) \Rightarrow (a)$. Applying (c) for $B \in O(S_2)$ gives

$$f^{\leftarrow}(B) = f^{\leftarrow}]B[^{S_2} \subseteq]f^{\leftarrow}(B)[^{S_1}_{semi-g},$$

so $F^{\leftarrow}(B) = f^{\leftarrow}(B) =]f^{\leftarrow}(B)[^{S_1}_{semi-g} \in semigo(S_1)$. Hence, (f, F) is semi-g-continuous.
Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then:

1. The following are equivalent:

(a) (f,F) is semi-g-irresolute.

(b) $]F^{\rightarrow}A[^{S_2}_{semi-g}\subseteq F^{\rightarrow}]A[^{S_1}_{semi-g}$, $\forall A\in\varphi_1$.

(c) $f^{\leftarrow}]B[^{S_2}_{semi-g}\subseteq]f^{\leftarrow}B[^{S_1}_{semi-g}$, $\forall B\in\varphi_2$.

2. The following are equivalent:

(a) (f,F) is semi-g-co-irresolute.

(b) $f^{\rightarrow}[A]^{S_1}_{semi-g}\subseteq[f^{\rightarrow}A]^{S_2}_{semi-g}$, $\forall A\in\varphi_1$.

(c) $[F^{\leftarrow}B]^{S_1}_{semi-g}\subseteq F^{\leftarrow}[B]^{S_2}_{semi-g}$, $\forall B\in\varphi_2$.

We prove (1), leaving the dual proof of (2) to the interested reader.
$(a)\Rightarrow(b)$. Take $A\in\varphi_1$. Then

$$f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{semi-g}\subseteq f^{\leftarrow}(F^{\rightarrow}A)\subseteq A$$

by [[5], Theorem 2.24 (2 a)]. Now $f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{semi-g}=F^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{semi-g}\in semigo(S_1)$ by semi-g-irresolute, so $f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{semi-g}\subseteq]A[^{S_1}_{semi-g}$ and applying [[5], Theorem 2.24 (2 b)] gives

$$]F^{\rightarrow}A[^{S_2}_{semi-g}\subseteq F^{\rightarrow}(f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{semi-g}\subseteq F^{\rightarrow}]A[^{S_1}_{semi-g},$$

which is the required inclusion.
$(b)\Rightarrow(c)$. Take $B\in\varphi_2$. Applying inclusion (b) to $A=f^{\leftarrow}B$ and using [[5], Theorem 2.24 (2 b)] gives

$$]B[^{S_2}_{semi-g}\subseteq]F^{\rightarrow}(f^{\leftarrow}B)[^{S_2}_{semi-g}\subseteq F^{\rightarrow}]f^{\leftarrow}B[^{S_1}_{semi-g}.$$

Hence, $f^{\leftarrow}]B[^{S_2}_{semi-g}\subseteq f^{\leftarrow}F^{\rightarrow}]f^{\leftarrow}B[^{S_1}_{semi-g}\subseteq]f^{\leftarrow}B[^{S_2}_{semi-g}$ by [[5], Theorem 2.24 (2 a)].
$(c)\Rightarrow(a)$. Applying (c) for $B\in semigo(S_2)$ gives

$$f^{\leftarrow}B=f^{\leftarrow}]B[^{S_2}_{semi-g}\subseteq]f^{\leftarrow}B[^{S_1}_{semi-g},$$

so $F^{\leftarrow}B=f^{\leftarrow}B=]f^{\leftarrow}B[^{S_1}_{semi-g}\in semigo(S_1)$. Hence, (f,F) is semi-g-irresolute.

Let $(S_j,\varphi_j,\tau_j,k_j,\sigma_j)$, for $j\in\{1,2\}$, be complemented ditopology and $(f,F):(S_1,\varphi_1)\to(S_2,\varphi_2)$ be complemented difunction. If (f,F) is semi-g-continuous then (f,F) is semi-g-cocontinuous. Since (f,F) is complemented, $(F^{'},f^{'})=(f,F)$. From [[5], Lemma 2.20], $\sigma_1((f^{'})^{\leftarrow}(B))=f^{\leftarrow}(\sigma_2(B))$ and $\sigma_1((F^{'})^{\leftarrow}(B))=F^{\leftarrow}(\sigma_2(B))$ for all $B\in\varphi_2$. The proof is clear from these equalities.

Let $(S_j,\varphi_j,\tau_j,k_j,\sigma_j)$, $j=1,2$, complemented ditopology and $(f,F):(S_1,\varphi_1)\to(S_2,\varphi_2)$ be complemented difunction. If (f,F) is semi-g-irresolute then (f,F) is semi-g-co-irresolute. Clear. A complemented ditopological texture space $(S,\varphi,\tau,k,\sigma)$ is called semi-g-compact if every cover of S by semi-g-open has a finite subcover. Here we recall that $C=\{A_j:j\in J\}$, $A_j\in\varphi$ is a cover of S if $\bigvee C=S$.

Let $(S,\varphi,\tau,k,\sigma)$ be a complemented ditopological texture space. Then:

1. Every semi-g-compact is compact.

2. Every g-compact is semi-g-compact.

Clear. If $(S,\varphi,\tau,k,\sigma)$ is semi-g-compact and $L=\{F_j:j\in J\}$ is a family of semi-g-closed sets with $\cap L=\phi$, then $\cap\{F_j:j\in J^{'}\}=\phi$ for $J^{'}\subseteq J$ finite. Suppose that $(S,\varphi,\tau,k,\sigma)$ is semi-g-compact and let $L=\{F_j:j\in J\}$ be a family of semi-g-closed sets with $\cap L=\phi$. Clearly $C=\{\sigma(F_j):j\in J\}$ is a family of semi-g-open sets. Moreover,

$$\bigvee C = \bigvee \{\sigma(F_j) : j \in J\} = \sigma(\cap\{F_j : j \in J\}) = \sigma(\phi) = S,$$

and so we have $J^{'} \subseteq J$ finite with $\bigvee\{\sigma(F_j) : j \in J^{'}\} = S$. Hence $\cap\{F_j : j \in J^{'}\} = \phi$. Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be an semi-g-irresolute difunction. If $A \in \varphi_1$ is semi-g-compact then $f^{\to}A \in \varphi_2$ is semi-g-compact. Take $f^{\to}A \subseteq \bigvee_{j\in J} G_j$, where $G_j \in semigo(S_2)$, $j \in J$. Now by [[5], Theorem 2.24 (2 a) and Corollary 2.12 (2)] we have

$$A \subseteq F^{\leftarrow}(f^{\to}A) \subseteq F^{\leftarrow}(\bigvee_{j\in J} G_j) = \bigvee_{j\in J} F^{\leftarrow}G_j.$$

Also, $F^{\leftarrow}G_j \in semigo(S_1)$ because (f, F) is semi-g-irresolute. So by the semi-g-compactness of A there exists $J^{'} \subseteq J$ finite such that $A \subseteq \cup_{j\in J'} F^{\leftarrow}G_j$. Hence

$$f^{\to}A \subseteq f^{\to}(\cup_{j\in J'} F^{\leftarrow}G_j) = \cup_{j\in J'} f^{\to}(F^{\leftarrow}G_j) \subseteq \cup_{j\in J'} G_j$$

by [[5], Corollary 2.12 (2) and Theorem 2.24 (2 b)]. This establishes that $f^{\to}A$ is semi-g-compact.

Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a surjective semi-g-irresolute difunction. Then, if $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ is semi-g-compact so is $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$. This follows by taking $A = S_1$ in Theorem 3 and noting that $f^{\to}S_1 = f^{\to}(F^{\leftarrow}S_2) = S_2$ by [[5], Proposition 2.28 (1 c) and Corollary 2.33 (1)].

A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called semi-g-stable if every semi-g-closed set $F \in \varphi\backslash\{S\}$ is semi-g-compact in S. Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:

1. Every semi-g-stable is stable.

2. Every g-stable is semi-g-stable.

Clear. Let $(S, \varphi, \tau, k, \sigma)$ be semi-g-stable. If G is an semi-g-open set with $G \neq \phi$ and $D = \{F_j : j \in J\}$ is a family of semi-g-closed sets with $\cap_{j\in J} F_j \subseteq G$ then $\cap_{j\in J'} F_j \subseteq G$ for a finite subsets $J^{'}$ of J. Let $(S, \varphi, \tau, k, \sigma)$ be semi-g-stable, let G be an semi-g-open set with $G \neq \phi$ and $D = \{F_j : j \in J\}$ be a family of semi-g-closed sets with $\cap_{j\in J} F_j \subseteq G$. Set $K = \sigma(G)$. Then K is semi-g-closed and satisfies $K \neq S$. Hence K is semi-g-compact. Let $C = \{\sigma(F) | F \in D\}$. Since $\cap D \subseteq G$ we have $K \subseteq \bigvee C$, that is C is an semi-g-open cover of K. Hence there exists $F_1, F_2, ..., F_n \in D$ so that

$$K \subseteq \sigma(F_1) \cup \sigma(F_2) \cup ... \cup \sigma(F_n) = \sigma(F_1 \cap F_2 \cap ... \cap F_n).$$

This gives $F_1 \cap F_2 \cap ... \cap F_n \subseteq \sigma(K) = G$, so $\cap_{j\in J'} F_j \subseteq G$ for a finite subsets $J^{'} = \{1, 2, ..., n\}$ of J. Let $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$, $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be two complemented ditopological texture spaces with $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ is semi-g-stable, and $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be an semi-g-bi-irresolute surjective difunction. Then $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is semi-g-stable. Take $K \in semigc(S_2)$ with $K \neq S_2$. Since (f, F) is semi-g-co-irresolute, so $f^{\leftarrow}K \in semigc(S_1)$. Let us prove that $f^{\leftarrow}K \neq S_1$. Assume the contrary. Since $f^{\leftarrow}S_2 = S_1$, by [[5], Lemma 2.28 (1 c)] we have $f^{\leftarrow}S_2 \subseteq f^{\leftarrow}K$, whence $S_2 \subseteq K$ by [[5], Corollary 2.33 (1 ii)] as (f, F) is surjective. This is a contradiction, so $f^{\leftarrow}K \neq S_1$. Hence $f^{\leftarrow}(K)$ is semi-g-compact in $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ by semi-g-stability. As (f, F) is semi-g-irresolute, $f^{\to}(f^{\leftarrow}K)$ is semi-g-compact for the ditopology (τ_2, k_2) by Theorem 3, and by [[5], Corollary 2.33 (1)] this set is equal to K. This establishes that $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is semi-g-stable.

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