CENTRALIZER AND NORMALIZER OF B-ALGEBRAS

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Abstract. In this paper, we introduce the concepts of centralizer and normalizer of B-algebras, and we investigate some of their properties. In particular, we prove that if H is a subalgebra of a B-algebra X, then the centralizer $C(H)$ of H is a subalgebra of X, which affirms to the result of P.J. Allen, J. Neggers, and H.S. Kim that the center $Z(X)$ is a subalgebra of X. Moreover, if H is normal in X, then $C(H)$ is normal in X, which affirms to the result of A. Walendziak that $Z(X)$ is normal in X.

1 Introduction In 2002, J. Neggers and H.S. Kim [6] introduced the notion of B-algebras. A *B-algebra* is an algebra $(X; *, 0)$ of type (2, 0) (that is, a nonempty set X with a binary operation $*$ and a constant 0) satisfying the following axioms: (I) $x * x = 0$, (II) $x * 0 = x$, and (III) $(x * y) * z = x * (z * (0 * y))$, for all $x, y, z \in X$. A B-algebra $(X; *, 0)$ is *commutative* [6] if $x * (0 * y) = y * (0 * x)$ for all $x, y \in X$. In [7], J. Neggers and H.S. Kim introduced the notions of a subalgebra and normality of B-algebras and some of their properties are established. A nonempty subset N of X is called a *subalgebra* of X if $x * y \in N$ for any x, $y \in N$. It is called *normal* in X if for any $x * y, a * b \in N$ implies $(x*a)*(y*b) \in N$. A normal subset of X is a subalgebra of X. Walendziak [9] characterized normality in a B-algebra. A subalgebra N is normal in X if and only if $x*(x*y) \in N$ for any $x \in X$, $y \in N$. Throughout this paper, X means a B-algebra $(X; *, 0)$. There are several properties of B-algebras as established by some researchers [1−9]. The following properties are used in this paper, for any $x, y, z \in X$, (P1) $0 * (0 * x) = x$ [6], (P2) $x * y = 0 * (y * x)$ [8], (P3) $x * (y * z) = (x * (0 * z)) * y$ [6], (P4) $x * y = x * z$ implies $y = z$ [2], and (P5) $(x * y) * (0 * y) = x$ [6].

2 Centralizer and Normalizer of B-algebras In this section, we introduce centralizer and normalizer of B-algebras. We start with some examples of B-algebras.

Example 2.1. The algebra $(\mathbb{Z}; *, 0)$ is a B-algebra, where $*$ is defined by $x * y = x - y$ for all $x, y \in \mathbb{Z}$.

Example 2.2. [6] Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with the following table of operation:

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Then $(X; *, 0)$ is a B-algebra.

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Definition 2.3. The *centralizer* $C(x)$ *of* x *in* X is defined by $C(x) = \{y \in X\colon\ y * (0 * x) = x * (0 * y)\}$.

Example 2.4. Let X be the B-algebra in Example 2.2. Then $C(1) = \{0,\ 1,\ 2\}$ and $C(3) = \{0, 3\}$.

Theorem 2.5. *$C(x)$ is a subalgebra of X for all $x \in X$.*

Proof. Let $x \in X$. Clearly, $0 \in C(x)$ and so $C(x) \neq \varnothing$. Let $a,\ b \in C(x)$. Then $a * (0 * x) = x * (0 * a)$ and $b * (0 * x) = x * (0 * b)$.
Claim 1: $b * x = (0 * x) * (0 * b)$.
Now, $b*(0*x) = x*(0*b)$ implies that $0*(b*(0*x)) = 0*(x*(0*b))$. By (P2), $(0*x)*b = (0*b)*x$. Multiplying both sides by $0*b$, we get $(0*b)*((0*x)*b) = (0*b)*((0*b)*x)$, and multiplying both sides by b, we get $b*[(0*b)*((0*x)*b)] = b*[(0*b)*((0*b)*x)]$. By (P3), $b*[((0*b)*(0*b))*(0*x)] = [b*(0*((0*b)*x))]*(0*b)$. By (I) and (P2), $b*(0*(0*x)) = [b*(x*(0*b))]*(0*b)$. Applying (P1) and (III), we get $b*x = ((b*b)*x)*(0*b)$. By (I), we have $b*x = (0*x)*(0*b)$. This proves claim 1. Now, $a*(0*x) = x*(0*a)$, (P1) (P2), (P3), and claim 1 imply that

$$\begin{aligned} x * (0 * (a * b)) &= x * (b * a) \\ &= (x * (0 * a)) * b \\ &= (a * (0 * x)) * b \\ &= a * (b * x) \\ &= a * ((0 * x) * (0 * b)) \\ &= [a * (0 * (0 * b))] * (0 * x) \\ &= (a * b) * (0 * x). \end{aligned}$$

Therefore, $a * b \in C(x)$ and so $C(x)$ is a subalgebra of X. □

In Example 2.4, the set $C(1)$ is normal in X, while $C(3)$ is not normal in X since $2 * (2 * 3) = 4 \notin C(3)$. This means that $C(x)$ need not be normal in X.

Definition 2.6. Let H be a nonempty subset of X. The *centralizer* $C(H)$ *of* H *in* X is defined by $C(H) = \{y \in X\colon\ y * (0 * x) = x * (0 * y) \text{ for all } x \in H\}$.

Example 2.7. Let X be the B-algebra in Example 2.2. If $H_1 = \{0, 1, 2\}$, $H_2 = \{0,\ 3\}$, then $C(H_1) = H_1$ and $C(H_2) = H_2$.

Remark 2.8. *Let H be a nonempty subset of X. Then* $C(H) = \bigcap_{x \in H} C(x)$.

In [1], the center $Z(X)$ is a subalgebra of X. The following corollary affirms this fact.

Corollary 2.9. *Let H be a nonempty subset of X. Then $C(H)$ is a subalgebra of X.*

Proof. A direct consequence of Remark 2.8 and Theorem 2.5. □

In Example 2.7, the set $C(H_1)$ is normal in X, while $C(H_2)$ is not normal in X. This means that $C(H)$ need not be normal in X even if H is a subalgebra of X. However, if H is normal in X, then $C(H)$ is normal in X. The proof of this is given in the last part of this paper.

Remark 2.10. *Let H be a nonempty subset of X. Then* $\bigcup_{x \in H} C(x)$ *need not be a subalgebra of X.*

To see this remark, consider the B-algebra X in Example 2.2. Let $H = \{1, 3\}$. Then $C(1) \cup C(3) = \{0, 1, 2, 3\}$. Since $1 * 3 = 4 \notin C(1) \cup C(3)$, $C(1) \cup C(3)$ is not a subalgebra of X.

Lemma 2.11. *For any x, y, $z \in X$,*

i. $x * (x * y) = z$ *if and only if* $(0 * x) * ((0 * x) * z) = y$,

ii. $(x * y) * ((x * y) * z) = x * [x * ((0 * y) * ((0 * y) * z))]$.

Proof. By (P2), (P3), (P4), (P5), and (I), we have

$$\begin{aligned} x * (x * y) = z &\Leftrightarrow (0 * x) * (x * (x * y)) = (0 * x) * z \\ &\Leftrightarrow (0 * x) * [(0 * x) * (x * (x * y))] = (0 * x) * ((0 * x) * z) \\ &\Leftrightarrow [(0 * x) * (0 * (x * (x * y)))] * (0 * x) = (0 * x) * ((0 * x) * z) \\ &\Leftrightarrow [(0 * x) * ((x * y) * x)] * (0 * x) = (0 * x) * ((0 * x) * z) \\ &\Leftrightarrow [((0 * x) * (0 * x)) * (x * y)] * (0 * x) = (0 * x) * ((0 * x) * z) \\ &\Leftrightarrow (0 * (x * y)) * (0 * x) = (0 * x) * ((0 * x) * z) \\ &\Leftrightarrow (y * x) * (0 * x) = (0 * x) * ((0 * x) * z) \\ &\Leftrightarrow y = (0 * x) * ((0 * x) * z). \end{aligned}$$

This proves (i). By (III) and (P2), we have

$$\begin{aligned} (x * y) * ((x * y) * z) &= x * [((x * y) * z) * (0 * y)] \\ &= x * \{[x * (z * (0 * y))] * (0 * y)\} \\ &= x * \{x * \{(0 * y) * [0 * (z * (0 * y))]\}\} \\ &= x * [x * ((0 * y) * ((0 * y) * z))]. \end{aligned}$$

This proves (ii). □

The previous lemma is useful for the succeeding results. We now introduce the concept of normalizer of B-algebras.

Definition 2.12. Let H and K be nonempty subsets of X. For every $x \in X$, we define H_x as the set $H_x = \{x * (x * h) : h \in H\}$. The *normalizer of H in K*, denoted by $N_K(H)$, is defined by $N_K(H) = \{x \in K : H_x = H\}$. If $K = X$, then $N_X(H)$ is called the *normalizer of H*, denoted by $N(H)$. If $H = \{x\}$, then we write $N(x)$ in place of $N(\{x\})$.

Theorem 2.13. *Let H be a nonempty subset of X and K be a subalgebra of X. Then $N_K(H)$ is a subalgebra of X.*

Proof. By (P1), $H_0 = H$. Since K is a subalgebra, $0 \in K$. Thus, $0 \in N_K(H)$ and so $N_K(H) \neq \varnothing$. Let $x, y \in N_K(H)$. Then $x, y \in K$ and $H_x = H = H_y$.
Claim 1: $0 * y \in N_K(H)$.
Since K is a subalgebra, $0 * y \in K$. Let $a \in H$. Since $H = H_y$, $y * (y * a) = h_1$ for some $h_1 \in H$. By Lemma 2.11(i), $a = (0 * y) * ((0 * y) * h_1) \in H_{0*y}$. Thus, $H \subseteq H_{0*y}$. Let $b \in H_{0*y}$. Then $b = (0 * y) * ((0 * y) * h_2)$ for some $h_2 \in H$. By Lemma 2.11(i), $y * (y * b) = h_2 \in H = H_y$. Hence, $y * (y * b) = y * (y * h_3)$ for some $h_3 \in H$. By (P4), $b = h_3 \in H$. Thus, $H_{0*y} \subseteq H$. Therefore, $H = H_{0*y}$. This proves claim 1.
Claim 2: $x * y \in N_K(H)$.
Since K is a subalgebra, $x * y \in K$. Let $a \in H_{x*y}$. Then $a = (x * y) * ((x * y) * h_4)$

for some $h_4 \in H$. By Lemma 2.11(ii), $a = x * [x * ((0 * y) * ((0 * y) * h_4))]$. By claim 1, $(0*y)*((0*y)*h_4) \in H_{0*y} = H$. Hence, $a \in H_x = H$. Thus, $H_{x*y} \subseteq H$. Let $b \in H$. Then $b \in H_x$, that is, $b = x*(x*h_5)$ for some $h_5 \in H$. Since $H = H_{0*y}$, $h_5 = (0*y)*((0*y)*h_6)$ for some $h_6 \in H$. Hence, $b = x * [x * ((0 * y) * ((0 * y) * h_6))]$. By Lemma 2.11(ii), $b = (x * y) * ((x * y) * h_6) \in H_{x*y}$. Thus, $H \subseteq H_{x*y}$. Hence, $H = H_{x*y}$. This proves claim 2. Therefore, $N_K(H)$ is a subalgebra of X. □

Corollary 2.14. *Let H be a nonempty subset of X. Then N(H) is a subalgebra of X.*

Proposition 2.15. *C(x) = N(x) for all $x \in X$.*

In view of Remark 2.10 and Proposition 2.15, $\bigcup_{x\in H} N(x)$ need not be a subalgebra of X.

Theorem 2.16. *Let H be a subalgebra of X.*

i. H is normal in X if and only if N(H) = X,

ii. H is normal in N(H),

iii. N(H) is the largest subalgebra of X in which H is normal.

Proof. Let H be a subalgebra of X.

i. Suppose H is normal in X. Clearly, $N(H) \subseteq X$. Let $x \in X$. Then $x * (x * h) \in H$ for all $h \in H$. Hence, $H_x \subseteq H$. Let $h \in H$. Then $(0 * x) * ((0 * x) * h) \in H$. Thus, $(0 * x) * ((0 * x) * h) = h'$ for some $h' \in H$. By Lemma 2.11(i), $h = x * (x * h') \in H_x$. Hence, $H \subseteq H_x$. Thus, $H = H_x$, that is, $x \in N(H)$. Therefore, $N(H) = X$. Conversely, let $h \in H$ and $x \in X$. Since $N(H) = X$, $x * (x * h) \in H$. Therefore, H is normal in X.

ii. Let $x \in H$. Since H is a subalgebra of X, $H_x \subseteq H$. Let $h \in H$. Since H is a subalgebra of X, $0 * x \in H$ and so $(0 * x) * ((0 * x) * h) \in H$. Thus, $(0 * x) * ((0 * x) * h) = h'$ for some $h' \in H$. By Lemma 2.11(i), $h = x * (x * h') \in H_x$. Hence, $H \subseteq H_x$. Thus, $H = H_x$, that is, $x \in N(H)$. Therefore, $H \subseteq N(H)$. By Corollary 2.14, $N(H)$ is a subalgebra of X. Since $H \subseteq N(H)$, H is a subalgebra of $N(H)$. In view of Definition 2.12, H is normal in $N(H)$.

iii. Let H be normal in a subalgebra K of X. Let $k \in K$. Since H is normal in K, $k * (k * h) \in H$ for all $h \in H$. Hence, $H_k \subseteq H$. Let $h \in H$. Since H is normal in K and K is a subalgebra, $(0 * k) * ((0 * k) * h) \in H$. Thus, $(0 * k) * ((0 * k) * h) = h'$ for some $h' \in H$. By Lemma 2.11(i), $h = k * (k * h') \in H_k$. Hence, $H \subseteq H_k$. Thus, $H = H_k$, that is, $k \in N(H)$. Therefore, $K \subseteq N(H)$.

□

Corollary 2.17. *C(0) = N(0) = X.*

Corollary 2.18. *Let H and K be subalgebras of X. Then H is normal in K if and only if $H \subseteq K \subseteq N(H)$.*

Corollary 2.19. *Let H be a nonempty subset of X. Then $C(H) \subseteq N(H)$.*

Let $H = \{0, 1, 2\}$ be the subset of the B-algebra X in Example 2.2. Then $C(H) = H \neq X = N(H)$.

3 Some Properties of Automorphisms of B-algebras In this section, we consider all B-isomorphisms of X onto itself. A mapping $\varphi : X \to Y$ is called a *B-homomorphism* [7] if $\varphi(x*y) = \varphi(x)*\varphi(y)$ for any $x, y \in X$. A B-homomorphism φ is called a *B-monomorphism*, *B-epimorphism*, or *B-isomorphism* if φ is one-to-one, onto, or a bijection, respectively. A B-isomorphism $\varphi : X \to X$ is called a *B-automorphism*. We define $Aut(X)$ as the set of all B-automorphisms of X. We recall from [6] that if $(X; \circ, e)$ is a group with identity e, then $(X; *, 0 = e)$ is a B-algebra, where $*$ is defined by $x * y = x \circ y^{-1}$. Since $Aut(X)$ is a group under composition of functions, $(Aut(X); \circledcirc, id_X)$ is a B-algebra, where $\circledcirc$ is defined by $f \circledcirc g = f \circ g^{-1}$ for all $f, g \in Aut(X)$, where $\circ$ denotes the composition of functions.

Theorem 3.1. *Let $x \in X$. Define φ_x: $X \to X$ by $\varphi_x(y) = x * (x * y)$ for all $y \in X$. Then*

i. $\varphi_x \in Aut(X)$,

ii. $\varphi_x \circ \varphi_{0*y} = \varphi_{x*y}$ *and* $\varphi_x \circ \varphi_y = \varphi_{x*(0*y)}$ *for all* $x, y \in X$,

iii. $\varphi_0 = id_X$,

iv. $(\varphi_x)^{-1} = \varphi_{0*x}$,

v. for all $\psi \in Aut(X)$, $\psi \circ \varphi_x \circ \psi^{-1} = \varphi_{\psi(x)}$.

Proof. i. Clearly, φ_x is well-defined. Let $a, b \in X$. Now, by (P1), (P3), (I), (II), and (III), we have

$$\begin{aligned}
\varphi_x(a * b) &= x * (x * (a * b)) \\
&= x * [(x * (0 * b)) * a] \\
&= (x * (0 * a)) * (x * (0 * b) \\
&= [(x * (0 * a)) * ((0 * x) * (0 * x))] * (x * (0 * b)) \\
&= [((x * (0 * a)) * x) * (0 * x)] * (x * (0 * b)) \\
&= \{[x * (x * (0 * (0 * a)))] * (0 * x)\} * (x * (0 * b)) \\
&= ((x * (x * a)) * (0 * x)) * (x * (0 * b)) \\
&= (x * (x * a)) * [(x * (0 * b)) * (0 * (0 * x))] \\
&= (x * (x * a)) * ((x * (0 * b)) * x) \\
&= (x * (x * a) * [x * (x * (0 * (0 * b)))] \\
&= (x * (x * a) * (x * (x * b)) \\
&= \varphi_x(a) * \varphi_x(b).
\end{aligned}$$

Hence, φ_x is a B-homomorphism.
Also, φ_x is onto since by (P1), (P2), (P3), (I), and (III), we get

$$\begin{aligned}
\varphi_x((0 * x) * ((0 * x) * a)) &= x * [x * ((0 * x) * ((0 * x) * a))] \\
&= x * [x * (((0 * x) * (0 * a)) * (0 * x))] \\
&= x * [(x * (0 * (0 * x))) * ((0 * x) * (0 * a))] \\
&= x * [0 * ((0 * x) * (0 * a))] \\
&= x * ((0 * a) * (0 * x)) \\
&= (x * (0 * (0 * x))) * (0 * a) \\
&= 0 * (0 * a) \\
&= a.
\end{aligned}$$

Suppose $\varphi_x(a) = \varphi_x(b)$. Then $x * (x * a) = x * (x * b)$. By (P4), $a = b$. Hence, φ_x is one-to-one. Therefore, $\varphi_x \in Aut(X)$.

ii. Let $a \in X$. Then by (III) and (P2), we have

$$\begin{aligned}
\varphi_{x*y}(a) &= (x * y) * ((x * y) * a) \\
&= x * [((x * y) * a) * (0 * y)] \\
&= x * \{[x * (a * (0 * y))] * (0 * y)\} \\
&= x * \{x * \{(0 * y) * [0 * (a * (0 * y))]\}\} \\
&= x * [x * ((0 * y) * ((0 * y) * a))] \\
&= x * (x * \varphi_{0*y}(a)) \\
&= \varphi_x(\varphi_{0*y}(a)) \\
&= (\varphi_x \circ \varphi_{0*y})(a).
\end{aligned}$$

Hence, $\varphi_{x*y} = \varphi_x \circ \varphi_{0*y}$. By (P1), $\varphi_x \circ \varphi_y = \varphi_x \circ \varphi_{0*(0*y)}$. Since $\varphi_{x*y} = \varphi_x \circ \varphi_{0*y}$, we have $\varphi_x \circ \varphi_{0*(0*y)} = \varphi_{x*(0*y)}$.

iii-v Straightforward.

□

The φ_x of Theorem 3.1 is called an *inner B-automorphism* of X. We define $Inn(X)$ as the set of all inner B-automorphisms of X.

Theorem 3.2. *Inn(X) is a normal subalgebra of Aut(X).*

Proof. By Theorem 3.1(iii), $id_X = \varphi_0 \in Inn(X)$ and so $Inn(X) \neq \varnothing$. By Theorem 3.1(i), $Inn(X) \subseteq Aut(X)$. Let φ_x, $\varphi_y \in Inn(X)$. Then by Theorem 3.1(iv, ii), $\varphi_x \circledcirc \varphi_y = \varphi_x \circ \varphi_y^{-1} = \varphi_x \circ \varphi_{0*y} = \varphi_{x*y} \in Inn(X)$. Thus, $Inn(X)$ is a subalgebra of $Aut(X)$. Let $\psi \in Aut(X)$ and $\varphi_x \in Inn(X)$. Then by Theorem 3.1(v), we have $\psi \circledcirc (\psi \circledcirc \varphi_x) = \psi \circledcirc (\psi \circ \varphi_x^{-1}) = \psi \circ (\psi \circ \varphi_x^{-1})^{-1} = \psi \circ (\varphi_x \circ \psi^{-1}) = \varphi_{\psi(x)} \in Inn(X)$. Therefore, $Inn(X)$ is a normal subalgebra of $Aut(X)$. □

The following theorem is labeled as the First Isomorphism Theorem for B-algebras and can be found in [7]. We also note that the kernel of a B-homomorphism is normal [7].

Theorem 3.3. *Let φ: $X \to Y$ be a B-homomorphism of X into Y. Then X/Ker $\varphi \cong$ Im φ.*

Theorem 3.4. *Let H be a subalgebra of X. Then C(H) is normal in N(H) and N(H)/C(H) $\cong$ a subalgebra of Aut(H).*

Proof. Define f: $N(H) \to Aut(H)$ by $f(x) = \varphi_x$ for all $x \in N(H)$, where φ_x: $H \to H$ is defined by $\varphi_x(h) = x * (x * h)$ for all $h \in H$. Clearly, f is well-defined. Let $x, y \in N(H)$. Then $f(x * y) = \varphi_{x*y} = \varphi_x \circ \varphi_{0*y} = \varphi_x \circ \varphi_y^{-1} = \varphi_x \circledcirc \varphi_y = f(x) \circledcirc f(y)$. Therefore, f is a B-homomorphism. By the First Isomorphism Theorem for B-algebras, $N(H)/\text{Ker } f \cong \text{Im } f$.
Claim: If $x * (x * h) = h$ for all $h \in H$, then $x \in C(H)$.
Suppose $x * (x * h) = h$ for all $h \in H$. Multiplying both sides by $0 * x$, we get $(x * (x * h)) * (0 * x) = h * (0 * x)$. By (III), $x * ((0 * x) * (0 * (x * h))) = h * (0 * x)$. Applying (P2), we obtain $x * ((0 * x) * (h * x)) = h * (0 * x)$. Hence, by (P3), $x * [((0 * x) * (0 * x)) * h] = h * (0 * x)$.

Thus, by (I), $x*(0*h)=h*(0*x)$, that is, $x\in C(H)$. This proves the claim. Now, by Corollary 2.19 and the claim,

$$\begin{aligned}\text{Ker } f &= \{x\in N(H) : f(x)=id_H\}\\ &= \{x\in N(H) : \varphi_x(h)=id_H(h) \text{ for all } h\in H\}\\ &= \{x\in N(H) : x*(x*h)=h \text{ for all } h\in H\}\\ &= \{x\in N(H) : x\in C(H)\}\\ &= C(H).\end{aligned}$$

Therefore, $C(H)$ is a normal in $N(H)$ and $N(H)/C(H)\cong$ a subalgebra of $Aut(H)$.

In [9], the center $Z(X)$ is normal in X. The following corollaries affirm this fact.

Corollary 3.5. *Z(X) is normal in X and X/Z(X) ≅ Inn(X).*

Proof. Let $H=X$ in Theorem 3.4. Then $N(X)=X$ and $C(X)=Z(X)$ and so $Z(X)$ is normal in X. Since Im $f=Inn(X)$, $X/Z(X)\cong Inn(X)$.

Corollary 3.6. *If H is normal in X, then so is C(H).*

Proof. This follows from Theorems 3.4 and 2.16(i).

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