FACILITY LOCATION PROBLEM FOR SUPPLY CENTER OF SCHOOL LUNCH

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1 Abstract

Recently providing suitable lunch to the student of elementary school etc is very important problem. To realize suitable lunch, we should consider the construction of supply center at the suitable place at first and so we consider the following facility location problem. There are schools in an urban area. We consider the construction site of new supply center providing lunch for these schools. The trader delivers ingredients to the supply center every morning. After receiving these ingredients, the supply center starts to make lunch. Lunch for all schools should be ready on delivery time. The delivery cars must deliver lunch to be till lunch time of each school. For that purpose, we divide schools into groups corresponding to delivery cars. Considering rectilinear distance from the finite possible sites of center to trader and all schools, we choose the best site of the center by minimizing the latest delivery time of lunch among all schools. Under the above setting, we propose an effective solution procedure to find the most suitable site of the center.

Next, we consider the above problem into the case that possible construction sites are restricted to the point of disjoint rectangular areas. We extend the solution procedure of finite possible sites and manage to construct solution procedure.

Finally, we summarize the result of the paper and discuss future research problems.

2 Introduction

There exists huge facility location models from the successful research by J. Elzinga and D. W. Hearn [1] and Hamacher et al., [2] tried to classify them using similar codes to queueing and scheduling. This paper considers a new model to determine the optimal site of a center to provide school lunch in a certain area. School lunch is important to provide necessary nutrition’s to students in elementary schools etc. In order to deliver school lunch to all schools till lunch time, the site of center is a key point considering distance to all schools and preparing materials. Usually possible candidate sites are limited, we consider finite case and infinite but restricted area case. Section 3 formulates our problem and first we consider the finite case. Based on the results in section 3, next we consider possible candidate are limited on some rectangular regions in section 4. Finally section 5 summarizes the results in this paper and discusses further research problems.

3 Problem formulation

We consider the following facility location problem:

(1) There are m schools, $S_1, S_2, \ldots, S_m$ in an urban area. We consider the construction site of new supply center providing lunch for these schools among n possible sites $F_1, F_2, \ldots, F_n$.

(2) The trader delivers ingredients to the supply center every morning. After receiving these ingredients, the supply center starts to make lunch. Lunch for all schools should be ready on delivery time. The delivery cars must deliver lunch to be in lunch time of each school. For that purpose, we divide schools into groups corresponding to r delivery cars T_t $(t = 1, 2, \ldots, r)$.

(3) Considering rectilinear distance from each possible site of center to trader and all schools, we choose the best site of the center by minimizing the latest delivery time of lunch among all schools.

First we calculate a rectilinear distances d_{ij} from each possible site F_i $(i = 1, 2, \ldots, n)$ to each school S_j $(j = 1, 2, \ldots, m)$. Note that $d_{ij} = |c_x^i - s_x^j| + |c_y^i - s_y^j|$ where $F_i = (c_x^i, c_y^i), S_j = (s_x^j, s_y^j)$. Sorting d_{ij} $(j = 1, 2, \ldots, m)$ for each F_i, let result be

$$d_{i,i(1)} \leq d_{i,i(2)} \leq \cdots \leq d_{i,i(k)} \leq \cdots \leq d_{i,i(m)}$$

where $i(k)$ denotes the k-th furthest school number from candidate site F_i, Further we assume that m is a multiplier of r, that is, $m = pr$ for some positive integer p without any loss of generality by adding necessary dummy schools of distance 0 from site F_i .

Then for each F_i we divide schools into r trucks as follows: Choose r longest distances and assign school $S_{i(m-t+1)}$ to delivery cars T_t $(t = 1, 2, \ldots, r)$. Then let be $\bar{d}_{ik} = 2d_{ii(k)}$ $(k = 1, 2, \ldots, m - r)$ for each remaining school $S_{i(k)}$ $(k = 1, 2, \ldots, m - r)$.

3.1 Dividing schools into r groups when candidate site F_i is fixed

Step 1: Set $B_{i(t)} = d_{i,i(m-t+1)} + \bar{d}_{it}, (t = 1, 2, \ldots, r - 1)$ $B_{i(r)} = d_{i,i(m-r+1)} + \bar{d}_{ii(m-r)}$.
$k = m - r$ and $G(t) = S_{i(m-t+1)}$ $(t = 1, 2, \ldots, r - 1)$ $G(r) = \{S_{i(m-r+1)}, S_{i(m-r)}\}$.
Go to step 2.

Step 2: Let $k = k - 1$. If $k = 0$, terminate. Otherwise go to step 3.

Step 3: Let $B(m) \leftarrow \min\{B_{i(u)} | u = 1, 2, \ldots, r\}$ and its minimizer be $t(k)$.
Then set $B_{i(t(k))} = B_{i(t(k))} + \bar{d}_{ii(k)}, G(t(k)) = G(t(k)) \cup S_{i(k)}$. Return to Step 2.

Note that final $B_{i(u)}$ divided by the standard speed describes the total delivery time using T_u to group of schools $G(u)$ $(u = 1, 2, \ldots, r)$. Though heuristic, the above dividing method tries to make burden even, that is, minimizing the maximum burden among delivery cars. Let the maximum burden using the above dividing method for candidate F_i be $M(i)$.

3.2 Choosing suitable candidate site for supply center of school lunch

Here we assume the staring time of making lunch is fixed and so finishing time preparing lunch is also fixed and so minimizer of $M(i)$ $(i = 1, 2, \ldots, n)$ after calculating $M(i)$ is a suitable site for a center of school lunch. We skip to consider a distance from trader to the candidate site since the start we delete the candidate sites too far from the trader site due to the big delay in the start of cooking lunch.

4 Possible candidate sites in some rectangular regions

In this section, we assume that any site in some rectangular regions is a possible candidate site. Note that if each rectangular region is shrinking into one point, then it is the case in section 3. Let vertices for each subregion $R(v), v = 1, \ldots, h$ be v_A, v_B, v_C, v_D where h is a number of subregions (see Figure 1). In Figure 1, $v_{Ax}, v_{Bx}, v_{Cx}, v_{Dx}$ are x coordinates of vertices v_A, v_B, v_C, v_D respectively. Also $v_{Ay}, v_{By}, v_{Cy}, v_{Dy}$ are y coordinates of vertices v_A, v_B, v_C, v_D respectively.

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Fig.1 Four vertices of $R(v)$

First we divide the urban region X into sub regions as follows: Draw vertical lines

$$x = v_{Ax}, v_{Bx}, v_{Cx}, v_{Dx}, v = 1, 2, \ldots, h, x = s_x^j, j = 1, 2, \ldots, m, x = X_0, X_1$$

and horizontal lines

$$y = v_{Ay}, v_{By}, v_{Cy}, v_{Dy}, v = 1, 2, \ldots, h, y = s_y^j, j = 1, 2, \ldots, m, y = Y_0, Y_1$$

Let the result be rectangular sub-regions $SR(1), SR(2), \ldots, SR(k), \ldots, SR(q)$ numbered by anti-clock-wisely and q is the total number of sub-regions $q = O((h + m)^2)$. Let the four vertices of the sub-region $SR(k)$ be A_k, B_k, C_k, D_k like Figure 2.

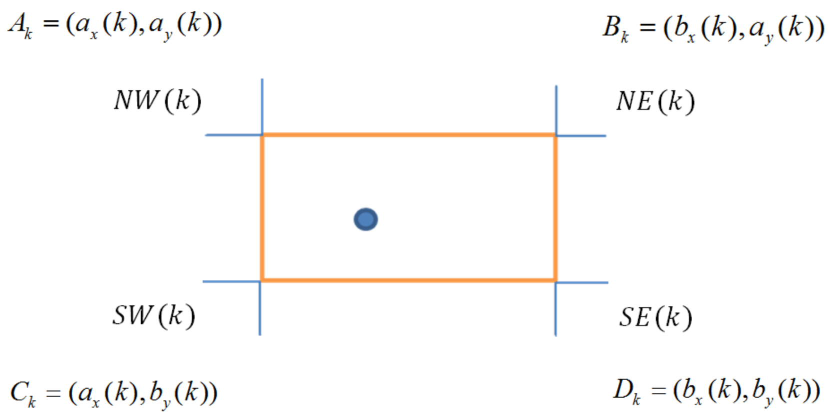

Fig.2 Four vertices of $SR(k)$

If we choose the center LC as (x, y) in $SR(k)$, distance from the center LC to schools is calculated by dividing schools into four subgroups $NW(k), NE(k), SW(k), SE(k)$ depending on the direction to $RS(k)$ where $a_x(k) \leq x \leq b_x(k), b_y(k) \leq y \leq a_y(k)$. That is,

$$\begin{aligned}
NW(k) &= \{S_j | s_x^j \leq a_x(k), s_y^j \geq a_y(k)\} \\
NE(k) &= \{S_j | s_x^j \geq b_x(k), s_y^j \geq a_y(k)\} \\
SW(k) &= \{S_j | s_x^j \leq a_x(k), s_y^j \leq b_y(k)\} \\
SE(k) &= \{S_j | s_x^j \geq b_x(k), s_y^j \leq b_y(k)\}
\end{aligned}$$

and for $S_j \in NW(k)$, that distance from LC is the distance from A_k to $S_j + x - a_x(k) + a_y(k) - y$, for $S_j \in NE(k)$, that distance is from B_k to $S_j + b_x(k) - x + a_y(k) - y$, for $S_j \in SW(k)$, that distance is from C_k to $S_j + x - a_x(k) + y - b_y(k)$ and for $S_j \in SE(k)$, that distance is from D_k to $S_j + b_x(k) - x + y - b_y(k)$.

Therefore we first apply the method in section 2 to $4h$ possible candidate sites $A_k, B_k, C_k, D_k, k = 1, 2, \ldots, h$. Then we check whether the situation is improved or not if the candidate site moves to inside $SR(k), k = 1, 2, \ldots, h$ by using the above result on the distance from LC to schools. After that, we obtain the suitable site of LC.

5 Example

We illustrate our method by using an example. Our campus is located in Sanda city in Japan and so we apply our method to data about Sanda city. As an example, we consider to construct the center for school lunch. Currently there exist 39 schools and the center provides school lunch to these schools. Delivery cars are assumed to be 13 and so each car delivers school lunch to 3 schools as an average. Among 3 candidate sites A,B,C, we expect the candidate A is the most suitable since the furthest distance among all schools from A is the minimum. But after applying our method, we find candidate site C is the most preferable.

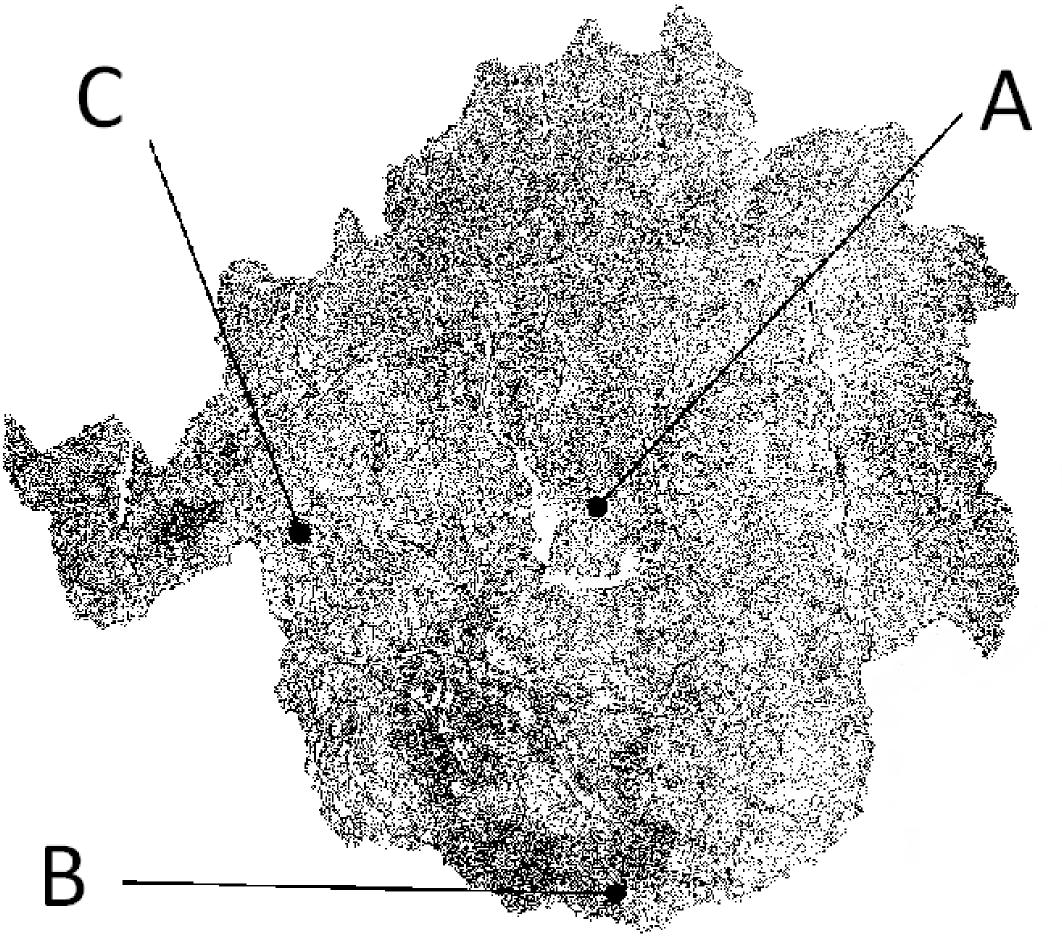

Fig.3 Candidate sites of Sanda city

Table1 Distance from candidate site of A to school

School No.	Distance(m)	School No.	Distance(m)	School No.	Distance(m)
1	1790	14	6948	27	8702
2	1860	15	7123	28	8702
3	2842	16	7299	29	8843
4	3158	17	7509	30	9194
5	3895	18	7965	31	9194
6	4632	19	8001	32	9229
7	4772	20	8036	33	9404
8	5579	21	8036	34	9544
9	5720	22	8106	35	9755
10	5895	23	8141	36	9860
11	6843	24	8422	37	9895
12	6878	25	8422	38	10001
13	6878	26	8632	39	11369

Table2 Distance from dealer to candidate site of A

Dealer No.	Distance(m)
1	4316
2	5755
3	6211
4	6421
5	9264
6	9755
7	9860
8	10176
9	10352
10	10632
11	11159
12	11299

Table3 Migration length of delivery cars

Deliver No.	School A	School B	School C	Total Distance(m)
1	8702	17264	3580	29546
2	8702	16844	6316	31862
3	8843	16844	3720	29407
4	9194	16282	9544	35020
5	9194	16212	11158	36564
6	9229	16702	11440	36741
7	9404	16702	9264	34740
8	9544	16002	7790	33336
9	9755	15930	5684	31369
10	9860	15018	13686	38564
11	9895	14598	13756	38249
12	10001	14246	13756	38003
13	11369	13896	11790	37055

Table4 Distance from candidate site of B to school

School No.	Distance(m)	School No.	Distance(m)	School No.	Distance(m)
1	1544	14	5439	27	8316
2	1684	15	5755	28	8422
3	1755	16	6176	29	8527
4	2526	17	6597	30	8702
5	3334	18	6737	31	8808
6	3614	19	6737	32	9088
7	4492	20	6772	33	9650
8	4492	21	6948	34	11053
9	4807	22	7088	35	11229
10	4807	23	7615	36	11229
11	4842	24	8246	37	11334
12	4878	25	8281	38	11931
13	5193	26	8281	39	12071

Table5 Distance from dealer to candidate site of B

Dealer No.	Distance(m)
1	4211
2	4562
3	4913
4	5123
5	5369
6	5685
7	5930
8	6000
9	6281
10	6772
11	7790
12	8913

Table6 Migration length of delivery cars

Deliver No.	School A	School B	School C	Total Distance(m)
1	8316	16562	3510	28388
2	8422	16562	3368	28352
3	8527	16492	3088	28107
4	8702	15230	8984	32916
5	8808	14176	9756	32740
6	9088	13896	9684	32668
7	9650	13544	9614	32808
8	11053	13474	6668	31195
9	11229	13474	5052	29755
10	11229	13194	7228	31651
11	11334	12352	8984	32670
12	11931	11510	9614	33055
13	12071	10878	10386	33335

Table7 Distance from candidate site of C to school

School No.	Distance(m)	School No.	Distance(m)	School No.	Distance(m)
1	625	14	5714	27	7500
2	1679	15	5790	28	7720
3	2143	16	5946	29	7825
4	2351	17	5965	30	7839
5	2518	18	6071	31	7893
6	2526	19	6106	32	7946
7	2589	20	6106	33	9357
8	2696	21	6176	34	9580
9	3228	22	6554	35	9650
10	3679	23	6679	36	12282
11	4554	24	6750	37	12492
12	4911	25	6807	38	16352
13	5579	26	6948	39	16492

Table8 Distance from dealer to candidate site of C

Dealer No.	Distance(m)
1	702
2	1018
3	2807
4	4492
5	7334
6	7720
7	7860
8	8246
9	8422
10	8702
11	9194
12	9439

Table9 Migration length of delivery cars

Deliver No.	School A	School B	School C	Total Distance(m)
1	7500	13896	5178	26574
2	7720	13614	5392	26726
3	7825	13500	6456	27781
4	7839	13358	7358	28555
5	7893	13108	9822	30823
6	7946	12352	11158	31456
7	9357	12212	5178	26747
8	9580	12212	5052	26844
9	9650	12142	5036	26828
10	12282	11930	4702	28914
11	12492	11892	4286	28670
12	16352	11580	3358	31290
13	16492	11428	1250	29170

6 Conclusion

We have discussed on the suitable site for the center of school lunch. Our model includes grouping of schools and delivery order. Our method does not find optimal grouping and delivery order shown in the above. Though our problem maybe NP hard and so to find optimal solution is difficult, more refinement method should be considered. One is to make a clear solution method to the infinite possible candidates in section 3. Further more realistic model should be considered such as A-distance ([4],[5]) existence of barrier, fuzzy version points ([3]) etc. However the most important thing is to deliver the delicious lunch to school students as fast as possible not to lose freshness.

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